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Natural Frequency of Viso-Elastic Square Plate with Thickness Variation

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ABSTRACT

Visco- Elastic Plates are being increasingly used in the aeronautical and aerospace industry as well as in other fields of modern technology. To use them a good understanding of their structural and dynamical behavior is needed. In the modern technology, the plates of variable thickness are widely used in engineering applications. The present work is to develop for the use of research workers in space technology, mechanical sciences and nuclear energy where certain components of the structure have to operate under the elevated temperature. The aim of present paper is to study the bi-linearly thermal effect on vibration of visco-elastic square plate whose thickness varies parabolically in x-direction. A frequency equation is derived by using Rayleigh-Ritz technique. Two modes of frequency are calculated by the latest computational software for the various values of taper parameter and thermal gradient.

Introduction

During the past four decades, vibration of plates has become an important subject in engineering applications. There are several papers about plate vibrations in open technical literature. Square plates have many engineering applications. These are commonly found in spacecrafts, missiles, land base vehicles, off-shore platforms, and underwater acoustic transducers.

In recent years, interest in the effect of temperature on solid bodies has highly increased because of rapid development in space technology, high speed atmospheric flights and in nuclear energy applications. Further, in mechanical systems where certain parts of machine have to operate under elevated temperature, its effect is far from negligible. The reason for this is that during heating up period of structures exposed to high intensity heat fluxes, the material properties under go significant vibrations. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Plates with thickness variability are of great importance in a wide variety of engineering applications.

Leissa [1] gave different models on the vibration of plates. Recently, Gupta and Lalit kumar [2] studied Thermal effects on the vibration of non-homogeneous visco-elastic rectangular plate of linearly varying thickness. Gupta and Anupam Khanna [3] discussed thermal effect on vibrations of parallelogram plate of linearly varying thickness. A. Khanna, A. Kumar and M. Bhatia [4] recently presented an analysis on two dimensional thermal effect with two dimensional varying thickness of visco-elastic square plate. The effect of thermal gradient on the frequencies of an orthotropic plate of linearly varying thickness has been discussed by Tomar and Gupta [5]. Vibration of rectangular plates by the Ritz method was given by Young [6]. Tomar and Gupta [7] discussed the effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions.

An analysis is presented on the vibration of clamped visco-elastic rectangular plate with parabolic thickness variations by Gupta and Anupam Khanna [8]. Singh and Saxena [9] presented an analysis on transverse vibration of rectangular plate with bi-directional thickness variation. Gupta and Kaur [10] studied the effect of thermal gradient on free vibration of visco-elastic rectangular plates with linearly thickness variation in both directions. Vibration behavior and simplified design of thick rectangular plates with variable thickness considered by Sasajima, Kakudate and Narita [11]. An interesting analysis of the free vibration of rectangular plate is given by Leissa [12]. Gupta and Khanna [13-14] discussed the free vibration of clamped visco-elastic rectangular plate having bi-direction thickness variations.

It is assumed that the plate is clamped on all the four edges and its temperature varies linearly in both the directions. Due to temperature variation, we assume that non homogeneity occurs in Modulus of Elasticity. For various numerical values of thermal gradient and taper constants; frequency for the first two modes of vibration are calculated with the help of latest software.

Equation of Motion

The governing differential equation of transverse motion of a visco-elastic plate of variable thickness in Cartesian co-ordinates [1]:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \qquad (1)$$

The expression for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\tilde{D}D_1\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\tilde{D}D_1\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\tilde{D}D_1(1-\nu)\frac{\partial^2 w}{\partial y \partial x} \end{aligned}\right\} \qquad (2)$$

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where $\tilde{D}$ is visco-elastic operator.

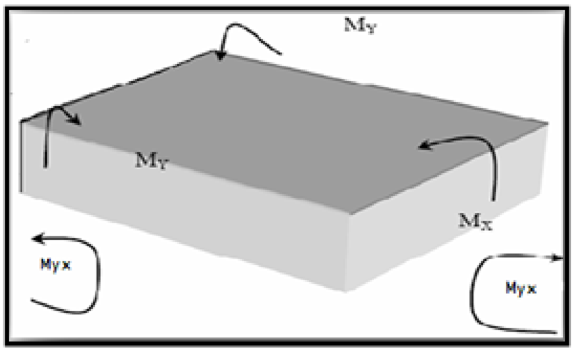


Fig I - Square plate with bending moments

On substitution the values M_x, M_y and M_{yx} from equation (2) in (1) and taking w, as a product of two function, equal to w(x,y,t)=W(x,y)T(t), equation (1) become:

$$[D_1(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4})+2\frac{\partial D_1}{\partial x}(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2})+2\frac{\partial D_1}{\partial y}(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y})$$

$$+\frac{\partial^2 D_1}{\partial x^2}(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2})+\frac{\partial^2 D_1}{\partial y^2}(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2})+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}]/\rho hW=-\frac{\ddot{T}}{\tilde{D}T} \quad (3)$$

Here dot denote differentiation with respect to t. taking both sides of equation (3) are equal to a constant p^2 (square of frequency), we have

$$[D_1\left(W,_{xxxx}+2W,_{xxyy}+W,_{yyyy}\right)+2D_{1},_x\left(W,_{xxx}+W,_{xyy}\right)+2D_{1,y}\left(W,_{yyy}+W,_{yxx}\right)+$$

$$D_{1,xx}(W,_{xx}+\nu W,_{yy})+D_{1,yy}(W,_{yy}+\nu W,_{xx})+2(1-\nu)D_{1,xy}W,_{xy}]-\rho hp^2W=0 \quad (4)$$

Eq. (4) is a differential equation of transverse motion for non-homogeneous plate of variable thickness. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1=Eh^3/12(1-\nu^2) \quad (5)$$

and corresponding two-term deflection function is taken as [5]

$$W=[(x/a)(y/a)(1-x/a)(1-y/a)]^2[A_1+A_2(x/a)(y/a)(1-x/a)(1-y/a)] \quad (6)$$

where A_1 and A_2 are constants to satisfy boundary conditions.

Assuming that the square plate of engineering material has a steady two dimensional linear temperature distribution i.e.

$$\tau=\tau_0(1-x/a)(1-y/a) \quad (7)$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this

$$E=E_0(1-\gamma\tau) \quad (8)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ. The modulus variation (5) become

$$E=E_0[1-\alpha(1-x/a)(1-y/a)] \quad (9)$$

where $\alpha=\gamma\tau_0(0\le\alpha<1)$, thermal gradient.

It is assumed that thickness also varies parabolic in x- directions as shown below:

$$h=h_0(1+\beta_1x^2/a^2) \quad (10)$$

where β_1 is taper parameters in x- directions respectively and $h=h_0$ at x=y=0.

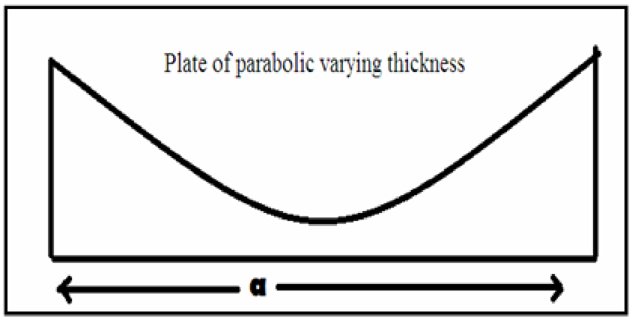


Fig II:- Plate with parabolic varying thickness

Put the value of E & h from equation (9) & (10) in the equation (5), one obtain

$$D_1=[E_0[1-\alpha(1-x/a)(1-y/a)]h_0^3(1+\beta_1x^2/a^2)^3]/12(1-\nu^2) \quad (11)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(V^*-T^*)=0 \quad (12)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{array}{ll} W=W,_x=0, & x=0,a \\ W=W,_y=0, & y=0,a \end{array}\right\} \quad (13)$$

Now assuming the non-dimensional variables as

$$X=x/a,\ Y=y/a,\ \overline{W}=W/a,\ \overline{h}=h/a \quad (14)$$

The kinetic energy T* and strain energy V* are [2]

$$T^*=(1/2)\rho p^2\overline{h_0}a^5\int_0^1\int_0^1[(1+\beta_1X^2)\overline{W^2}]dYdX \quad (15)$$

And

$$V^*=Q\int_0^1\int_0^1[1-\alpha(1-X)(1-Y)](1+\beta_1X^2)^3\{(\overline{W},_{XX})^2+(\overline{W},_{YY})^2+2\nu\overline{W},_{XX}\overline{W},_{YY}+2(1-\nu)(\overline{W},_{XY})^2\}dYdX \quad (16)$$

where, $Q=E_0h_0^3a^3/24(1-\nu^2)$

Using equations (15) & (16) in equation (12), one get

$$(V^{**}-\lambda^2T^{**})=0 \quad (17)$$

where,

$$V^{**}=\int_0^1\int_0^1[1-\alpha(1-X)(1-Y)](1+\beta_1X^2)^3\{(\overline{W},_{XX})^2+(\overline{W},_{YY})^2+2\nu\overline{W},_{XX}\overline{W},_{YY}+2(1-\nu)(\overline{W},_{XY})^2\}dYdX \quad (18)$$

and

$$T^{**}=\int_0^1\int_0^1[(1+\beta_1X^2)\overline{W^2}]dYdX \quad (19)$$

Here, $\lambda^2=12\rho p^2(1-\nu^2)a^2/E_0h_0^2$ is a frequency parameter.

Equation (19) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**}-\lambda^2T^{**})/\partial A_n=0 \quad \text{, n=1, 2} \quad (20)$$

On simplifying (20), we gets

$$bn_1A_1+bn_2A_2=0 \quad \text{, n=1, 2} \quad (21)$$

where bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (21) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{vmatrix} = 0 \qquad (22)$$

With the help of equation (22), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1 (Mode1) & λ_2 (Mode2) for different values of taper constant and thermal gradient for a clamped plate.

Result and Discussion

All calculations are carried out with the help of latest Matrix Laboratory computer software. Computation has been done for frequency of visco-elastic square plate for different values of taper constant β_1, thermal gradient α, at different points for first two modes of vibrations.

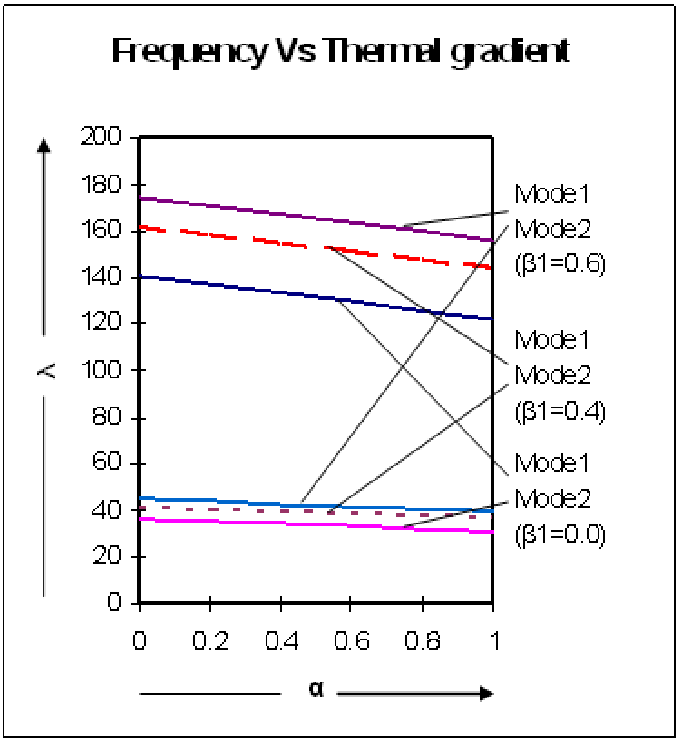
Table 1:- It is clearly seen that value of frequency decreases as thermal gradient increases from 0.0 to 1.0 for β_1=0, β_1=0.4 and β_1=0.6 for both modes of vibrations. Also, note that frequency increases fast as taper parameters (β_1) increase from 0.0 to 0.4 and 0.6 respectively.

Table 2:- Value of frequency increases with the increment in taper parameter β_1 for following cases

i) α= 0.2, iii) α =0.4 ii) α= 0.8

Interesting to note that frequency increases with the increment in β_1 from 0.0 to 1.0. Also, value in case (iii)and (ii) are more than from case (i).

Graph 1:- Frequency Vs Thermal gradient



Conclusion

So, main aim for our research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology.

Graph 2:-Frequency Vs Taper constant

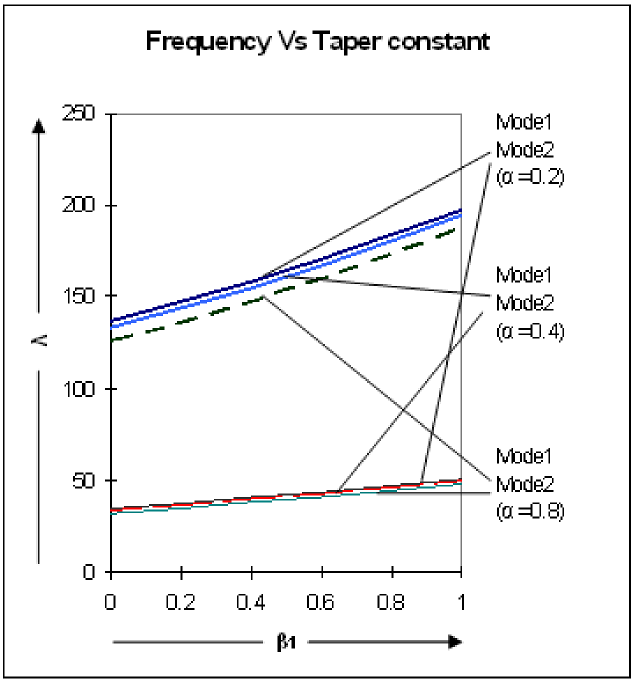


Table 1:- Frequency Vs Thermal gradient

A	β_1=0.0		β_1=0.4		β_1=0.6	
	Mode1	**Mode2**	**Mode1**	**Mode2**	**Mode1**	**Mode2**
0	140.88	35.99	162.18	41.57	174.40	44.71
0.2	137.32	35.08	158.74	40.68	171.01	43.82
0.4	133.65	34.15	155.22	39.77	167.54	42.92
0.6	129.88	33.19	151.62	38.84	164.00	41.98
0.8	126.01	32.19	147.92	37.88	160.39	41.03
1	122.01	31.17	144.15	36.90	156.05	40.07

Table 2:-Frequency Vs Taper constant

β_1	α =0.2		α =0.4		α =0.8	
	Mode1	Mode2	Mode1	Mode2	Mode1	Mode2
0	137.32	35.08	133.65	34.15	126.01	32.19
0.2	147.45	37.75	143.86	36.84	136.42	34.92
0.4	158.74	40.68	155.23	39.77	147.93	37.88
0.6	171.01	43.83	167.54	42.92	160.39	41.03
0.8	184.11	47.14	180.68	46.22	173.63	44.33
1	197.91	50.57	194.51	49.65	187.52	47.72

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