



# Model Developed for Single and Coupled Microstripline Structure Useful for the Study of Reflection & Transmission Co-efficient and Losses in Microstripline

Surya Deo Choudhary

Department of Electronics and Communication Engineering, NIET Greater Noida, India.

## ARTICLE INFO

### Article history:

Received: 30 April 2015;

Received in revised form:

18 November 2015;

Accepted: 23 November 2015;

### Keywords

Microstripline,  
Coupler, Structure, TEM,  
Quasi-TEM, Non-TEM mode,  
Phase Velocity,  
Even & Odd-modes,  
Conformal Transformation.

## ABSTRACT

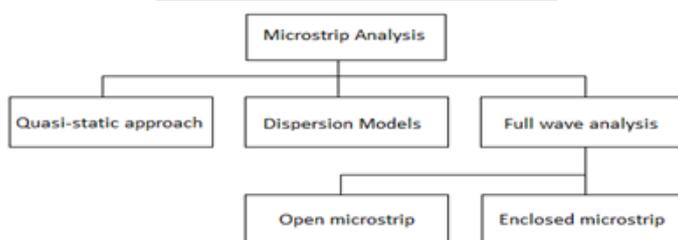
This paper presents the model which is developed for single and coupled microstripline structure for both the even and odd-mode of propagation of waves. Most important and common models are: Conformal transformation model, coupled analysis model, hybrid analysis model, numerical analysis by relaxation model, even and odd-mode models, which is the outcome of the coupled line model. H.A. Wheeler derived the characteristic impedance and propagation parameters. Numerical methods are adopted by Stenelhalfer and Sylvester based on pure TEM-mode, Quasi-TEM mode and Non-TEM mode.

© 2015 Elixir All rights reserved.

## Introduction

For the analysis and synthesis of single and coupled microstripline several investigators have developed various models based on pure TEM-mode, Quasi-TEM mode and Non-TEM mode. Microstripline has been analyzed by many investigators. Most of the initial work was based on the pure TEM or Quasi-TEM analysis. Based on this approach Wheeler derived the characteristic impedance and propagation parameters. Numerical methods adopted by Stenelhalfer and Sylvester have yielded most accurate results. Non-TEM analysis for determining dispersion in microstrip has been adopted by many investigators. Mitra et al. Thomas G. Bryant and J.A. Weiss used hybrid modes and numerical methods, while Getsinger and E.J. Delinger have given results based on simplified circuit models.

Various methods of microstrip analysis



## Methods of Microstrip Analysis

For both the single and coupled line methods of analysis may be divided into three groups basically. First is quasi static method. This analysis is adequate for designing circuits at frequency below X-band (8 GHz), where the strip width and height of the substrates are not longer than the wavelength in the substrate. Second is called dispersion method in which the deviation from the TEM-mode is accounted for quasi empirically. The third group is hybrid method; the account is to

be taken for the modes of propagation. Most important and common models are:

- Conformal transformation model
- Coupled analysis model
- Hybrid analysis model
- Numerical analysis by relaxation model
- Even and odd-mode models, which is the outcome of the coupled line model.

### Conformal Transformation Model

In case of lower frequency range wave propagation along the microstripline has been considered to be a pure TEM-mode by H.A. Wheeler. The characteristic impedance is given by:

$$Z_p = 1 / V_p C$$

Where,  $V_p$  = phase velocity of the wave

$C$  = capacitance per unit length of the line

By the use of set of conformal transformation the microstrip geometry is first converted into parallel plate geometry. For calculating capacitance real and imaginary parts of an analytic function of a complex variable satisfy the Laplace equation. In electrostatics the electric potential  $V(x,y)$  and the flux  $\Phi(x,y)$  of the displacement vector  $D (= \epsilon E)$  also have the same properties.

$$\nabla^2 V(x, y) = 0 \text{ and } \nabla^2 \Phi(x, y) = 0$$

$$-\partial v / \partial x = \partial \phi / \partial y \text{ and } -\partial v / \partial y = \partial \phi / \partial x$$

### Transformation Used In Single Microstripline Analysis

Conformal transformation suited for analyzing microstrip geometry had been proposed by H.A. Wheeler and the parallel strip geometry has analyzed by using Schwartz-Christoffel transformation defined by:

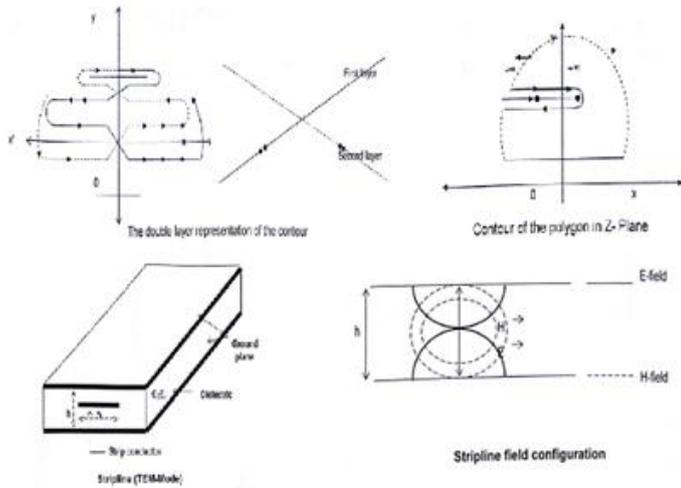
$$dZ/dt = f'(t) = A/(t-r_1)_{a1} (t-r_2)_{a2} \dots (t-r_k)_{ak}$$

Where  $Z = (x + jy)$

$t = (r + jw)$  are complex variables.

A is complex constant and  $r_1, r_2, \dots$  etc are real finite numbers.

Microstrip geometry can be obtained by inserting a plane of symmetry half way between the two strips and effect of the dielectric substrate has been also discussed by Wheeler. Figure shows the electric and magnetic field configuration.



Using different methods Wheeler has obtained the following transformation for the microstrip geometry:

Z-plane to t-plane transformation

$$Z = JP + t - 2 \tanh^{-1}(t/v)$$

t-plane to W-plane transformation

$$t = V \tanh(w/z)$$

**Characteristic Impedance**

The line capacitance for one quarter of a profile is equal to that for complete the profile and may be expressed in terms of M as:

$$(C / \epsilon_0) (w / 2h) = (n + 1 + \ln [2\pi ((m/\pi) + 0.94) ]/n)$$

Where  $n = \pi w / 2d$

Hence the characteristic impedance  $Z_0$  of the parallel stripline may be written as:

$$Z_0 / (n2h / w) = m / (m + 1 + \ln(2\pi(m/\pi) - 0.94))$$

Where  $\mu =$  intrinsic impedance for free space =  $377 \Omega$

The impedance of wide strip ( $w \gg d$ ) is given as:

$$Z_0 = ((n/2\pi)(\sqrt{2}/(\epsilon_r) + 1) [\ln(8h/w) + ((w/2h)^2)8 - ((\epsilon_r - 1)/(\epsilon_r + 1)) < \ln(\pi/2) + (1/(\epsilon_r) \ln(4/\pi^2))$$

For the narrow strip case ( $w \ll d$ ) the impedance is:

$$Z_0 = (n/\sqrt{\epsilon_r})(h/w)[1 + (h/\pi w) (2\ln 4 + (1 + (1/6)r) \ln(\pi e((w/2h) + 0.94)/2 + ((1/\epsilon_r) - (1/\epsilon_r)) \ln(e\pi^2/10))]^{-1}$$

**Effective Dielectric Constant**

The effective dielectric constant is a weighted mean of the dielectric constants of the two materials. If two dielectric materials in the capacitor are replaced by a material with a dielectric constant equal to the effective dielectric constant, the values of the capacitance remains unchanged. There are two cases of mixed dielectrics:

> The dielectric boundaries are located along the flux lines, and the configuration may be considered as two capacities with different dielectric in parallel.

> The dielectric boundaries are along the potential contours and the configuration may be considered as two capacitors are in series.

For capacitors that are partially filled with a dielectric and remaining parts filled with air is that of effective filling fraction and related with effective dielectric constant as:

$$\epsilon_{\text{reff}} = (1 - q) + q \epsilon_r - 1 + (\epsilon_r - 1)$$

In case of parallel combination q is identical with the actual filling fraction and in general the volume occupied by dielectric can be divided into an equivalent parallel volume  $V_p$  and an equivalent series volume  $V_s$ .  $V_d$  is the volume occupied by the dielectric out of a total volume  $V_1$  the actual filling fraction q is given as:

$$q = V_d/V$$

The parallel part of the filling fraction is

$$q'' = (V_d - V_s)/V$$

$$q' = q' - V_s/V$$

For small values of  $V_s$  series volume is only  $1/\epsilon_r$  times as effective as  $V_p$ . The effective filling fraction is therefore given as:

$$q = q'' + (q - q'')/\epsilon_r$$

$$q = q'/\epsilon_r + [1 - (1/\epsilon_r)] q'$$

**Model for Microstripline Coupler**

In addition to K.C.Gupta, S.K.Kaul, Bhat and Bhartia, P.Selvester, Karage and Haddad have developed models for coupled microstripline transmission systems. The method of analysis is taken into account the coupling between quasi TEM-mode along the microstripline and  $TM_{10}$  surface wave mode on the dielectric substrate with the metallization on the bottom surface reported by Hartwig et al and Jan et al. They assumed that a strong coupling exists between the two modes. At upper cut-off frequency ( $f_U$ ) the phase velocity of the quasi TEM-mode along the microstripline is equal to the phase velocity of  $TM_{10}$  surface on the substrate. At frequency below  $f_L$ , the effective dielectric constant for the coupled modes is obtained in terms of effective dielectric constant of the uncoupled modes by using the coupled mode theory.

$$\epsilon_{\text{reff} 1,2} = [(\sqrt{\epsilon_{\text{reff} 0}} + \sqrt{\epsilon_{\text{reff} TM}})/2 \pm \sqrt{(\epsilon_r 1,2 \epsilon_r 2,1) + (\sqrt{\epsilon_{\text{reff} 0}} - \sqrt{\epsilon_{\text{reff} TM})^2/4}]^2$$

Where,

$\epsilon_r 1,2 / \epsilon_m = \epsilon_{2,1} / \epsilon_m =$  Coupling Coefficient between the modes

And  $\epsilon_m = (\sqrt{\epsilon_{\text{reff} 0}} - \epsilon_{\text{reff} TM})$

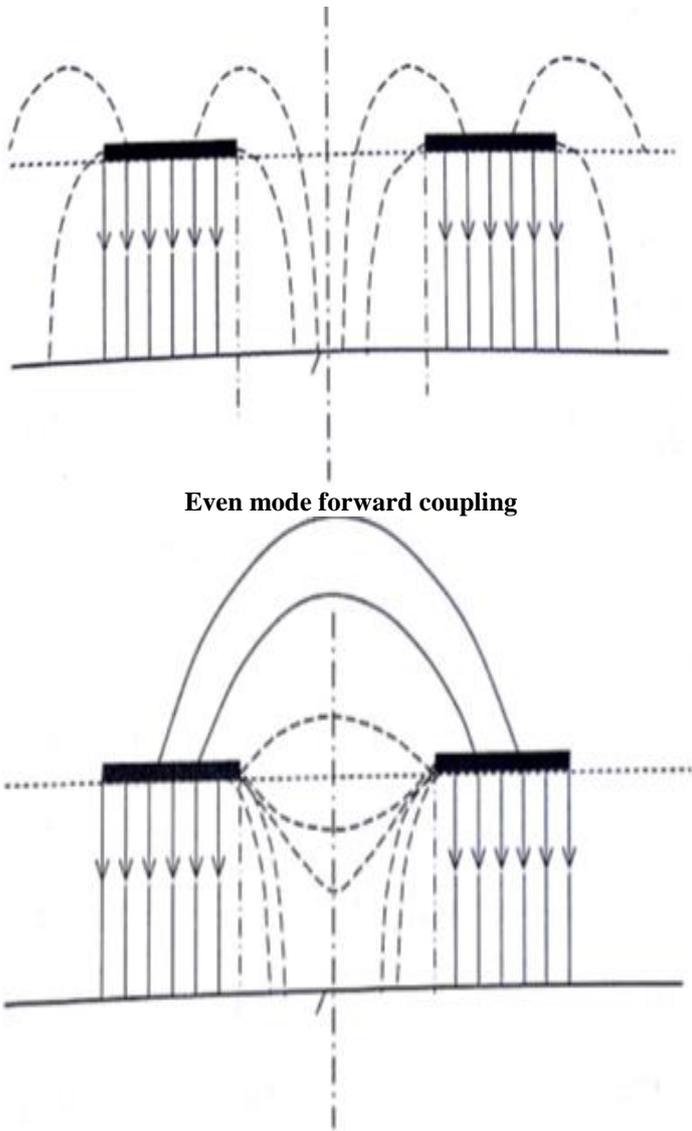
Coupling coefficient is given as,

$$\epsilon_{1,2}^2 = 0.22(\sqrt{\epsilon_r - 1})^2(w/h)^{3/4}(f/fl)^{4/3}$$

This model may be used up to a frequency of 0.5 GHz.

**Model for the Analysis of Microstripline Coupler for Even and Odd-Modes**

Because of the coupling of electromagnetic fields a pair of coupled line can support two different modes of propagation. The velocity of propagation of these two modes is equal when the lines are embedded in a homogeneous dielectric medium. The effective dielectric constants and the phase velocities are not equal for these modes. This non-synchronous feature deteriorates the performance of the circuits. The even and odd mode method is the most convenient method to describe the behavior of symmetrical coupled lines. Here phase velocity along a coupled pair of lines is expressed in terms of two modes corresponding to an even and odd symmetry about a plane which can be replaced by a magnetic and electric wall for the purpose of analysis.



Even mode forward coupling

Odd mode reverse coupling

Here the dotted line indicates the symmetry plane. If equal in phase voltages are applied to the ports 1 and 2 a maximum voltage occurs along line of symmetry, hence the junction may be open circuited at the symmetry line without affecting the field distribution and a magnetic wall may be located at this plane. If two equal and opposite voltages are applied to port 1 and 2, the voltage at the line of symmetry will be zero. Hence a short circuit may be located in the symmetry plane without affecting the field distribution. For each mode the analysis reduces to that of a two port network. By superposition the sum of two modes is equivalent to a single voltage applied to port 1. The two coupled mode lines are terminated by input and output lines of characteristic impedance  $Z_0$ . For couples to be matched at all frequencies it is necessary that the input impedance  $Z_{in}$  at the port 1 be always equal to  $Z_0$ . Superposing these two modes the input impedance is expressed as:

$$Z_{in} = (E_{1o} + E_{1e}) / (I_{1o} + I_{1e})$$

Where,  $I_{1o} = (E/Z_0) + Z_{1o}$   
 $I_{1e} = (E/Z_0) + Z_{1e}$   
 $E_{1o} = (EZ_{1o}/Z_0) + Z_{1o}$   
 $E_{1e} = (EZ_{1e}/Z_0) + Z_{1e}$

Where  $Z_{1e}$  and  $Z_{1o}$  are the input impedances for the coupled lines under the even and odd-modes excitation respectively. Using transmission line equation it can be shown that:

$$Z_{1e} = Z_{oe} (Z_o + JZ_{oe}\tan\theta_e) / (Z_{oe} + JZ_{oe}\tan\theta_e)$$

$$Z_{1o} = Z_{oo} (Z_{ov} + JZ_{oo}\tan\theta_o) / (Z_{oo} + JZ_{oo}\tan\theta_o)$$

Where  $Z_{oe}$  and  $Z_{oo}$  are even and odd-mode characteristic impedances of the coupled lines respectively.

$\Theta_e$  and  $\Theta_o$  are the electrical length of the micro strip. For even and odd modes:

$$\Theta_e = 2\pi fL/V_{pe}$$

$$\Theta_o = 2\pi fL/V_{po}$$

Where  $L$  is the physical length of the coupled lines and  $V_{pe}$  and  $V_{po}$  are the phase velocities for the two modes.

If  $E_1, E_2, E_3$  and  $E_4$  are the voltages appearing at the four ports and  $r_e$  and  $r_o$  and  $T_e$  and  $T_o$  are reflection and transmission coefficients. Then,

$$E_1 = E (r_e + r_o)/2$$

$$E_2 = E (r_e - r_o)/2$$

$$E_3 = E (T_e - T_o)/2$$

$$E_4 = E (T_e + T_o)/2$$

The transmission and reflection coefficients are expressed in terms of transfer matrices, relates input voltage and current to the output voltage and current by the relation.

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} \cos\theta_1 & JZ_{o1} \sin\theta_1 \\ (J/Z_{o1})\sin\theta_1 & \cos\theta_1 \end{vmatrix}$$

Using matched condition it can be shown that,

$$\Gamma_e = -\Gamma_o = [\{J(Z_{oe}/Z_{oo})^{1/2} - (Z_{oo}/Z_{oe})^{1/2}\} \sin\theta] / \Sigma$$

And

$$\Gamma_e = \Gamma_o = 2 / \Sigma$$

$$\Sigma = 2\cos\theta + J[\sqrt{(Z_{oe}/Z_{oo})} + \sqrt{(Z_{oo}/Z_{oe})}]\sin\theta$$

Then from above equations, we get

$$E_1 = E_3 = 0$$

$$E_2/E = Jc\sin\theta / \{\sqrt{(1-c^2)}\} \cos\theta + J\sin\theta$$

$$E_4/E = \sqrt{(1-c^2)} / \{\sqrt{(1-c^2)}\} \cos\theta + J\sin\theta$$

Where,

$$C = \{(Z_{oe}/Z_{oo}) - 1\} / \{(Z_{oe}/Z_{oo}) + 1\}$$

= Coupling Coefficient

The coupling is maximum when  $\Theta = \pi/2$ , that is the section of the micro strip is one quarter wavelength long.  $C$  is the non-band voltage coupling.  $E_1$  and  $E_2$  are out of phase of  $90^\circ$  at all frequencies.

### Coupled Mode Approach

In this approach the wave propagation is expressed in terms of the modes of propagation on individual uncoupled lines modified by the coupling due to mutual capacitances and inductances. This is applicable to symmetric and asymmetric coupled lines. In the mode of analysis the voltages on the line is written in terms of the currents on both lines and the self and mutual impedances. Similarly the current is written in terms of voltages and admittance. Elimination of currents or voltages yields the coupled equation. The solution of these coupled equations determines the propagation constants for the two modes.

### Discussion and Conclusion

The above study reveals that a model is developed for the study of single and coupled microstripline structures in both even and odd-modes. Numerical methods adopted by the Stenelhalfer and Sylvester works and applied satisfactorily. The topic discussed above provides useful guidelines for the design of different microstripline structures such as: Coupler, Directional Coupler, Isolator, Circulators etc. These results are also very useful for the study of reflection and transmission coefficient of the microstripline coupler in case of both even and odd-modes of propagation.

### References

- [1] P.H. Ladbrooke et al, Coupling Errors in cavity resonance measurements on MIC dielectrics, *IEEE trans. MTT* vol.21, page 560-562 (1973).
- [2] J.S. Hornbaski and Gopinath, Fourier analysis of a dielectric loaded wave guide with a microstriplines, *Electron letter*, vol.5 june 12, 1969, pp 265-267.
- [3] H.J. Carlin, A simplified circuit model for microstrip, *IEEE Trans. MTT*. Vol.21, page 589-591 (1973).
- [4] I.J. Bahl and R. Garg, Simple and Accurate formulae for microstrip with finite strip thickness. *Proc. IEEE* vol.65, page 1611-1612 (1977).
- [5] A.C. Cangellaris, The importance of skin effect in microstrip lines at high frequencies, in *IEEE MTT. Int. microwave symp. Dig*, New York, NY, May 25-27, 1988, pp.197-198.
- [6] P.H. Ladbrooke et al, Comments on a quick accurate method to measure the dielectric constant of MIC substrate, *IEEE Trans. MTT* vol. 21, page 570-571 (1973).
- [7] E.V. Loewenstein et al, Optical constants for Infrared materials and Crystalline Solids, *applied optics* vol.12, pp 398-406 (1973).
- [8] P.Grivet, The physics of transmission line at high and very high frequencies vol.1, academic press, page 47 (1970).
- [9] J.B. Knorr and A. Tufekcioglu, Special domain calculation of microstrip characteristics impedance, *IEEE Trans. MTT*. Vol.23,page 725-728 (1975).
- [10] H.E. Stinehlfer, An accurate calculation of uniform microstrip transmission line, *IEEE Trans. MTT*, vol.16, page 439-447 (1968).
- [11] J.Q. Howell, A quick accurate method to measure dielectric constants MIC substrates, *IEEE Trans. MTT*. Vol.21, page 142-143 (1993).
- [12] Marconi Instrumentation, vol.4, no.5, October 1974.
- [14] H.L.Sah, "Investigation of the characteristics of coupled striplines", 1996.
- [15] T.C. Edwards "Foundation for microstrip circuit design", John Wiley & Sons.
- [16] H.L.Sah "Investigation of characteristics of microstripline directional coupler's. *Institutes of Engineers(India)* vol. 80, 2000.
- [17] Sah, H.L. R.R. Kumar & K.B. Singh "A new horizon of communication using fibre technology",NSOE-03, April 2003, Meerut, India.
- [18] H.A.Wheeler, Transmission line properties of parallel strip separated by dielectric sheet, *IEEE.Tr.MTT-13*,page 172-185, 1965.
- [19] S.B.Cohn"Slotline on dielectric substrate" *IEEE Trans MTT-17*,1969, page 168-178.
- [20] R.P.Owens, J.E.Aitken and T.C.Edwards "Quasi static characteristics of microstrip on an Anisotropic Sapphire Substrate", *IEEE Trans. MTT*. Vol.24 No.8, page 499-505, 1976.
- [21] H.E.Green, The numerical solution of some important transmission line problems, *IEEE Trans, MTT* Vol.13, page 676-692,1965.
- [22]T.E.Van Deventer, L.P.B.Katehi and A.C.Cangellaris "An integral equation method for the evaluation of conductor and dielectric losses in high frequency interconnects, *IEEE rans. Microwave theory tech*,vol.37,pp 1964-1972, Dec 1989.
- [23] H.J.Carlin "A simplified circuit model for microstrip", *IEEE trans, MTT* vol.21,page 589-591, 1973.
- [24] I.J.Bahl and R.Garg "Simple and accurate formulae for microstrip with finite strip thickness *proc IEEE* vol 65, page 1611-1612, 1977.
- [25] P.H.Ladbrooke et al "Comment on a quick accurate method to measure the dielectric constant of MIC substrate *IEEE trans MTT* vol 21, page 570-571, 1973.