Available online at www.elixirpublishers.com (Elixir International Journal)

Discrete Mathematics

Elixir Dis. Math. 88 (2015) 36597-36601

ISSN: 2229-712X

Further Results on Cube Divisor Cordial Labeling

K. K. Kanani and M. I. Bosmia

Gujarat Technological University, Government Engineering College, Rajkot, Gujarat, India.

ARTICLE INFO

Article history:

Received: 14 September 2015;

Received in revised form:

20 November 2015;

Accepted: 25 November 2015;

Keywords

Cube Divisor Cordial Labeling,

Barycentric Subdivision,

Vertex Switching,

Degree Spliting.

ABSTRACT

A cube divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,\ldots,|V|\}$ such that an edge $e=uv$ is assigned the label 1 if $[f(u)]^3 \mid f(v)$ or $[f(v)]^3 \mid f(u)$ and the label 0 otherwise, then $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a cube divisor cordial labeling is called a cube divisor cordial graph. In this paper we prove that barycentric subdivision of the star $K_{1,n}$, switching of a vertex in cycle C_n, degree splitting graphs of $B_{n,n}$ and $P_n; n \neq 5$ admit cube divisor cordial labeling.

Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G=(V(G),E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

The latest updates of various Graph Labeling Techniques can be found in Gallian [2].

Notation 1.2. $e_f(i)$ = Number of edges with label $i; i = 0,1$.

Definition 1.3. Let $G=(V(G),E(G))$ be a simple graph and $f: V(G) \to \{1,2,\ldots,|V(G)|\}$ be a bijection. For each edge $e=uv$, assign the label 1 if $[f(u)] \mid f(v)$ or $[f(v)] \mid f(u)$ and the label 0 otherwise. The function f is called a *divisor cordial labeling* if $|e_f(0)-e_f(1)| \leq 1$. A graph which admits a divisor cordial labeling is called a *divisor cordial graph*.

The concept of divisor cordial labeling was introduced by Varatharajan et al. [10] and they proved the following results:

- The star graph $K_{1,n}$ is divisor cordial.
- $S(K_{1,n})$, the subdivision of the star $K_{1,n}$ is divisor cordial.

Vaidya and Shah [9] proved that:

- Switching of a vertex in cycle C_n admits divisor cordial labeling.
- Switching of a rim vertex in a wheel W_n admits divisor cordial labeling.

Definition 1.4. Let $G=(V(G),E(G))$ be a simple graph and $f: V(G) \to \{1,2,\ldots,|V(G)|\}$ be a bijection. For each edge $e=uv$, assign the label 1 if $[f(u)]^2 \mid f(v)$ or $[f(v)]^2 \mid f(u)$ and the label 0 otherwise. The function f is called a *square divisor cordial labeling* if $|e_f(0)-e_f(1)| \leq 1$. A graph which admits a square divisor cordial labeling is called a *square divisor cordial graph*.

The concept of square divisor cordial labeling was introduced by Murugesan et al. [6].

Vaidya and Shah [8] proved that:

- $DS(B_{n,n})$, the degree splitting graphs of the bistar $B_{n,n}$ is a square divisor cordial graph.
- $DS(P_n)$, the degree splitting graphs of the path P_n is a square divisor cordial graph.

Definition 1.5. Let $G=(V(G),E(G))$ be a simple graph and $f: V(G) \to \{1,2,\ldots,|V(G)|\}$ be a bijection. For each edge $e=uv$, assign the label 1 if $[f(u)]^3 \mid f(v)$ or $[f(v)]^3 \mid f(u)$ and the label 0 otherwise. The function f is called a *cube divisor cordial labeling* if $|e_f(0)-e_f(1)| \leq 1$. A graph which admits a cube divisor cordial labeling is called a *cube divisor cordial graph*.

The concept of cube divisor cordial labeling was introduced by Kanani and Bosmia[9] and they proved the following results:

- The complete graph K_n is cube divisor cordial if and only if $n=1,2,3,4$.
- The star graph $K_{1,n}$ is a cube divisor cordial graph if and only if $n=1,2,3$.

Tele: +919426449132, +919016914493

E-mail addresses: kananikkk@yahoo.co.in, cosmicmohit@gmail.com

- The complete bipartite graph $K_{2,n}$ is a cube divisor cordial graph.
- The complete bipartite graph $K_{3,n}$ is cube divisor cordial if and only if $n = 1, 2$.
- The bistar $B_{n,n}$ is a cube divisor cordial graph.
- Restricted $B^2_{n,n}$ is a cube divisor cordial graph.

A graph may be labeled by both the labeling techniques square divisor cordial labeling and cube divisor cordial labeling at a time, may be labeled by any one of them, or may not at all be labeled by any of them. In other words these two graph labeling techniques behave independently. This fact is exemplified in the following examples:

- The bistar $B_{n,n}$ is both square divisor cordial as proved in [8] and cube divisor cordial as proved in [4]
- The complete graph K_4 is not square divisor cordial as proved in [6] but it is cube divisor cordial as proved in [4].
- The complete bipartite graph $K_{3,6}$ is square divisor cordial as proved in [6] but it is not cube divisor cordial as proved in [4].
- The star $K_{1,n}$ $(n \geq 8)$ is not square divisor cordial as proved in [6] and it is not cube divisor cordial as proved in [4].

Definition 1.6. Let $G = (V(G), E(G))$ be a graph. Let $e = uv$ be an edge of G and w is not a vertex of G. The edge e is *sub divided* when it is replaced by the edges $e' = uw$ and $e'' = wv$.

Definition 1.7. Let $G = (V(G), E(G))$ be a graph. If every edge of graph G is subdivided, then the resulting graph is called *barycentric subdivision* of graph G.

In other words barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of original graph. The barycentric subdivision of any graph G is denoted by $S(G)$.

Definition 1.8. A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

Definition 1.9. [7] Let $G = (V(G), E(G))$ be a graph with $V = S_1 \cup S_2 \cup \ldots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V - \left(\bigcup_{i=1}^{t} S_i\right)$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, w_3, \ldots, w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

Main Results

Theorem 2.1. The barycentric subdivision $S(K_{1,n})$ of the star $K_{1,n}$ is a cube divisor cordial graph.

Proof: Let $K_{1,n}$ be the star with apex vertex v_0 and pendant vertices $v_1, v_2, \ldots, v_n$. Let $e_i = v_0 v_i$ for $i = 1, 2, \ldots, n$. To obtain barycentric subdivision $G = S(K_{1,n})$ of the star $K_{1,n}$ subdivide each edge of star $K_{1,n}$ by the vertices $w_1, w_2, \ldots, w_n$. Where each w_i is added between v_0 and v_i for $i = 1, 2, \ldots, n$. We note that $|V(G)| = 2n+1$ and $|E(G)| = 2n$.

Define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2n+1\}$ as follows:

$f(v_0) = 1$.

$f(v_i) = 2i+1;\ 1 \leq i \leq n$.

$f(w_i) = 2i;\ 1 \leq i \leq n$.

In view of above defined labeling pattern we have $e_f(0) = e_f(1) = n$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the barycentric subdivision $S(K_{1,n})$ of the star $K_{1,n}$ is a cube divisor cordial graph.

Illustration 2.2. Cube divisor cordial labeling of the graph $S(K_{1,8})$ is shown in *Figure 1*.

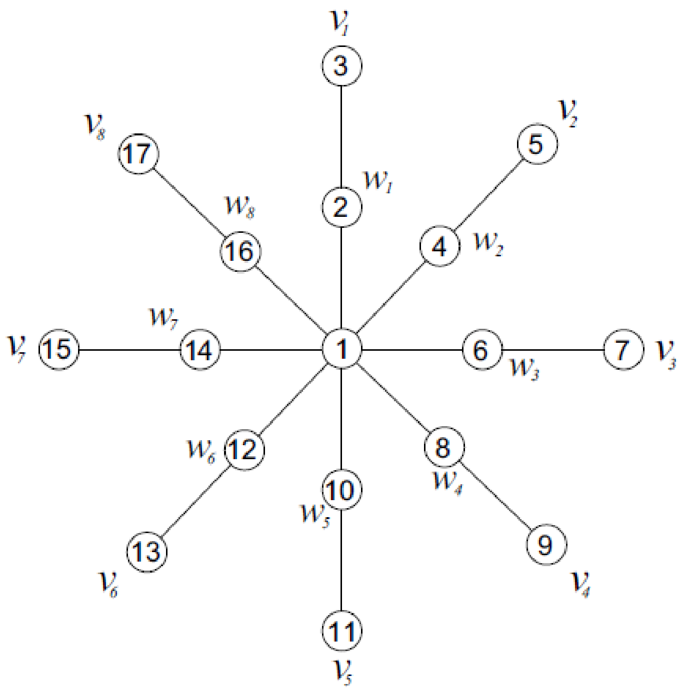

Figure 1: Cube divisor cordial labeling of $S(K_{1,8})$.

Theorem 2.3. The graph G_v obtained by switching of a vertex in cycle C_n is a cube divisor cordial graph.

Proof: Let $v_1, v_2, \ldots, v_n$ be the successive vertices of the cycle C_n and G_v denotes graph obtained by switching of vertex v of $G = C_n$. Without loss of generality let the switched vertex be v_1. We note that $|V(G_v)| = n$ and $|E(G_v)| = 2n-5$.

Define vertex labeling $f : V(G_v) \to \{1, 2, \ldots, n\}$ as follows:

$f(v_1) = 1$.

$f(v_i) = n + 2 - i$; 2£ i£ n.

In view of above defined labeling pattern we have $e_f(1) = n-3$, $e_f(0) = n-2$. Thus, $|e_f(0) - e_f(1)| \le 1$.

Hence, the graph G_v obtained by switching of a vertex in cycle C_n is a cube divisor cordial graph.

Illustration 2.4. The graph obtained by switching of a vertex in cycle C_8 and its cube divisor cordial labeling is shown in *Figure 2*.

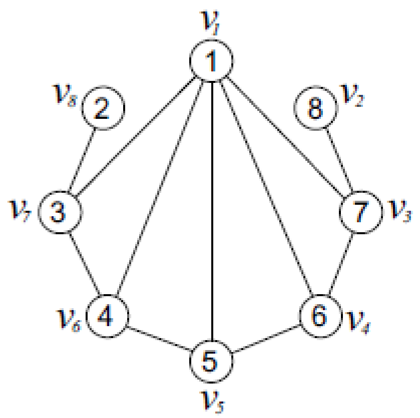

Figure 2: Switching of a vertex in cycle C_8 and its cube divisor cordial labeling.

Theorem 2.5. $DS(B_{n,n})$ is a cube divisor cordial graph.

Proof: Let $B_{n,n}$ be the star with $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \le i \le n\}$, where u_i, v_i are pendant vertices. Here $V(B_{n,n}) = V_1 \cup V_2$, where $V_1 = \{u_i, v_i : 1 \le i \le n\}$ and $V_2 = \{u, v\}$. Now in order to obtain $DS(B_{n,n})$ from $B_{n,n}$ we add w_1, w_2 corresponding to V_1, V_2. Let G be the $DS(B_{n,n})$ graph. Then $|V(G)| = 2n+4$ and $E(G) = \{uv, uw_2, vw_2\} \cup \{uu_i, vv_i, w_1u_i, w_1v_i : 1 \le i \le n\}$. So, $|E(G)| = 4n+3$.

To define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2n+4\}$ we consider two cases.

<u>Case 1:</u> $n = 1$.

So, $|V(G)| = 6$ and $|E(G)| = 7$.

Define

$f(u) = 1$, $f(v) = 5$.

$f(u_1) = 3$, $f(v_1) = 4$.

$f(w_1) = 2$, $f(w_2) = 6$.

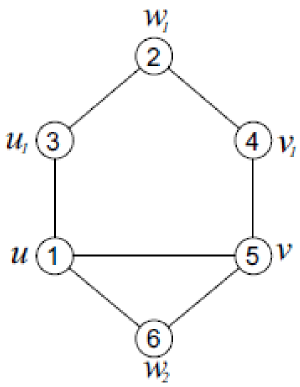

Figure 3: Cube divisor cordial labeling of $DS(B_{1,1})$.

Here, $e_f(0) = 4$, $e_f(1) = 3$. Thus, $|e_f(0) - e_f(1)| \le 1$.

<u>Case 2:</u> $n \ge 2$.

Define

$f(u) = 2$, $f(v) = 2n+3$.

$f(w_1) = 1$, $f(w_2) = 8$.

$f(u_i) = 1 + 2i; 1 \le i \le n$.

$f(v_i) = 2 + 2i; 1 \le i \le n, i \ne 3$.

$f(v_3) = 2n+4$.

In view of the above defined labeling pattern we have $e_f(0) = 2n+2$, $e_f(1) = 2n+1$. Thus, $|e_f(0) - e_f(1)| \le 1$.

Hence, $DS(B_{n,n})$ is a cube divisor cordial graph.

Illustration 2.6. Cube divisor cordial labeling of the graph $DS(B_{4,4})$ is shown in *Figure 4*.

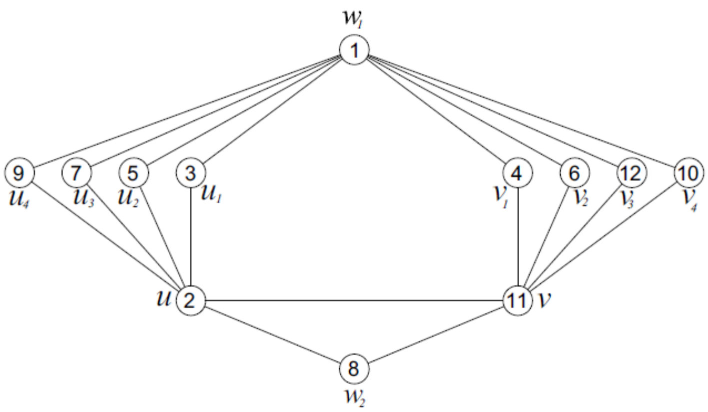

Figure 4: Cube divisor cordial labeling of $DS(B_{4,4})$.

Theorem 2.7. $DS(P_n)$ is a cube divisor cordial graph if and only if $n \ne 5$.

Proof: Let P_n be the path with $V(P_n) = \{v_i : 1 \le i \le n\}$. Here $V(P_n) = V_1 \cup V_2$, where $V_1 = \{v_1, v_n\}$ and $V_2 = \{v_i : 2 \le i \le n-1\}$. Now in order to obtain $DS(P_n)$ from P_n, we consider following two cases.

Case 1: $n = 3$.

We add w_1 to V_1. Then $|V(DS(P_3))| = 4$ and $E(DS(P_3)) = E(P_3) \cup \{v_1w_1, v_3w_1\}$. So $|E(DS(P_3))| = 4$. The cube divisor cordial labeling of $DS(P_3)$ is shown in *Figure 5*.

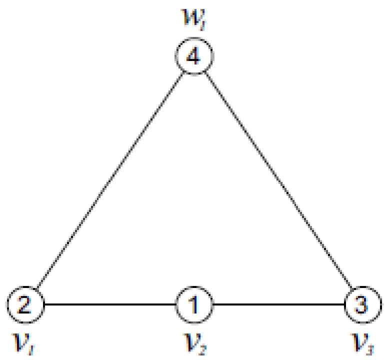

Figure 5: Cube divisor cordial labeling of $DS(P_3)$.

Here, $e_f(0) = e_f(1) = 2$. Thus, $|e_f(0) - e_f(1)| = 0$.
Hence, $DS(P_3)$ is a cube divisor cordial graph.

Case 2: $n \geq 4$.

We add w_1, w_2 corresponding to V_1, V_2. Then $|V(DS(P_n))| = n + 2$ and $E(DS(P_n)) = E(P_n) \cup \{v_1w_1, v_nw_1\} \cup \{w_2v_i : 2 \leq i \leq n-1\}$. So $|E(DS(P_n))| = 2n - 1$.

To define vertex labeling $f : V(DS(P_n)) \to \{1, 2, \ldots, n+2\}$ we consider following three subcases.

Subcase 1: $n = 4$.

The cube divisor cordial labeling of $DS(P_4)$ is shown in *Figure 6*.

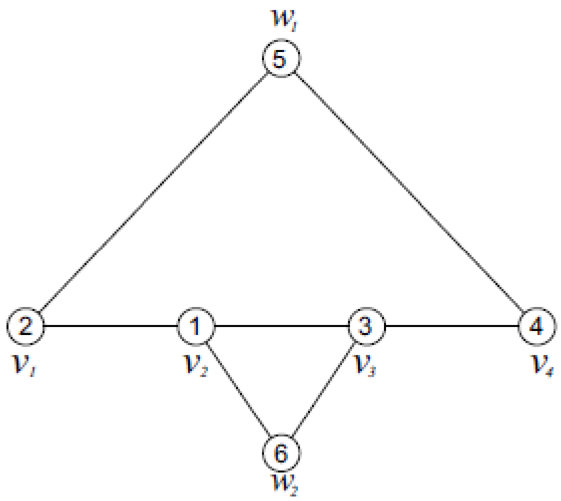

Figure 6: Cube divisor cordial labeling of $DS(P_4)$.

Here, $e_f(0) = 4$, $e_f(1) = 3$. Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $DS(P_4)$ is a cube divisor cordial graph.

Subcase 2: $n = 5$.
The graph $DS(P_5)$ is shown in *Figure 7*.

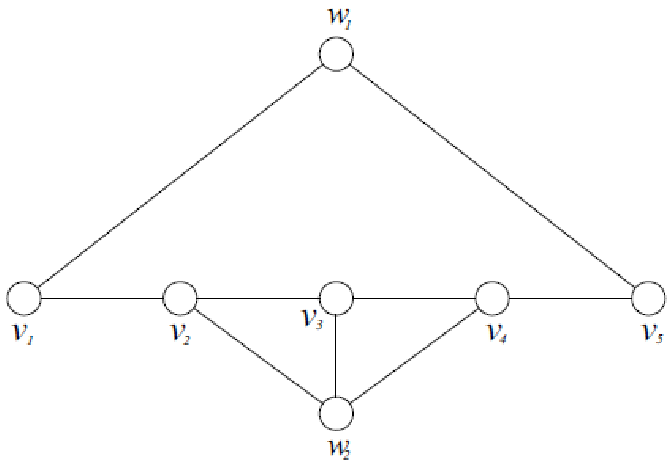

Figure 7: The graph $DS(P_5)$.

In any labeling pattern we get at most 3 edges having label 1. Therefore, $e_f(1) \leq 3$. So, $e_f(0) \geq 9 - 3 = 6$. Thus, $|e_f(0) - e_f(1)| \geq 3$.
Hence, $DS(P_5)$ is not a cube divisor cordial graph.

Subcase 3: $n \geq 6$.

Let p be the largest prime number such that $p \leq n + 2$.
Define
$f(w_1) = 8$, $f(w_2) = 1$.
$f(v_1) = 2$, $f(v_n) = p$.
Label the remaining vertices $v_2, v_3, \ldots, v_{n-1}$ successively from the set $\{3, 4, 5, 6, 7, 9, 10, 11, \ldots, p-1, p+1, \ldots, n+1, n+2\}$.
In view of above defined labeling pattern we have $e_f(0) = n$ and $e_f(1) = n - 1$. Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $DS(P_n)$ is a cube divisor cordial graph if and only if $n \neq 5$.

Illustration 2.8. Cube divisor cordial labeling of the graph $DS(P_9)$ is shown in *Figure 8*.

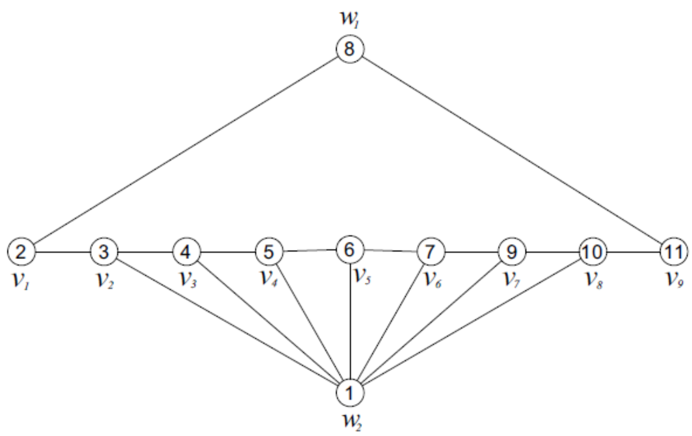

Figure 8: Cube divisor cordial labeling of $DS(P_9)$.

Concluding Remarks

As all the graphs do not admit cube divisor cordial labeling it is very interesting and challenging as well to investigate cube divisor cordial labeling for the graph or graph families which admit cube divisor cordial labeling. Here it has been proved that barycentric subdivision of the star $K_{1,n}$, switching of a vertex in cycle C_n, degree splitting graphs of $B_{n,n}$ and $P_n; n \neq 5$ are cube divisor cordial graphs.

References

[1] D. M. Burton, *Elementary Number Theory*, Brown Publishers, Second Edition, (1990).

[2] J A Gallian, A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, 17(2014) #DS6.

[3] J Gross and J Yellen, Handbook of graph theory, *CRC Press*, (2004).

[4] K. K. Kanani, M. I. Bosmia, On cube divisor cordial graphs, *International Journal of Mathematics and Computer Applications Research*, 5(4) (2015) 117-128.

[5] P. Lawrence Rozario Raj, R. Valli, Some new families of divisor cordial graphs *International Journal of Mathematics Trends and Technology*, 7(2) (2014).

[6] S. Murugesan, D. Jayaraman, J. Shiama, Square divisor cordial graphs, *International Journal of Computer Applications*, 64(22) (2013).

[7] P. Selvaraju, P. Balaganesan, J. Renuka and V. Balaj, Degree spliting graph on graceful, felicitous and elegant labeling, *International journal of Mathematical combinatorics*, 2(2012) 96-102.

[8] S. K. Vaidya, N. H. Shah, On square divisor cordial graphs, *Journal of Scientific Research*, 6(3) (2014) 445-455.

[9] S. K. Vaidya, N. H. Shah, Further results on divisor cordial labeling, *Annals of Pure and Applied Mathematics*, 4(2) (2013) 150-159.

[10] R.Varatharajan, S.Navanaeethakrishnan, K.Nagarajan, Divisor cordial graphs, *International Journal of Mathematical Combinatorics*, 4(2011) 15-25.