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(1,2)*-FG-Closed and (1,2)*-FG-Open Maps in Fuzzy Bitopological Spases

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ABSTRACT

In this chapter, we introduce (1,2)*-fg-closed maps, (1,2)*-fg-open maps, (1,2)*-fg*-closed maps and (1,2)*-fg*-open maps in Fuzzy bitopological spaces and obtain certain characterizations of these classes of maps. In last section, we introduce (1,2)*-fg*-homeomorphisms and prove that the set of all (1,2)*-fg*-homeomorphisms forms a group under the operation composition of functions.

1.Introduction

Malghan [2] introduced the concept of generalized closed maps in topological spaces. Devi [1] introduced and studied sg-closed maps and gs-closed maps. Recently, Sheik John [6] defined ω-closed maps and studied some of their properties. In this paper, we introduce (1,2)*-fg-closed maps, (1,2)*-fg-open maps, (1,2)*-fg*-closed maps and (1,2)*-fg*-open maps in fuzzy bitopological spaces and obtain certain characterizations of these classes of maps. In last section, we introduce (1,2)*-fg*-homeomorphisms and prove that the set of all (1,2)*-fg*-homeomorphisms forms a group under the operation composition of functions.

1.2.Preliminaries

Definition 1.2.1

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(i) (1,2)*-g-closed [5] if f(V) is (1,2)*-g-closed in Y, for every $\tau_{1,2}$-closed set V of X.

(ii) (1,2)*-sg-closed [4] if f(V) is (1,2)*-sg-closed in Y, for every $\tau_{1,2}$-closed set V of X.

(iii) (1,2)*-gs-closed [4] if f(V) is (1,2)*-gs-closed in Y, for every $\tau_{1,2}$-closed set V of X.

(iv) (1,2)*-ψ-closed [3] if f(V) is (1,2)*-ψ-closed in Y, for every $\tau_{1,2}$-closed set V of X.

We introduce the following definitions

Definition 1.2.2

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(i) (1,2)*-fsg-closed if f(V) is (1,2)*-fsg-closed in Y, for every $\tau_{1,2}$-closed set V of X.

(ii) (1,2)*-fgs-closed if f(V) is (1,2)*-fgs-closed in Y, for every $\tau_{1,2}$-closed set V of X.

(iii) (1,2)*-fψ-closed if f(V) is (1,2)*-fψ-closed in Y, for every $\tau_{1,2}$-closed set V of X.

1.3. (1,2)*-fg-CLOSED MAPS

Definition 1.3.1

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (1,2)*-fg-closed if the image of every $\tau_{1,2}$-closed set in X is (1,2)*-fg-closed in Y.

Proposition 1.3.2

For any $A \leq X$,

(i) (1,2)*-g-cl(A) is the smallest (1,2)*-fg-closed set containing A.

(ii) A is (1,2)*-fg-closed if and only if (1,2)*-g-cl(A) = A.

Proposition 1.3.3

For any two subsets A and B of X,

(i) If $A \leq B$, then (1,2)*-g-cl(A) $\leq$ (1,2)*-g-cl(B).

(ii) (1,2)*-g-cl($A \cap B$) $\leq$ (1,2)*-g-cl(A) $\cap$ (1,2)*-g-cl(B).

Proposition 1.3.4

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-fg-closed if and only if (1,2)*-g-cl(f(A)) $\leq$ f($\tau_{1,2}$-cl(A)) for every subset A of X.

Proof

Suppose that f is (1,2)*-fg-closed and $A \leq X$. Then $\tau_{1,2}$-cl(A) is $\tau_{1,2}$-closed in X and so f($\tau_{1,2}$-cl(A)) is (1,2)*-fg-closed in Y. We have f(A) $\leq$ f($\tau_{1,2}$-cl(A)) and by Propositions 1.3.2 and 1.3.3, (1,2)*-g-cl(f(A)) $\leq$ (1,2)*-g-cl(f($\tau_{1,2}$-cl(A))) = f($\tau_{1,2}$-cl(A)).Conversely, let

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A be any $\tau_{1,2}$-closed set in X. Then A = $\tau_{1,2}$-cl(A) and so f(A) = f($\tau_{1,2}$-cl(A)) ≥ (1,2)*-g-cl(f(A)), by hypothesis. We have f(A) ≤ (1,2)*-g-cl(f(A)). Therefore f(A) = (1,2)*-g-cl(f(A)). That is f(A) is (1,2)*-fg-closed by Proposition 1.3.2 and hence f is (1,2)*-g-closed.

Proposition 1.3.5

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a map such that (1,2)*-g-cl(f(A)) ≤ f($\tau_{1,2}$-cl(A)) for every subset A ≤ X. Then the image f(A) of a $\tau_{1,2}$-closed set A in X is (1,2)*-fg-closed in Y.

Proof

Let A be a $\tau_{1,2}$-closed set in X. Then by hypothesis (1,2)*-g-cl(f(A)) ≤ f($\tau_{1,2}$-cl(A)) = f(A) and so (1,2)*-g-cl(f(A)) = f(A). Therefore f(A) is (1,2)*-fg-closed in Y.

Theorem 1.3.6

A map f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1,2)*-fg-closed if and only if for each subset S of Y and each $\tau_{1,2}$-open set U containing $f^{-1}(S)$ there is an (1,2)*-fg-open set V of Y such that S ≤ V and $f^{-1}(V)$ ≤ U.

Proof

Suppose f is (1,2)*-fg-closed. Let S ≤ Y and U be an $\tau_{1,2}$-open set of X such that $f^{-1}(S)$ ≤ U. Then V = $(f(U^c))^c$ is an (1,2)*-fg-open set containing S such that $f^{-1}(V)$ ≤ U.

For the converse, let F be a $\tau_{1,2}$-closed set of X. Then $f^{-1}((f(F))^c) \le F^c$ and F^c is $\tau_{1,2}$-open. By assumption, there exists an (1,2)*-fg-open set V in Y such that $(f(F))^c \le V$ and $f^{-1}(V) \le F^c$ and so $F \le (f^{-1}(V))^c$. Hence $V^c \le f(F) \le f((f^{-1}(V))^c) \le V^c$ which implies f(F) = V^c. Since V^c is (1,2)*-fg-closed, f(F) is (1,2)*-fg-closed and therefore f is (1,2)*-fg-closed.

Proposition 1.3.7

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1,2)*-fsg-irresolute (1,2)*-fg-closed and A is an (1,2)*-fg-closed subset of X, then f(A) is (1,2)*-fg-closed in Y.

Proof

Let U be an (1,2)*-fsg-open set in Y such that f(A) ≤ U. Since f is (1,2)*-fsg-irresolute, $f^{-1}(U)$ is an (1,2)*-fsg-open set containing A. Hence $\tau_{1,2}$-cl(A) ≤ $f^{-1}(U)$ as A is (1,2)*-fg-closed in X. Since f is (1,2)*-fg-closed, f($\tau_{1,2}$-cl(A)) is an (1,2)*-fg-closed set contained in the (1,2)*-fsg-open set U, which implies that $\tau_{1,2}$-cl(f($\tau_{1,2}$-cl(A))) ≤ U and hence $\tau_{1,2}$-cl(f(A)) ≤ U. Therefore, f(A) is an (1,2)*-fg-closed set in Y.

The following example shows that the composition of two (1,2)*-fg-closed maps need not be a (1,2)*-fg-closed.

Example 1.3.8

Let (X,τ_1,τ_2) be a fuzzy bitopological space where X = {a, b, c}.

$\tau_1 = 0,1,\lambda = \frac{1}{a}+\frac{0}{b}+\frac{0}{c}, \mu = \frac{1}{a}+\frac{0}{b}+\frac{1}{c}$ and τ_2 = {0,1}.

τ_{12}-closed are $0,1,\lambda' = \frac{0}{a}+\frac{1}{b}+\frac{1}{c}, \mu' = \frac{0}{a}+\frac{1}{b}+\frac{0}{c}$. Then (1,2)*-fg closed are

$$0,1,\lambda' = \frac{0}{a}+\frac{1}{b}+\frac{1}{c}, \mu' = \frac{0}{a}+\frac{1}{b}+\frac{0}{c}, \frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c} \text{ where } 0 \le \alpha_1,\alpha_2,\alpha_3 \le 1, \alpha_2 \ne 0 \cdot$$

Let (Y,σ_1,σ_2) be a fuzzy bitopological space where Y = {a, b, c}.

$\sigma_1 = 0,1,\lambda = \frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ and σ_2 = {1,0}.

σ_{12}-closed are $0,1,\lambda' = \frac{0}{a}+\frac{1}{b}+\frac{1}{c},$ .Then (1,2)*-fg closed are

$$0,1,\lambda' = \frac{0}{a}+\frac{1}{b}+\frac{1}{c},, \frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c} \text{ where } 0 \le \alpha_1,\alpha_2,\alpha_3 \le 1 \cdot$$

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is an (1,2)*-fg-closed map.

Let (Z,η_1,η_2) be a fuzzy bitopological space where Z = {a, b, c}.

$\eta_1 = 0,1,\lambda = \frac{1}{a}+\frac{0.5}{b}+\frac{0}{c}$ and η_2 = {1, 0}.

η_{12}-closed are $0,1,\lambda' = \frac{0}{a}+\frac{0.5}{b}+\frac{1}{c},$ .Then (1,2)*-fg closed are

$$0,1,\lambda' = \frac{0}{a}+\frac{0.5}{b}+\frac{1}{c}, \frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c} \text{ where } 0 \le \alpha_1,\alpha_2,\alpha_3 \le 1, \alpha_3 \ne 0 \cdot$$

.Let $g:(Y,\sigma_1,\sigma_2)\to(Z.\eta_1,\eta_2)$ be the identity map. Then both f and g are (1,2)*-fg-closed maps but their composition g ο f : (X, τ_1, τ_2) → (Z, η_1, η_2) is not an (1,2)*-fg-closed map, since for the τ_{12} closed set $\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$ in X, (g ο f) ($\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$) = $\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$ which is not and (1,2)*-fg-closed set in Z.

Corollary 1.3.9

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be (1,2)*-fg-closed and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be (1,2)*-fg-closed and (1,2)*-fsg-irresolute, then their composition g ο f : (X, τ_1, τ_2) → (Z, η_1, η_2) is (1,2)*-fg-closed.

Proof

Let A be a $\tau_{1,2}$-closed set of X. Then by hypothesis f(A) is an (1,2)*-fg-closed set in Y. Since g is both (1,2)*-fg-closed and (1,2)*-fsg-irresolute by Proposition 1.3.5, g(f(A)) = (g ο f) (A) is (1,2)*-fg-closed in Z and therefore g ο f is (1,2)*-fg-closed.

Proposition 1.3.10

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be (1,2)*-fg-closed maps where Y is a $T_{(1,2)*\text{-}g}$-space. Then their composition g ο f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1,2)*-fg-closed.

Proof

Let A be a $\tau_{1,2}$-closed set of X. Then by assumption f(A) is (1,2)*-fg-closed in Y. Since Y is a $T_{(1,2)*\text{-}g}$-space, f(A) is $\sigma_{1,2}$-closed in Y and again by assumption g(f(A)) is (1,2)*-fg-closed in Z. That is (g ο f) (A) is (1,2)*-fg-closed in Z and so g ο f is (1,2)*-fg-closed.

Proposition 1.3.11

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is (1,2)*-fg-closed, g : (Y, σ_1, σ_2) → (Z, η_1, η_2) is (1,2)*-fg-closed (resp. (1,2)*-fg-closed, (1,2)*-fψ -closed, (1,2)*-fsg-closed and (1,2)*-fgs-closed) and Y is a $T_{(1,2)*\text{-}g}$-space, then their composition g ο f : (X, τ_1, τ_2) → (Z, η_1, η_2) is (1,2)*-fg-closed (resp. (1,2)*-fg-closed, (1,2)*-fψ -closed, (1,2)*-fsg-closed and (1,2)*-fgs-closed).

Proof

Similar to Proposition 1.3.10.

Proposition 1.3.12

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a (1,2)*-fuzzy closed map and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be an (1,2)*-fg-closed map, then their composition g ο f : (X, τ_1, τ_2) → (Z, η_1, η_2) is (1,2)*-fg-closed.

Proof

Similar to Proposition 1.3.10.

Remark 1.3.13

If f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is an (1,2)*-fg-closed and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) is (1,2)*-fuzzy closed, then their composition need not be an (1,2)*-fg-closed map as seen from the following example.

Example 1.3.14

Let (X,τ_1,τ_2) be a fuzzy bitopological space where X = {a, b, c}.

$\tau_1=0,1,\lambda=\frac{1}{a}+\frac{0}{b}+\frac{0}{c},\mu=\frac{0}{a}+\frac{0}{b}+\frac{1}{c}$ and τ_2 = {0,1}.

τ_{12}-closed are $0,1,\lambda'=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},\mu'=\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$ .Then (1,2)*-fg closed are

$0,1,\lambda'=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},\mu'=\frac{0}{a}+\frac{1}{b}+\frac{0}{c},\frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c}$ where $0\le\alpha_1,\alpha_2,\alpha_3\le 1,\alpha_2\ne 0\cdot$

Let (Y,σ_1,σ_2) be a fuzzy bitopological space where Y = {a, b, c}.

$\sigma_1=0,1,\lambda=\frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ and σ_2 = {0,1}.

σ_{12}-closed are $0,1,\lambda'=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},$ .Then (1,2)*-fg closed are

$0,1,\lambda'=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},,\frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c}$ where $0\le\alpha_1,\alpha_2,\alpha_3\le 1\cdot$

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity map. Then f is an (1,2)*-fg-closed map.

Let (Z,η_1,η_2) be a fuzzy bitopological space where Z = {a, b, c}.

$\eta_1=0,1,\lambda=\frac{1}{a}+\frac{0}{b}+\frac{0}{c},\mu=\frac{0}{a}+\frac{1}{b}+\frac{1}{c}$ and η_2 = {0,1}.

η_{12}-closed are $0,1,\lambda'=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},\mu'=\frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ .Then (1,2)*-fg closed are

$0,1,\lambda'=\frac{0}{a}+\frac{1}{b}+\frac{1}{c},\mu'=\frac{1}{a}+\frac{0}{b}+\frac{0}{c}$.

Let $(Z.\eta_1,\eta_2)$ be the identity map. Then (1,2)*-fuzzy closed maps but their composition g o f : (X, τ_1, τ_2) $\rightarrow$ (Z, η_1, η_2) is not an (1,2)*-fg-closed map, since for the τ_{12} closed set $\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$ in X, (g o f) ($\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$) = $\frac{0}{a}+\frac{1}{b}+\frac{0}{c}$ which is not and (1,2)*-fg-closed set in Z.

Theorem 1.3.15

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) $\rightarrow$ (Z, η_1, η_2) be two maps such that their composition g o f : (X, τ_1, τ_2) $\rightarrow$ (Z, η_1, η_2) is an (1,2)*-fg-closed map. Then the following statements are true.

(i) If f is (1,2)*-fuzzy continuous and surjective, then g is (1,2)*-fg-closed.
(ii) If g is (1,2)*-fg-irresolute and injective, then f is (1,2)*-fg-closed.
(iii) If f is (1,2)*-f$\hat{g}$-continuous, surjective and (X, τ) is a (1,2)*-T_ω-space, then g is (1,2)*-fg-closed.
(iv) If g is strongly (1,2)*-fg-continuous and injective, then f is (1,2)*-fuzzy closed.

Proof

(i)Let A be a $\sigma_{1,2}$-closed set of Y. Since f is (1,2)*-fuzzy continuous, $f^{-1}(A)$ is $\tau_{1,2}$-closed in X and since g o f is (1,2)*-fg-closed, (g o f)($f^{-1}(A)$) is (1,2)*-fg-closed in Z. That is g(A) is (1,2)*-fg-closed in Z, since f is surjective. Therefore g is an (1,2)*-fg-closed map.
(ii) Let B be a $\tau_{1,2}$-closed set of X. Since g o f is (1,2)*-fg-closed, (g o f) (B) is (1,2)*-fg-closed in Z. Since g is (1,2)*-fg-irresolute, g^{-1}((g o f)(B)) is (1,2)*-fg-closed set in Y. That is f(B) is (1,2)*-fg-closed in Y, since g is injective. Thus f is an (1,2)*-fg-closed map.
(iii) Let C be a $\sigma_{1,2}$-closed set of Y. Since f is (1,2)*-f$\hat{g}$-continuous, $f^{-1}(C)$ is (1,2)*-f$\hat{g}$-closed in X. Since X is a (1,2)*-T_ω-space, $f^{-1}(C)$ is $\tau_{1,2}$-closed in X and so as in (i), g is an (1,2)*-fg-closed map.
(iv) Let D be a $\tau_{1,2}$-closed set of X. Since g o f is (1,2)*-fg-closed, (g o f)(D) is (1,2)*-fg-closed in Z. Since g is strongly (1,2)*-fg-continuous, g^{-1}((g o f)(D)) is $\sigma_{1,2}$-closed in Y. That is f(D) is $\sigma_{1,2}$-closed set in Y, since g is injective. Therefore f is a (1,2)*-fuzzy closed map.

In the next theorem we show that (1,2)*-fuzzy normality is preserved under (1,2)*- fuzzy continuous, (1,2)*-fg-closed maps.

Theorem 1.3.16

A set A of X is (1,2)*-fg-open if and only if F $\leq$ $\tau_{1,2}$-int(A) whenever F is (1,2)*-fsg-closed and F $\leq$ A.

Theorem 1.3.17

If f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is a (1,2)*-fuzzy continuous, (1,2)*-fg-closed map from a (1,2)*-fuzzy normal space X onto a space Y, then Y is (1,2)*-fuzzy normal.

Proof

Let A and B be two disjoint $\sigma_{1,2}$-closed subsets of Y. Since f is (1,2)*-fuzzy continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2}$-closed sets of X. Since X is (1,2)*-fuzzy normal, there exist disjoint $\tau_{1,2}$-open sets U and V of X such that $f^{-1}(A) \leq U$ and $f^{-1}(B) \leq V$. Since f is (1,2)*-fg-closed, by Theorem 1.3.6, there exist disjoint (1,2)*-fg-open sets G and H in Y such that A $\leq$ G, B $\leq$ H, $f^{-1}(G) \leq U$ and $f^{-1}(H) \leq V$. Since U and V are disjoint, $\sigma_{1,2}$-int(G) and $\sigma_{1,2}$-int(H) are disjoint $\sigma_{1,2}$-open sets in Y. Since A is $\sigma_{1,2}$-closed, A is (1,2)*-fsg-closed and therefore we have by Theorem 1.3.16, A $\leq$ $\sigma_{1,2}$-int(G). Similarly B $\leq$ $\sigma_{1,2}$-int(H) and hence Y is (1,2)*-fuzzy normal.

Analogous to an (1,2)*-fg-closed map, we have defined an (1,2)*-fg-open map as follows:

Definition 1.3.18

A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is said to be an (1,2)*-fg-open map if the image f(A) is (1,2)*-fg-open in Y for each $\tau_{1,2}$-open set A in X.

Proposition 1.3.19

For any bijection f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2), the following statements are equivalent:

(i) f^{-1} : (Y, σ_1, σ_2)$\rightarrow$ (X, τ_1, τ_2) is (1,2)*-fg-continuous.
(ii) f is (1,2)*-fg-open map.
(iii) f is (1,2)*-fg-closed map.

Proof

(i) $\Rightarrow$ (ii). Let U be an $\tau_{1,2}$-open set of X. By assumption, $(f^{-1})^{-1}(U)$ = f(U) is (1,2)*-fg-open in Y and so f is (1,2)*-fg-open.
(ii) $\Rightarrow$ (iii). Let F be a $\tau_{1,2}$-closed set of X. Then F^c is $\tau_{1,2}$-open set in X. By assumption, $f(F^c)$ is (1,2)*-fg-open in Y. That is $f(F^c) = (f(F))^c$ is (1,2)*-fg-open in Y and therefore f(F) is (1,2)*-fg-closed in Y. Hence f is (1,2)*-fg-closed.
(iii) $\Rightarrow$ (i). Let F be a $\tau_{1,2}$-closed set of X. By assumption, f(F) is (1,2)*-fg-closed in Y. But f(F) = $(f^{-1})^{-1}(F)$ and therefore f^{-1} is (1,2)*-fg-continuous.

Theorem 1.3.20

Assume that the collection of all (1,2)*-fg-open sets of Y is closed under arbitrary union. Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) be a map. Then the following statements are equivalent:

(i)f is an (1,2)*-fg-open map.
(ii)For a subset A of X, f($\tau_{1,2}$-int(A)) $\leq$ (1,2)*-fg-int(f(A)).
(iii)For each x $\in$ X and for each $\tau_{1,2}$-neighborhood U of x in X, there exists an (1,2)*-fg-neighborhood W of f(x) in Y such that W $\leq$ f(U).

Proof

(i) $\Rightarrow$ (ii). Suppose f is (1,2)*-fg-open. Let A $\leq$ X. Then $\tau_{1,2}$-int(A) is $\tau_{1,2}$-open in X and so f($\tau_{1,2}$-int(A)) is (1,2)*-fg-open in Y. We have f($\tau_{1,2}$-int(A)) $\leq$ f(A). Therefore by Proposition 1.3.2, f($\tau_{1,2}$-int(A)) $\leq$ (1,2)*-g-int(f(A)).

(ii) $\Rightarrow$ (iii). Suppose (ii) holds. Let x $\in$ X and U be an arbitrary $\tau_{1,2}$-neighborhood of x in X. Then there exists an $\tau_{1,2}$-open set G such that x $\in$ G $\leq$ U. By assumption, f(G) = f($\tau_{1,2}$-int(G)) $\leq$ (1,2)*-g-int(f(G)). This implies f(G) = (1,2)*-g-int(f(G)). By Proposition 1.3.2, we have f(G) is (1,2)*-fg-open in Y. Further, f(x) $\in$ f(G) $\leq$ f(U) and so (iii) holds, by taking W = f(G).

(iii) $\Rightarrow$ (i). Suppose (iii) holds. Let U be any $\tau_{1,2}$-open set in X, x $\in$ U and f(x) = y. Then y $\in$ f(U) and for each y $\in$ f(U), by assumption there exists an (1,2)*-g-neighborhood W_y of y in Y such that $W_y \leq$ f(U). Since W_y is an (1,2)*-g-neighborhood of y, there exists an (1,2)*-fg-open set Vy in Y such that y $\in$ $V_y \leq W_y$. Therefore, f(U) = $\cup$ {V_y : y $\in$ f(U)} is an (1,2)*-fg-open set in Y. Thus f is an (1,2)*-fg-open map.

Theorem 1.3.21

A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is (1,2)*-fg-open if and only if for any subset S of Y and for any $\tau_{1,2}$-closed set F containing f^{-1}(S), there exists an (1,2)*-fg-closed set K of Y containing S such that f^{-1}(K) $\leq$ F.

Proof

Similar to Theorem 1.3.6.

Corollary 1.3.22

A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is (1,2)*-fg-open if and only if f^{-1}((1,2)*-g-cl(B)) $\leq$ $\tau_{1,2}$-cl(f^{-1}(B)) for each subset B of Y.

Proof

Suppose that f is (1,2)*-fg-open. Then for any B $\leq$ Y, f^{-1}(B) $\leq$ $\tau_{1,2}$-cl(f^{-1}(B)). By Theorem1.3.21, there exists an (1,2)*-fg-closed set K of Y such that B $\leq$ K and f^{-1}(K) $\leq$ $\tau_{1,2}$-cl(f^{-1}(B)). Therefore, f^{-1}((1,2)*-fg-cl(B)) $\leq$ (f^{-1}(K)) $\leq$ $\tau_{1,2}$-cl(f^{-1}(B)), since K is an (1,2)*-fg-closed set in Y.

Conversely, let S be any subset of Y and F be any $\tau_{1,2}$-closed set containing f^{-1}(S). Put K = (1,2)*-g-cl(S). Then K is an (1,2)*-fg-closed set and S $\leq$ K. By assumption, f^{-1}(K) = f^{-1}((1,2)*-g-cl(S)) $\leq$ $\tau_{1,2}$-cl(f^{-1}(S)) $\leq$ F and therefore by Theorem 1.3.21, f is (1,2)*-fg-open.

Finally in this section, we define another new class of maps called (1,2)*-fg*-closed maps which are stronger than (1,2)*-fg-closed maps.

Definition 1.3.23

A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is said to be (1,2)*-fg*-closed if the image f(A) is (1,2)*-fg-closed in Y for every (1,2)*-fg-closed set A in X.

Remark 1.3.24

Since every $\tau_{1,2}$-closed set is an (1,2)*-fg-closed set we have (1,2)*-fg*-closed map is an (1,2)*-fg-closed map. The converse is not true in general as seen from the following example.

Example 1.3.25

Let (Y,σ_1,σ_2) be a fuzzy bitopological space where Y = {a, b, c}.

$\sigma_1 = 0{,}1, \lambda = \frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ and σ_2 = {0,1}.

σ_{12}-closed are $0{,}1, \lambda' = \frac{0}{a}+\frac{1}{b}+\frac{1}{c},$ .Then (1,2)*-fg closed are

$0{,}1, \lambda' = \frac{0}{a}+\frac{1}{b}+\frac{1}{c},, \frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c}$ where $0 \leq \alpha_1,\alpha_2,\alpha_3 \leq 1\cdot$

Let (Z,η_1,η_2) be a fuzzy bitopological space where Z = {a, b, c}.

$\eta_1 = 0{,}1, \lambda = \frac{1}{a}+\frac{0{,}5}{b}+\frac{0}{c}$ and η_2 = {0,1}.

η_{12}-closed are $0{,}1, \lambda' = \frac{0}{a}+\frac{0.5}{b}+\frac{1}{c},$ .Then (1,2)*-fg closed are

$0{,}1, \lambda' = \frac{0}{a}+\frac{0.5}{b}+\frac{1}{c}, \frac{\alpha_1}{a}+\frac{\alpha_2}{b}+\frac{\alpha_3}{c}$ where $0 \leq \alpha_1,\alpha_2,\alpha_3 \leq 1, \alpha_3 \neq 0\cdot$

Let $g:(Y,\sigma_1,\sigma_2)\rightarrow(Z.\eta_1,\eta_2)$ be the identity map. Then g is (1,2)*-fg closed map but not (1,2)*-fg*-closed map. Since $\frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ is (1,2)*-fg-closed set in X, but its image under g is $\frac{1}{a}+\frac{0}{b}+\frac{0}{c}$ which is not (1,2)*-fg-closed set in Z.

Proposition 1.3.26

A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is (1,2)*-fg*-closed if and only if (1,2)*-g-cl(f(A)) $\leq$ f((1,2)*-g-cl(A)) for every subset A of X.

Proof

Similar to Proposition 1.3.4.

Analogous to (1,2)*-fg*-closed map we can also define (1,2)*-fg*-open map.

Proposition 1.3.27

For any bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

(i) $f^{-1} : (Y, \sigma_1, \sigma_2) \rightarrow (X, \tau_1, \tau_2)$ is (1,2)*-fg-irresolute.

(ii) f is (1,2)*-fg*-open map.

(iii) f is (1,2)*-fg*-closed map.

Proof

Similar to Proposition 1.3.19.

Proposition 1.3.28

If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-fsg-irresolute and (1,2)*-fg-closed, then it is an (1,2)*-fg*-closed map.

Proof

The proof follows from Proposition 1.3.7.

1.4. (1,2)*-Fg*-Homeomorphisms

The notion of (1,2)*-fuzzy homeomorphisms plays a very important role in fuzzy bitopological spaces. By definition, an (1,2)*-fuzzy homeomorphism between two fuzzy bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) is a bijective map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ when f and f^{-1} are (1,2)*-fuzzy continuous.

We introduce the following definition:

Definition 1.4.1

A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be

(i) (1,2)*-fg-homeomorphism if f is both (1,2)*-fg-continuous and (1,2)*-fg-open.

(ii) (1,2)*-fg*-homeomorphism if both f and f^{-1} are (1,2)*-fg-irresolute.

We denote the family of all (1,2)*-fg*-homeomorphisms of a fuzzy bitopological space (X, τ_1, τ_2) onto itself by (1,2)*-fg*h(X).

Theorem 1.4.2

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a bijective (1,2)*-fg-continuous map. Then the following are equivalent:

(i) f is an (1,2)*-fg-open map.

(ii) f is an (1,2)*-fg-homeomorphism.

(iii) f is an (1,2)*-fg-closed map.

Proof

Follows from Proposition 1.3.19.

Proposition 1.4.3

If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ are (1,2)*-fg*-homeomorphisms, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is also (1,2)*-fg*-homeomorphism.

Proof

Let U be (1,2)*-fg-open set in (Z, η_1, η_2). Now, $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) = f^{-1}(V)$, where $V = g^{-1}(U)$. By hypothesis, V is (1,2)*-fg-open in Y and so again by hypothesis, $f^{-1}(V)$ is (1,2)*-fg-open in X. Therefore, $g \circ f$ is (1,2)*-fg-irresolute.

Also for an (1,2)*-fg-open set G in X, we have $(g \circ f)(G) = g(f(G)) = g(W)$, where $W= f(G)$. By hypothesis f(G) is (1,2)*-fg-open in Y and so again by hypothesis, g(f(G)) is (1,2)*-fg-fopen in Z. That is $(g \circ f)(G)$ is (1,2)*-fg-open in Z and therefore $(g \circ f)^{-1}$ is (1,2)*-fg-irresolute. Hence $g \circ f$ is a (1,2)*-fg*-homeomorphism.

Theorem 1.4.4

The set (1,2)*-fg*-h(X) is a group under the composition of maps.

Proof

Define a binary operation $* :$ (1,2)*-fg*-h(X) $\times$ (1,2)*-fg*-h(X) $\rightarrow$ (1,2)*-fg*-h(X) by $f * g = g \circ f$ for all f, g $\in$ (1,2)*-fg*-h(X) and $\circ$ is the usual operation of composition of maps. Then by Proposition 1.4.3, $g \circ f \in$ (1,2)*-fg*-h(X). We know that the composition of maps is associative and the identity map $I : (X, \tau_1, \tau_2) \rightarrow (X, \tau_1, \tau_2)$ belonging to (1,2)*-fg*-h(X) serves as the identity element. If f $\in$ (1,2)*-fg*-h(X), then $f^{-1} \in$ (1,2)*-fg*-h(X) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of (1,2)*-fg*-h(X). Therefore, ((1,2)*-fg*- h(X), $\circ$) is a group under the operation of composition of maps.

Theorem 1.4.5

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be an (1,2)*-fg*-homeomorphism. Then f induces an (1,2)*-fuzzy isomorphism from the group (1,2)*-fg*-h(X) on to the group (1,2)*-fg*-h(Y).

Proof

Using the map f, we define a map $\theta_f :$ (1,2)*-fg*-h(X) $\rightarrow$ (1,2)*-fg*-h(Y) by $\theta_f(h) = f \circ h \circ f^{-1}$ for every h $\in$ (1,2)*-fg*-h(X). Then θ_f is a bijection. Further, for all $h_1, h_2 \in$ (1,2)*-fg*-h(X), $\theta_f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f(h_1) \circ \theta_f(h_2)$. Therefore, θ_f is a (1,2)*-fuzzy homomorphism and so it is an (1,2)*-fuzzy isomorphism induced by f.

Theorem 1.4.6

(1,2)*-fg*-homeomorphism is an equivalence relation in the collection of all bitopological spaces.

Proof

Reflexivity and symmetry are immediate and transitivity follows from Proposition 1.4.3.

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