Available online at www.elixirpublishers.com (Elixir International Journal)

Electrical Engineering

Elixir Elec. Engg. 87 (2015) 35519-35521

ISSN: 2229-712X

Identification of Wiener Systems

Adil Brouri
ENSAM, Moulay Ismail University, L2MC, AEEE Depart, Meknes, Morocco.

ARTICLE INFO

Article history:
Received: 28 August 2015;
Received in revised form:
23 September 2015;
Accepted: 29 September 2015;

Keywords
Wiener model,
Frequency identification,
Nonparametric linear subsystem.

ABSTRACT
Wiener systems identification is studied in the presence of possibly infinite-order linear dynamics and nonparametric nonlinear element. The latter can be noninvertible and of arbitrary-shape. Using simple constant and sine excitations, and getting benefit from model plurality, the problem identification problem is made. All estimators are shown to be consistent.

Introduction

The Wiener model is a series connection of a linear dynamic bloc and a memoryless nonlinearity (Fig. 1). When both parts are parametric, the identification problem has been dealt with using several methods e.g. [1]-[4]. Multi-stage methods, involving two or several stages, have been proposed in e.g. [4].

In this paper, the problem of identifying Wiener systems is addressed. Unlike many previous works, the model structure of the linear subsystem is entirely unknown. Furthermore, the system nonlinearity is of arbitrary-shape and can be noninvertible. This is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of order p and unknown parameters, i.e. $x = h(w) \approx \sum_{k=0}^{p} c_k w^k$.

The present strategy is allowed to interest a wide range of the system nonlinearity. The identification problem amounts to determining an accurate estimate of the (nonparametric) frequency response $G(j\omega)$, for a set of frequencies $(\omega_1 K\ \omega_m)$, and the nonlinearity. The present identification method is a two-stage: the system nonlinearity is identified first, using simple constant inputs, and based upon in the second stage to identify the linear subsystem.

The paper is organized as follows: the identification problem is formulated in Section 2; the nonlinear operator identification is coped with in Section 3; the linear subsystem frequency response determination is investigated in Section 4.

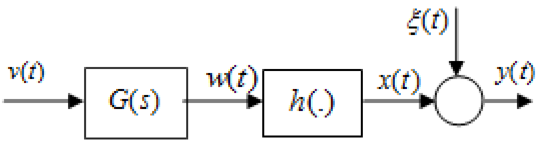

Figure 1. Wiener model structure

Identification problem statement

Standard Wiener systems consist of a linear dynamic subsystem $G(s)$ followed in series by a memoryless nonlinear element $h(.)$ (Fig. 1). The above model is analytically described by the following equations:

$$w(t) = g(t) * v(t) . \qquad (1a)$$

$$x(t) = h(w(t)) . \qquad (1b)$$

$$y(t) = h(w(t)) + \xi(t) . \qquad (1c)$$

where $g(t) = L^{-1}(G(s))$ is the inverse Laplace transform of $G(s)$; the symbol * refers to the convolution operation; $w(t)$ is the internal signal; $x(t)$ is the undisturbed output.

the only measurable signals are the system input $v(t)$ and output $y(t)$. The equation error $\xi(t)$ is a zero-mean stationary sequence of independent random variables; it accounts for external noise, it is supposed to be ergodic (so that arithmetic averages can be substituted to probabilistic means whenever this is necessary). The linear part is submitted to the following assumptions:

A1. $G(s)$ is BIBO stable (because system identification is carried out in open loop).

The system nonlinearity $h(.)$ is of arbitrary-shape. This latter is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of order p.

A2. There exist a given subinterval I such that:

$x = h(w) \approx \sum_{k=0}^{p} c_k w^k$, where $C = [c_0 \dots c_p]^T$ is the coefficients vector of $h(.)$.

Except for the above assumptions, the system is arbitrary, particularly $h(.)$ can be nonparametric outside of the subinterval I and the linear subsystem may have any structure.

We aim at designing an identification scheme that is able to provide a model estimate $\left(\hat{G}(j\omega_k), \hat{h}(.)\right)$ that represents well the system when. Since $w(t)$ and $x(t)$ are not measurable, the system identification should be fully based upon measurements of the input $v(t)$ and the output system $y(t)$. Therefore, the considered identification problem does not have a unique solution: if the model $(G(s), h(w))$ represents a solution then, any model of the form $(G(s)/k, h(kw))$ is also a solution (where k is any nonzero real). This naturally leads to the question: what particular model should we focus on? This

Tele: 212-6-66-12-19-66
E-mail addresses: a.brouri@ensam-umi.ac.ma

question will be answered later. Such a lack of uniqueness, will be exploited (in Section 3) to cope with the uncertainty on the amplitude of the internal signals $w(t)$ and $x(t)$.

System nonlinearity identification

In this section, we want to treat the problem of identifying a set of points belonging to non-linearity. In Section 2 it was shown that, if k is any nonzero real, so any model of the form $\left(G(s)/k, h(kw)\right)$ is representative of the system. Accordingly, the system to be identified is described by the transfer function:

$$\bar{G}(s) = \frac{G(s)}{G(0)} \tag{2a}$$

and the nonlinearity:

$$\bar{h}\left(w(t)\right) = h\left(G(0)w(t)\right) \tag{2b}$$

Then, $\bar{G}(0) = 1$. Under these conditions, if $v(t)$ is constant then the steady-state undisturbed output $x(t)$ depends only on the input value and the nonlinearity $\bar{h}(.)$.

The considered Wiener system and satisfying the properties 2a-2b will be denoted $\left(G(s), h(.)\right)$.

Then, the considered system described by Equations 1a-1c is excited by simple constant inputs:

$$v(t) = V_j \text{ for } j = 1K\ N \tag{3}$$

where the number N is arbitrarily chosen by the user. Accordingly, as the linear subsystem $G(s)$ is asymptotically stable, it follows that the steady-state of the internal signal $w(t)$ is constant i.e. $w(t) \underset{t\to\infty}{\to} W_j$, and is written using Equations 1a and 2a :

$$W_j = V_j \text{ for } j = 1K\ N \tag{4}$$

In which case, the undisturbed output $x(t)$ is also constant (in the steady-state) i.e. $x(t) \underset{t\to\infty}{\to} X_j$. Then, it readily follows from Equations 1b and 4a that X_j the undisturbed output system can be expressed as follows:

$$X_j = h\left(W_j\right) \text{ for } j = 1K\ N \tag{5}$$

Finally, notice that the steady-state undisturbed output X_j $(j = 1K\ N)$ can simply be estimated using the fact that $y(t) = w(t) + \xi(t)$ and $\xi(t)$ is zero-mean. Specifically, W_j can be recovered by averaging $y(t)$ on a sufficiently large interval. Hence, a number of points of the nonlinear function $h(.)$ can thus be accurately estimated by repeating the above experiment successively for V_1 to V_N.

Note that each input value V_j $(j = 1K\ N)$ is kept on during lT_r seconds making possible for the output signal to settle down within each interval of the form $\left[(j-1)lT_r\ jlT_r\right]$ with $j = 1K\ N$, where T_r should be comparable to the system rise time . Practically, one can determine a suitable value of N by observing any step response of the system. Then, the undisturbed output estimates can be given as follows;

$$\hat{X}_j(p) = \frac{1}{pT_r}\int_{(j-1)pT_r}^{jpT_r} y(t)dt \text{ for } j = 1K\ N \tag{6}$$

Then, a set of points $\left(V_j, h(V_j)\right) = \left(V_j, X_j\right)$ (with $j = 1K\ N$) belonging to nonlinearity $h(.)$ can be accurately estimated.

On the other hand, these results allow to determine the parameters vector $C = \left[c_0 \ldots c_p\right]^T$ by exciting by a set of points within to the subinterval I.

Linear subsystem identification

In this section, an identification method is proposed to obtain estimates of the complex gain corresponding to the two linear subsystem $G(s)$ at the frequencies $k\omega$ $(k = 0,1,K)$ whatever $\omega > 0$.

All along this Section, the identified system is submitted to a given sine input:

$$v(t) = v_0 + V \sin(\omega t) \tag{7}$$

where the amplitude $V > 0$ and v_0 is any point chosen such that: $v(t) \in I$. Let T be the corresponding period $(T = 2\pi/\omega)$.

As the linear subsystem $G(s)$ is asymptotically stable with unit static gain (using Equation 2a), it follows from Equation 1a that the internal signal $w(t)$ turns out to be (in steady state):

$$w(t) = v_0 + V\left|G(j\omega)\right|\sin\left(\omega t + \varphi(\omega)\right) \tag{8}$$

with $\varphi(\omega) = \arg\left(G(j\omega)\right)$. Also, it is readily obtained using assumption A2 and Equations (1b) and (8):

$$x(t) \approx \sum_{k=0}^{p} c_k w(t)^k = \sum_{k=0}^{p} c_k \left(v_0 + V\left|G(j\omega)\right|\sin\left(\omega t + \varphi(\omega)\right)\right)^k \tag{9}$$

The term $w(t)^k$ in Equation (9) can be developed as follows:

$$\left(v_0 + V\left|G(j\omega)\right|\sin\left(\omega t + \varphi(\omega)\right)\right)^k = \sum_{r=0}^{k} C_r^k v_0^{\,r}\left(V\left|G(j\omega)\right|\sin\left(\omega t + \varphi(\omega)\right)\right)^{k-r} \tag{10}$$

where the value of the binomial coefficient C_r^k is given explicitly by:

$$C_r^k = \frac{k!}{(k-r)!\,r!} \tag{11}$$

Accordingly, it follows from Equation 9-11 that:

$$x(t) = \sum_{k=0}^{p} c_k \sum_{r=0}^{k} C_r^k v_0^{\,r}\left(V\left|G(j\omega)\right|\right)^{k-r}\left(\sin(\omega t + \varphi(\omega))\right)^{k-r} \tag{12}$$

where the parameters vector $C = \left[c_0 \ldots c_p\right]^T$ is known. The only parameters unknown in the latter equation are the linear subsystem parameters (i.e. the modulus gain $\left|G(j\omega)\right|$ and the phase $\varphi(\omega)$). Finally, it is readily obtained using Equations 1c and (12):

$$y(t) = \sum_{k=0}^{p} c_k \sum_{r=0}^{k} C_r^k v_0^{\,r}\left(V\left|G(j\omega)\right|\right)^{k-r}\left(\sin(\omega t + \varphi(\omega))\right)^{k-r} + \xi(t) \tag{13}$$

On the other hand, one can notice that the steady-state undisturbed output $x(t)$ is periodic of same period T as the input, and $\xi(t)$ is a zero-mean ergodic white noise, the effect of the latter can be filtered considering the following trans-period averaging of the output:

$$\hat{x}_L(t) = \frac{1}{L}\sum_{k=1}^{L} y\left(t + (k-1)T\right), \quad 0 \le t < T \tag{14}$$

for some (large enough) integer L. Finally, the estimates of the modulus gain $\left|G(j\omega)\right|$ and the phase $\varphi(\omega)$ can be easily obtained using the equations 12-14.

Conclusion

The problem of system identification is addressed for Wiener systems where the linear subsystem may be parametric or not, finite order or not. The system nonlinearity is of arbitrary-shape and can be noninvertible. This is only supposed to be well approximated, within any subinterval belonging to the working interval, with a polynomial of order *p* and unknown parameters

The identification problem is dealt with using a two-stage approach combining frequency. Data acquisition in presence of constant inputs is performed in the first stage following the procedure of Section 3. Then, an accurate estimate of a set of points belonging to the nonlinearity can be accurately estimated.

Finally, the transfer function response is identified in the second stage using the algorithm described Section 4 and the estimator (14). To the author's knowledge no previous study has solved the identification problem for a so large class of Wiener systems.

References

[1] L. Vanbeylen, R. Pintelon, and J. Schoukens, Blind maximum-likelihood identification of wiener systems, IEEE Transactions on Signal Processing, vol. 57 (8), 2009, pp. 3017–3029.

[2] A. Wills, T.B. Schön, L. Ljung, B. Ninness, Blind Identification of Wiener Models, IFAC World Congress, Milan, Italy, 2011.

[3] W. Greblicki, and M. Pawlak, Nonparametric System Identification, Cambridge University Press, 2008.

[4] M. Lovera, T. Gustafsson T., and M. Verhaegen, Recursive subspace identification of linear and non-linear Wiener state-space models, Automatica, vol. 36, 2000, pp.1639–1650.

[5] L. Ljung. System identification. Theory for the user. Prentice Hall, NJ, USA, 1999.