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Multi Item EOQ Model with Average Budget Constraint under Fuzzy Environment

K Venkatasubbaiah and M Srinivasulu*

Department of Mechanical Engineering, Andhra University, Visakhapatnam, Andhra Pradesh, India.

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ABSTRACT

In the realistic situation, the total expenditure on the inventory may be limited to certain extent for a store house to maintain inventory of multiple items having independent ordering costs and holding costs. That is, less than a predetermined maximum permissible amount which may be vague to certain extent. In fact all the parameters in an inventory model are normally variable, uncertain and imprecise. These fuzzy variables like objective goal, costs and constraints are considered with linear and parabolic membership functions in fuzzy logic and the model is solved by fuzzy non-linear programming method using Lagrange multipliers and illustrated with numerical examples.

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Introduction

Ever since Zadeh used the concept of fuzzy set theory to solve the decision making problems, several authors worked on fuzzy mathematical programming (Zimmermann, 1985). Negoita and Sularia (1976) were the first to apply fuzzy set theory to linear programming. Trappey and Richard (1988) were presented the fuzzy non-linear programming (FNLP). In all the cases the authors considered ambiguous situations, vague parameters, loose restrictions or non-exact objectives.

K. Das, T. K. Roy and M. Maiti (2000) developed a multi-item inventory model with constant demand and infinite replenishment under the restrictions on storage area, total average shortage cost and total average inventory investment cost using a geometric programming approach. S Mondal and M Maiti (2002) proposed multi item fuzzy EOQ models using generic algorithm with mutation and whole arithmetic crossover under fuzzy objective goal and resources with/without fuzzy parameters in the objective function. Jafar Rezaei and Mansoor Davoodi (2005) extended a multi-item fuzzy inventory model under total production cost, total storage space and number of orders constraints and solved with a Genetic Algorithm.

Adil Baykasoglu and Tolunay Gocken (2007), proposed a fuzzy multi-item economic order quantity (EOQ) problem by employing different fuzzy ranking methods. All of the parameters of the multi-item EOQ problem are defined as triangular fuzzy numbers. A multi-item EOQ model is considered by Debdulal Panda, Samarjit Kar and Manoranjan Maiti (2008) in which the cost parameters are of fuzzy/hybrid nature under two types of resources — (a) resources as fuzzy quantities; (b) resources as fuzzy and fuzzy-random quantities. Further, Adil Baykasoğlu and Tolunay Göçken (2009) extended a multi-item EOQ problem with fuzzy parameters as triangular fuzzy numbers by making use of fuzzy ranking functions and the particle swarm optimization algorithm.

In the present work, the EOQ model with average budget constraint has been considered in fuzzy environment by considering the inventory costs like ordering costs and inventory carrying costs, objective goal and constraints as fuzzy variables and is solved by fuzzy non-linear programming method using Lagrange multipliers (Trappey and Richard, 1988). Considering the nature of the fuzzy parameters in the EOQ model, we assume membership functions to be non-decreasing for fuzzy inventory costs and non-increasing for fuzzy goal and constraints.

These parameters have been represented by four different combinations of linear and parabolic membership functions. It has also been observed from practice that the exact shapes of the various possibility distributions are not very critical to the success or failure of fuzzy system.

The model is illustrated with numerical examples and the results have been compared with those of crisp model. The sensitivity analysis on the optimum order quantity and average cost for the variations in the violations of inventory costs, investment amount and constraints have been presented. It gives the effect of violations of fuzzy variables on the optimum values of decision parameters.

As it is said by JFC Trappey et al, a conventional problem is modeled in the crisp mathematical programming form if the problem has brief and unambiguous mathematical definition. If there are discrete or continuous changes of parameter values in the original model, then sensitivity analysis or parametric analysis will respectively specify the possible effects on the final solution. When the ambiguous problem has random or stochastic properties, then stochastic optimization can be applied.

A special class of problems exists that does not suit the characteristics of the above problem types. This class of problems with non-stochastic uncertainty or ill-formed vagueness establishes the need for using a new problem modeling approach. Ths approach has to

Tele:

E-mail addresses: m_srinivasulu@vizagsteel.com

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be able to properly describe or model the ill-formed, non-stochastic vagueness of the problem. In this case, fuzzy set theory seems to be the needed approach to substantiate the kind of vagueness found in some mathematical programming problems (Zimmermann, 1983).

As long as the EOQ model is considered, there may be some vagueness to certain extent will exist in case of inventory carrying cost or holding cost and ordering cost or acquisitioning cost. Also in a realistic situation, total expenditure for an inventory model may be limited. That is, less than a pre-determined maximum permissible amount which may be vague to certain extent. Its value, instead of being at a fixed level varies within a range. Similarly we may have several limitations such as warehouse area, budget and inventory levels etc, which are vague to certain extent. The vagueness in the above parameters is introduced by making them fuzzy in nature (Roy, TK and M. Maiti, 1985).

Membership Functions for Fuzzy Variables:

In the present work, a multi item EOQ model with limited average budget constraint has been considered with fuzzy variables (goal, costs and constraints) and these variables are represented by following four different combinations of linear and parabolic membership functions:

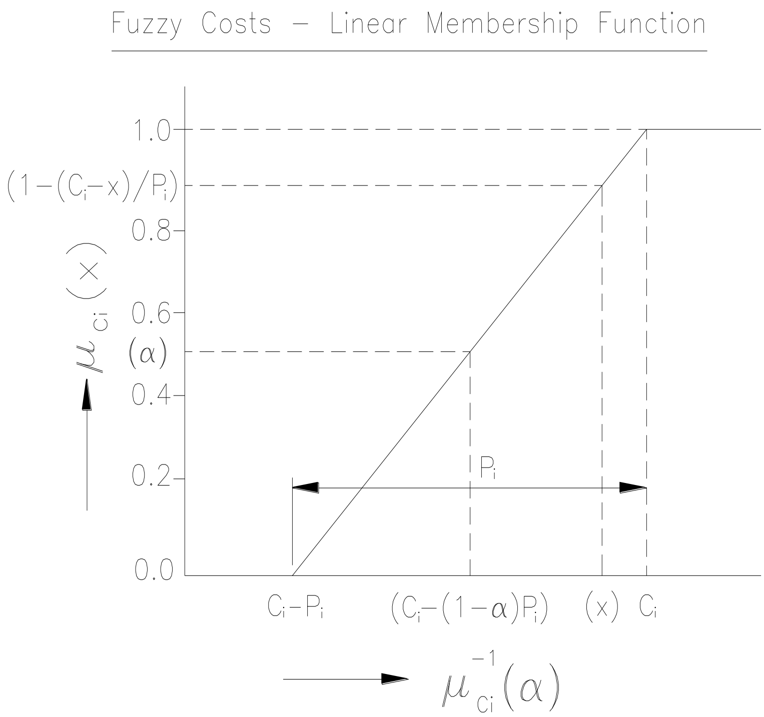
1. Fuzzy costs, goal and constraints – linear
2. Fuzzy costs – linear and Fuzzy goal / constraints – parabolic
3. Fuzzy costs – Parabolic and Fuzzy goal / constraints – linear
4. Fuzzy costs, goal and constraints – parabolic

Considering the nature of the above fuzzy parameters, we assume membership functions to be non-decreasing for fuzzy inventory costs and non-increasing for fuzzy goal and constraints.

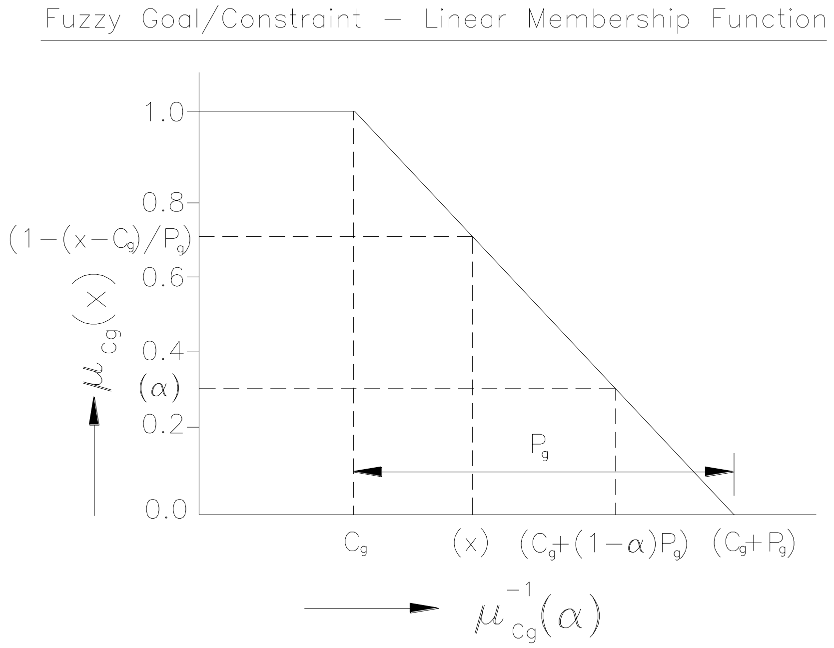
Linear membership functions

Fuzzy costs (C_i)

$\mu_{C_i}(x) = 1$ for $x > C_i$, ($C_i = C_c$, C_o),
$= 1 - (C_i - x)/P_i$ for $C_i - P_i \leq x \leq C_i$,
$= 0$ for $x < C_i - P_i$.



Fuzzy goal (C_g): $\mu_{C_g}(x) = 1$ for $x < C_g$,
$= 1 - (x - C_g)/P_g$ for $C_g \leq x \leq C_g + P_g$,
$= 0$ for $x > C_g + P_g$.



Fuzzy Constraint (M): $\mu_M(x) = 1$ for $x < M$,

$= 1 - (x - M)/P$ for $M \le x \le M + P$,

$= 0$ for $x > M + P$.

where $\mu_{Ci}(x)$, $\mu_{Cg}(x)$ and $\mu_M(x)$ are membership functions for fuzzy inventory costs (carrying coats, C_c and Ordering Costs, C_o), total cost (goal) and constraint respectively. And P_i (P_c and P_o), P_g and P are the maximally acceptable violations of the aspiration levels C_i (C_c , C_o), C_g and M.

hence $\mu_{Ci}^{-1}(\alpha) = C_i - (1\text{-}\alpha) P_i$,

$\mu_{Cg}^{-1}(\alpha) = C_g + (1\text{-}\alpha) P_g$ and

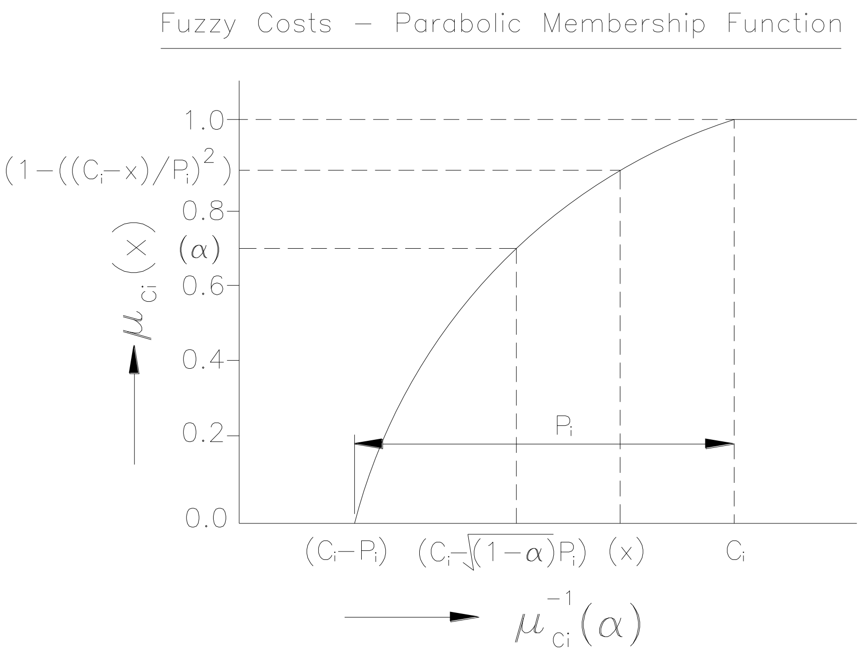
$\mu_M^{-1}(\alpha) = M + (1\text{-}\alpha) P$, which are the new values of the corresponding fuzzy variables at the determined aspiration level 'α'.

Parabolic membership functions

Fuzzy costs (C_i): $\mu_{Ci}(x) = 1$ for $x > C_i$, ($C_i = C_c$, C_o),

$= 1 - ((C_i - x)/P_i)^2$ for $C_i - P_i \le x \le C_i$,

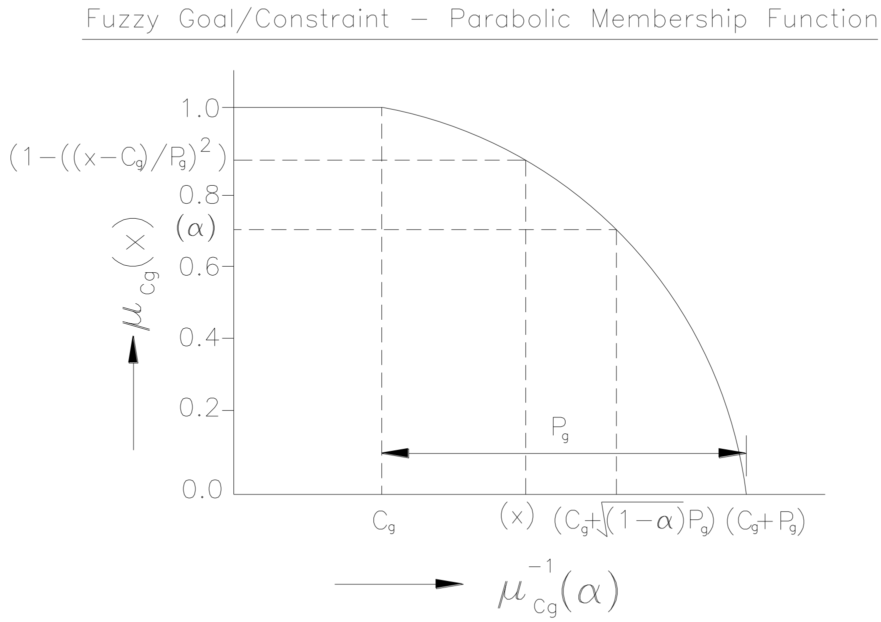
$= 0$ for $x < C_i - P_i$.



Fuzzy goal (C_g): $\mu_{Cg}(x) = 1$ for $x < C_g$,

$= 1 - ((x - C_g)/P_g)^2$ for $C_g \le x \le C_g + P_g$,

$= 0$ for $x > C_g + P_g$.



Fuzzy Constraint (M): $\mu_M(x) = 1$ for $x < M$,
$= 1 - ((x - M)/P)^2$ for $M \le x \le M + P$,
$= 0$ for $x > M + P$.

where $\mu_{Ci}(x)$, $\mu_{Cg}(x)$ and $\mu_M(x)$ are membership functions for fuzzy inventory costs (carrying coats, C_c and Ordering Costs, C_o), total cost (goal) and constraint respectively. And P_i (P_c and P_o), P_g and P are the maximally acceptable violations of the aspiration levels C_i (C_c, C_o), C_g and M.

hence $\mu_{Ci}^{-1}(\alpha) = C_i - \sqrt{(1-\alpha)}\, P_i$,
$\mu_{Cg}^{-1}(\alpha) = C_g + \sqrt{(1-\alpha)}\, P_g$ and
$\mu_M^{-1}(\alpha) = M + \sqrt{(1-\alpha)}\, P$, which are the new values of the corresponding fuzzy variables at the determined aspiration level 'α'.

Milti-Item EOQ model with Average Budget Constraint

In a crisp model, for single item EOQ, the problem is to choose the order level Q(>0) which minimizes the average total cost TC (Q) per unit time.

That is Min TC (Q) $= C_c C Q/2 + C_o Z/Q$
For Q > 0. ---- (1)

Where C_c = Carrying cost in % of the unit cost per unit time
C = Unit Cost
C_o = Ordering cost per period
Z = Demand per unit time

Now to solve the complex problem of multi item inventory with average investment constraint, the crisp model is:

Min TC (Q) $= \sum_{i=1,n} [C_c C_i Q_i / 2 + C_{oi} Z_i / Q_i]$ --- (2)
Subject to $\sum_{i=1,n} C_i Q_i /2 \le M$, $Q_i > 0$
Where n = no. of items and
M = Maximum allowable average budget on inventory in Rs.

In fuzzy set theory, the fuzzy variables are defined by their membership functions which may be linear or non-linear. According to Bellman, Zadeh, Trappey and Roy TK and M Maiti, the problem is transformed to:

Max α
Subject to: $\sum_{i=1,n} [\mu_{Cc}^{-1}(\alpha)\, C_i Q_i / 2 + \mu_{Co}^{-1}(\alpha)\, Z_i / Q_i] \le \mu_{Cg}^{-1}(\alpha)$ --- (3)
$\sum_{i=1,n} C_i Q_i / 2 \le \mu_M^{-1}(\alpha)$,
Q > 0,
$\alpha \in (0,1)$

Here α is an additional variable which is known as aspiration level. $\mu_{Cc}(x)$, $\mu_{Co}(x)$, $\mu_{Cg}(x)$ and $\mu_M(x)$ are membership functions of carrying cost, ordering cost, fuzzy goal and fuzzy constraint respectively.

Now let us consider the different combinations of membership functions:

All Fuzzy variables are Linear

$\mu_{Cc}(x)$, $\mu_{Co}(x)$, $\mu_{Cg}(x)$ and $\mu_M(x)$ are as defined in 2.1 above. Now the Lagrangian function is:

$L(\alpha, Q, \lambda) = \alpha - \lambda_1[\sum_{i=1,n}(C_c - (1-\alpha)P_c)C_i Q_i/2 + (C_{Oi} - (1-\alpha)P_{Oi})Z_i/Q_i - (C_g + (1-\alpha)P_g)] - \lambda_2[\sum_{i=1,n} C_i Q_i/2 - M - (1-\alpha) P]$ --- (4)

The corresponding Kuhn-Tucker conditions are:

$1 - \sum_{i=1,n} \lambda_1 P_c C_i Q_i / 2 - \lambda_1 \sum_{i=1,n} P_{Oi} Z_i / Q_i - \lambda_1 P_g - \lambda_2 P = 0$ --- (i)
$\lambda_1 [(C_c - (1-\alpha)P_c) C_i / 2 - (C_{Oi} - (1-\alpha)P_{Oi}) Z_i / Q_i^2] + \lambda_2 C_i / 2 = 0$ --- (ii)
$\sum_{i=1,n} [(C_c - (1-\alpha)P_c) C_i Q_i / 2 + (C_{Oi} - (1-\alpha)P_{Oi})Z_i / Q_i] - (C_g + (1-\alpha)P_g) = 0$ --- (iii)
$\sum_{i=1,n} [C_i Q_i / 2] - M - (1-\alpha) P = 0$ --- (iv)

Where, λ_1 and λ_2 are Lagrange multipliers.

From (4).(ii)

$$Q_i = \sqrt{\frac{2\,(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i}{((C_c - (1-\alpha)\,P_c) + \lambda)\,C_i}} \quad --- \quad (5)$$

Where, $\lambda = \lambda_2 / \lambda_1$.

From equations (5) and (4).(iv), we get:

$$\lambda = \frac{\left[\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}\,\right]^2}{2(M + (1-\alpha)\,P)^2} - (C_c - (1-\alpha)P_c) = 0 \quad --- \quad (6)$$

Re-substituting λ in equation (5):

$$Q^* = \frac{2\,(M + (1-\alpha^*)\,P}{\left[\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha^*)\,P_{Oi})\,Z_i\,C_i}\,\right]} \sqrt{\frac{(C_{Oi} - (1-\alpha^*)\,P_{Oi})\,Z_i}{C_i}} \quad --- \quad (7)$$

Where α^* is a root of:

$$\sum_{i=1,n} \left\{ \frac{(C_c - (1-\alpha)P_c)\,(M + (1-\alpha)\,P \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}}{\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}} + \frac{\sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}\;\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}}{2\,(M + (1-\alpha)\,P)} \right\} - (C_g + (1-\alpha)P_g) = 0 \quad --- \quad (8)$$

C_c & C_o – Linear and C_g & M - Parabolic

$\mu_{Cc}(x)$, $\mu_{Co}(x)$ are as defined in 2.1 and $\mu_{Cg}(x)$ and $\mu_M(x)$ are as defined in 2.2 above.

The corresponding optimal ordering quantities are:

$$Q^* = \frac{2\,(M + \sqrt{(1-\alpha^*)}\,P}{\left[\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha^*)\,P_{Oi})\,Z_i\,C_i}\,\right]} \sqrt{\frac{(C_{Oi} - (1-\alpha^*)\,P_{Oi})\,Z_i}{C_i}} \quad --- \quad (9)$$

Where α^* is a root of:

$$\sum_{i=1,n} \left\{ \frac{(C_c - (1-\alpha)P_c)\,(M + \sqrt{(1-\alpha)}\,P \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}}{\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}} + \frac{\sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}\;\sum_{i=1,n} \sqrt{(C_{Oi} - (1-\alpha)\,P_{Oi})\,Z_i\,C_i}}{2\,(M + \sqrt{(1-\alpha)}\,P)} \right\} - (C_g + \sqrt{(1-\alpha)}P_g) = 0 \quad --- \quad (10)$$

C_c & C_o – Parabolic and C_g & M - Linear

$\mu_{Cc}(x)$, $\mu_{Co}(x)$ are as defined in 2.2 and $\mu_{Cg}(x)$ and $\mu_M(x)$ are as defined in 2.1 above.

The corresponding optimal ordering quantities are:

$$Q^* = \frac{2\,(M + (1-\alpha^*)\,P}{\left[\sum_{i=1,n} \sqrt{(C_{Oi} - \sqrt{(1-\alpha^*)}\,P_{Oi})\,Z_i\,C_i}\,\right]} \sqrt{\frac{(C_{Oi} - \sqrt{(1-\alpha^*)}\,P_{Oi})\,Z_i}{C_i}} \quad --- \quad (11)$$

Where α^* is a root of:

$$\sum_{i=1,n} \left\{ \frac{(C_c - \sqrt{(1-\alpha)}P_c)\,(M + (1-\alpha)\,P \sqrt{(C_{Oi} - \sqrt{(1-\alpha)}\,P_{Oi})\,Z_i\,C_i}}{\sum_{i=1,n} \sqrt{(C_{Oi} - \sqrt{(1-\alpha)}\,P_{Oi})\,Z_i\,C_i}} + \frac{\sqrt{(C_{Oi} - \sqrt{(1-\alpha)}\,P_{Oi})\,Z_i\,C_i}\;\sum_{i=1,n} \sqrt{(C_{Oi} - \sqrt{(1-\alpha)}\,P_{Oi})\,Z_i\,C_i}}{2\,(M + (1-\alpha)\,P)} \right\} - (C_g + (1-\alpha)P_g) = 0 \quad --- \quad (12)$$

All Fuzzy variables are Parabolic

$\mu_{Cc}(x)$, $\mu_{Co}(x)$ are as defined in 2.2 and $\mu_{Cg}(x)$ and $\mu_M(x)$ are as defined in 2.1 above.

The corresponding optimal ordering quantities are:

$$Q^* = \frac{2\left(M + \sqrt{1-\alpha^*}\,P\right)}{\left[\sum_{i=1,n} \sqrt{\left(C_{Oi} - \sqrt{1-\alpha^*}\,P_{Oi}\right) Z_i C_i}\,\right]} \sqrt{\frac{\left(C_{Oi} - \sqrt{1-\alpha^*}\,P_{Oi}\right) Z_i}{C_i}} \quad \text{---} \quad (13)$$

Where α^* is a root of:

$$\sum_{i=1,n} \left\{ \frac{\left(C_c - \sqrt{1-\alpha}\,P_c\right)\left(M + \sqrt{1-\alpha}\,P\right)\sqrt{\left(C_{Oi} - \sqrt{1-\alpha}\,P_{Oi}\right) Z_i C_i}}{\sum_{i=1,n} \sqrt{\left(C_{Oi} - \sqrt{1-\alpha}\,P_{Oi}\right) Z_i C_i}} + \frac{\sqrt{\left(C_{Oi} - \sqrt{1-\alpha}\,P_{Oi}\right) Z_i C_i}\;\sum_{i=1,n} \sqrt{\left(C_{Oi} - \sqrt{1-\alpha}\,P_{Oi}\right) Z_i C_i}}{2\left(M + \sqrt{1-\alpha}\,P\right)} \right\} - \left(C_g + \sqrt{1-\alpha}\,P_g\right) = 0 \quad \text{---} \quad (14)$$

Numerical Experience

To illustrate the Multi Item model, let us assume the following:

C_c	=	25 %	P_c	= 5 %
M	=	Rs. 23,000,	P	= Rs. 1000
C_g	=	Rs. 17,500,	P_g	= Rs. 500

No. of Items = 3.

Item No.	Z, Rs.	C, Rs.	C_o, Rs.	P_o, Rs.
1.	10,000	50	75	20
2.	5,000	100	125	25
3.	15,000	75	100	25

Results of Fuzzy Models

A software algorithm had developed to find the optimum parameters with the given data. The following are the optimum parameters for the data assumed above:

Model No.	Item No.	C_c	C_o	Q*	M*	α*	C_g*
3.1	1 2 3	23.58 23.58 23.58	69.32 117.90 92.90	230.499 150.303 266.838	23284.006	0.716	17642.104
3.2	1 2 3	23.775 23.775 23.775	70.10 118.875 93.875	232.732 151.537 269.326	23494.98	0.755	17748.537
3.3	1 2 3	23.419 23.419 23.419	68.675 117.094 92.094	228.555 149.22 264.67	23100.008	0.90	17557.398
3.4	1 2 3	23.586 23.586 23.586	69.343 117.929 92.929	230.492 150.291 266.827	23282.857	0.92	17647.334

Result & Discussion

For multi-item model the possible number of crisp models is given by 2^{n+1}, where 'n' is the number of items. So, for a problem of 3 items, the number of crisp models is 32. It is a very laborious process to calculate all the 32 models. The fuzzy analysis replaces this time consuming parametric studies and the optimum results are obtained easily.

In table 4.1, when $\mathbf{P_g}$ is about 500, 25% change in $\mathbf{P_g}$ induces 0.017%, 0.046%, 0.026%, 0.176% and a negligible percent change in $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_g^*}$ and **α*** respectively. As $\mathbf{P_g}$ increases from 0 to 14,225, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$ and $\mathbf{Q_3^*}$ are almost invariable and the costs attain their highest allowable values for the large values of $\mathbf{P_g}$**.**

In table 4.2, when $\mathbf{P_c}$ is about 5, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_g^*}$ and **α*** are changes by 0.04%, 0.1%, 0.06%, 0.045% and 1.38% respectively. The ordering costs are increasing with the decrease of holding cost and as a result average cost decreases.

In table 4.3, when, $\mathbf{P_{O1}}$ is about 20, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_g^*}$ and **α*** are changes by 0.77%, 0.17%, 0.2%, 0.034% and 1.38% respectively. Here holding cost, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_{O2}^*}$ and $\mathbf{C_{O3}^*}$ are increases and $\mathbf{Q_1^*}$, $\mathbf{C_{O1}^*}$ and $\mathbf{C_g^*}$ are decreases with the increase of $\mathbf{P_{O1}}$.

In table 4.4, when, $\mathbf{P_{O2}}$ is about 25, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_g^*}$ and **α*** are changes by 0.2%, 0.56%, 0.18%, 0.03% and 1.38% respectively. Here holding cost, $\mathbf{Q_1^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_{O1}^*}$ and $\mathbf{C_{O3}^*}$ are increases and $\mathbf{Q_2^*}$, $\mathbf{C_{O2}^*}$ and $\mathbf{C_g^*}$ are decreases with the increase of $\mathbf{P_{O2}}$.

In table 4.5, when, $\mathbf{P_{O3}}$ is about 25, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$, $\mathbf{C_g^*}$ and **α*** are changes by 0.33%, 0.26%, 0.58%, 0.06% and 1.38% respectively. Here holding cost, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{C_{O1}^*}$ and $\mathbf{C_{O2}^*}$ are increases and $\mathbf{Q_3^*}$, $\mathbf{C_{O3}^*}$ and $\mathbf{C_g^*}$ are decreases with the increase of $\mathbf{P_{O2}}$.

In table 4.6, when, **P** is about 1000, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{Q_3^*}$ and $\mathbf{C_g^*}$ are changes by 0.3%, 0.28%, 0.28% and 0.01% respectively. Here holding cost, $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$ and $\mathbf{Q_3^*}$ are increases slightly and the costs attain their maximum values and $\mathbf{C_g^*}$ is decreases with the increase of **P**.

Sensitivity Analysis

The sensitivity analysis of the model 3.1 above is given in the following tables:

Table 4.1. Sensitivity analysis of P_g:

P_g	α^*	Q_1^*	Q_2^*	Q_3^*	C_c^*	C_{O1}^*	C_{O2}^*	C_{O3}^*	C_g^*	M^*
0	0.68	230.69	150.60	267.14	23.42	68.70	117.12	92.12	17500.00	23315.01
125	0.69	230.63	150.52	267.05	23.47	68.88	117.35	92.35	17538.25	23306.01
250	0.70	230.58	150.44	266.97	23.51	69.04	117.55	92.55	17574.50	23298.01
375	0.71	230.54	150.37	266.91	23.54	69.18	117.72	92.72	17609.13	23291.01
500	0.72	230.50	150.30	266.84	23.58	69.32	117.90	92.90	17642.00	23284.01
625	0.72	230.46	150.23	266.77	23.61	69.46	118.07	93.07	17673.13	23277.01
4025	0.83	229.77	149.18	265.70	24.16	71.66	120.82	95.82	18172.20	23167.01
7425	0.88	229.47	148.73	265.24	24.40	72.60	122.00	97.00	18391.06	23120.01
10825	0.91	229.30	148.48	264.98	24.53	73.14	122.67	97.67	18506.81	23093.01
14225	0.92	229.18	148.32	264.81	24.62	73.48	123.10	98.10	18581.22	23076.01

Table 4.2. Sensitivity analysis of P_c:

P_c	α^*	Q_1^*	Q_2^*	Q_3^*	C_c^*	C_{O1}^*	C_{O2}^*	C_{O3}^*	C_g^*	M^*
0	0.63	231.01	151.14	267.65	25.00	67.62	115.77	90.77	17684.50	23369.00
1.25	0.66	230.85	150.88	267.40	24.57	68.14	116.42	91.42	17671.50	23343.00
2.5	0.68	230.72	150.66	267.19	24.20	68.58	116.97	91.97	17660.50	23321.01
3.75	0.70	230.60	150.47	267.10	23.87	68.98	117.47	92.47	17650.50	23301.01
5.00	0.72	230.50	150.30	266.84	23.58	69.32	117.90	92.90	17642.00	23284.01
6.25	0.73	230.40	150.15	266.68	23.32	69.64	118.30	93.30	17634.00	23268.01
10.25	0.77	230.15	149.76	266.30	22.66	70.44	119.30	94.30	17614.00	23228.01
14.25	0.80	229.97	149.48	266.01	22.16	71.02	120.02	95.01	17599.50	23199.01
18.25	0.82	229.83	149.26	265.79	21.79	71.48	120.60	95.60	17588.00	23176.01
22.25	0.84	229.71	149.09	265.61	21.48	71.84	121.05	96.05	17579.00	23158.01

Table 4.3.Sensitivity analysis of P_{O1}:

P_{O1}	α^*	Q_1^*	Q_2^*	Q_3^*	C_c^*	C_{O1}^*	C_{O2}^*	C_{O3}^*	C_g^*	M^*
0	0.66	239.22	149.06	264.20	23.29	75.00	116.47	91.47	17670.50	23341.01
5	0.67	236.77	149.41	264.94	23.37	73.37	116.87	91.87	17662.50	23325.01
10	0.69	234.50	149.73	265.62	23.45	71.90	117.25	92.25	17655.00	23310.01
15	0.70	232.42	150.02	266.25	23.52	70.56	117.60	92.60	17648.00	23296.01
20	0.72	230.50	150.30	266.84	23.58	69.32	117.90	92.90	17642.00	23284.01

25	0.73	228.72	150.55	267.37	23.64	68.20	118.20	93.02	17636.00	23272.01
36	0.75	225.25	151.04	268.42	23.75	66.04	118.77	93.77	17624.50	23249.01
47	0.77	222.30	151.46	269.30	23.85	64.24	119.27	94.27	17614.50	23229.01
58	0.79	219.76	151.81	270.07	23.94	62.70	119.70	94.70	17606.00	23212.01
69	0.80	217.50	152.14	270.75	24.01	61.34	120.05	95.05	17599.00	23198.01

Table 4.4. Sensitivity analysis of P_{O2}

P_{O2}	α*	Q_1*	Q_2*	Q_3*	C_c*	C_{O1}*	C_{O2}*	C_{O3}*	C_g*	M*
0	0.66	228.20	154.43	264.31	23.31	68.24	125.00	91.55	17669.00	23338.01
6.25	0.68	228.84	153.27	265.02	23.38	68.54	122.98	91.92	17661.50	23323.01
12.50	0.69	229.44	152.20	265.68	23.45	68.82	121.14	92.27	17654.50	23309.01
18.75	0.70	229.99	151.22	266.28	23.52	69.08	119.45	92.60	17648.00	23296.01
25.00	0.72	230.50	150.30	266.84	23.58	69.32	117.90	92.90	17642.00	23284.01
31.25	0.73	230.96	149.45	267.34	23.64	69.96	116.50	93.20	17636.00	23272.01
51.25	0.76	232.25	147.13	268.76	23.79	70.18	112.65	93.97	17620.50	23241.01
71.25	0.78	233.29	145.26	269.89	23.92	70.68	109.61	94.60	17608.00	23216.01
91.25	0.80	234.15	143.72	270.84	24.02	71.08	107.11	95.10	17598.00	23196.01
111.25	0.82	234.86	142.44	271.61	24.10	71.42	105.09	95.52	17589.50	23179.01

Table 4.5. Sensitivity analysis of P_{O3}

P_{O3}	α*	Q_1*	Q_2*	Q_3*	C_c*	C_{O1}*	C_{O2}*	C_{O3}*	C_g*	M*
0	0.61	226.02	148.00	275.72	23.05	67.20	115.25	100.00	17695.00	23390.01
6.25	0.64	227.41	148.71	272.96	23.21	67.86	116.07	97.77	17678.50	23357.01
12.5	0.67	228.59	149.32	270.62	23.35	68.42	116.77	95.89	17664.50	23329.01
18.75	0.69	229.61	149.85	268.60	23.47	68.90	117.37	94.28	17652.50	23305.01
25	0.72	230.50	150.30	266.84	23.58	69.32	117.90	92.90	17642.00	23284.01
31.25	0.73	231.27	150.70	265.29	23.67	69.70	118.37	91.72	17632.50	23265.01
46.25	0.77	232.78	151.47	262.29	23.85	70.42	119.27	89.41	17614.50	23229.01
61.25	0.80	233.92	152.06	260.00	23.99	70.98	119.97	87.69	17600.50	23201.01
76.25	0.82	234.82	152.52	258.02	24.10	71.42	120.52	86.35	17589.50	23179.01
91.25	0.84	235.57	152.92	256.71	24.19	71.76	120.95	85.22	17581.00	23162.01

Table 4.6. Sensitivity analysis of P

P	α*	Q_1*	Q_2*	Q_3*	C_c*	C_{O1}*	C_{O2}*	C_{O3}*	C_g*	M*
0	0.70	227.62	148.52	263.55	23.49	68.98	117.47	92.47	17650.50	23000.00

250	0.70	228.37	148.99	264.41	23.52	69.08	117.60	92.60	17648.00	23074.00
500	0.71	229.10	149.44	265.24	23.54	69.16	117.70	92.70	17646.00	23146.00
750	0.70	229.81	149.88	266.05	23.56	69.24	117.80	92.80	17644.00	23216.00
1000	0.72	230.50	150.30	266.84	23.58	69.32	117.90	92.90	17642.00	23284.01
1250	0.72	231.17	150.72	267.60	23.60	69.40	118.00	93.00	17640.00	23350.01
5650	0.77	240.61	156.56	278.39	23.56	70.46	119.32	94.32	17613.50	24282.59
10050	0.81	247.25	160.68	285.99	24.03	71.14	120.17	95.17	17596.50	24939.72
14450	0.83	252.47	163.93	291.96	24.15	71.60	120.75	95.75	17585.00	25456.60
18850	0.85	256.59	166.50	296.68	24.24	71.96	121.20	96.20	17576.00	25865.34

Conclusions

In the present work, the real life multi-item inventory problem with average budget constraint in fuzzy environment has been formulated and the results have been presented. Here, we have considered two types of membership functions i.e. linear and parabolic, to represent the nature of variations in inventory costs, objective goal and the constraints. Fuzzy models with four combinations of membership functions are observed for the optimum values. Among these four combinations the third combination, i.e. costs-parabolic membership functions and constraints – linear membership functions, is showing the lowest optimum values.

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