Available online at www.elixirpublishers.com (Elixir International Journal)

Statistics

Elixir Statistics 88 (2015) 36362-36366

ISSN: 2229-712X

# Bayesian Analysis of Shape Parameter of Frechet distribution using Non-Informative Prior

Wajiha Nasir[1,*], Maria Qudsia[2] and Muhammad Aslam[1]

[1]Department of Basic Sciences, Riphah International University Islamabad, Pakistan.

[2]Department of Statistics, University of Sargodha. Pakistan.

**ARTICLE INFO**

**Article history:**

Received: 26 September 2015;
Received in revised form:
06 November 2015;
Accepted: 11 November 2015;

**Keywords**

Non-informative,
Uniform,
Jeffreys,
Loss Functions,
Simulation,
Bayes posterior risk.

**ABSTRACT**

In this paper we work on Frechet distribution with Bayesian paradigm. Posterior distribution is obtained by using Uniform, Jeffreys and generalization of non-informative priors. We use the quadrature numerical integration to solve the posterior distribution. Bayes estimator and their risk have been obtaining four loss functions. The performances of Bayes estimators are compared by using Monte Carlo simulation study.



## Introduction

Extreme value distribution are creating their importance in modeling extreme events such as floods, stock exchange, wind speed etc. There are three types of extreme value distribution and Frechet distribution is of second type of extreme value distribution. Gumbel (1965) estimates the parameters of Frechet distribution having three parameters by using method of moments and maximum likelihood estimation. Hood et al (1990) has estimates parameters for Frechet distribution by using maximum likelihood estimation for doubly censored data. Engelund and Rackwitz (1992) has determined the predictive distribution functions of extreme value distribution by using non-informative priors. Hood et al (1993) has estimated m-th maxima for Frechet distribution under different sampling techniques. Parameters of Frechet distribution has been estimated by maximum likelihood estimation under type-II censored data by Abbas and Tang (2013). Nasir and Aslam (2015) has estimated shape parameter of Frechet distribution by using informative priors via different loss functions.

The probability density function of Frechet distribution is

$$f\left(x;\alpha,\beta\right)=\frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1}e^{-\left(\frac{\beta}{x}\right)^{\alpha}},\text{where } \alpha,\beta>0$$

Where α is shape parameter and β is scale parameter.

We will work on the case for estimation of shape parameter by using non-informative priors.

## Posterior distribution

Posterior distribution is derived by using Uniform, Jeffreys and generalization of non-informative priors.

The uniform prior for shape parameter α is

$$p\left(\alpha\right)\propto 1,\text{ where } \alpha>0$$

The posterior distribution under uniform prior for parameter α for the data $x_1,x_2,\ldots,x_n$ is

$$p\left(\alpha\middle|x\right)=\frac{1}{k_1}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}$$

Where

$$k_1=\int_0^{\infty}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha$$

We will solve it numerically because it is not in close form.

The Jeffreys Prior for shape parameter is

$$p\left(\alpha\right)\propto\frac{1}{\alpha},\text{where } \alpha>0$$

The posterior distribution under Jeffreys prior for parameter α for the data $x_1,x_2,\ldots,x_n$ is

$$p\left(\alpha\middle|x\right)=\frac{1}{k_2}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}$$

$$k_2=\int_0^{\infty}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha$$

We will solve it numerically because it is not in close form.

The generalization of non-informative prior (GNIP) for shape parameter α is

$$p\left(\alpha\right)\propto\frac{1}{\alpha^{a}},\text{where } \alpha>0$$

The posterior distribution under generalization of non-informative prior for parameter α for the data $x_1,x_2,\ldots,x_n$ is

Tele:
E-mail addresses: wajiha.nasir19@gmail.com

$$p\left(\alpha|x\right)=\frac{1}{k_3}\alpha^{n-a}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}$$

Where

$$k_3=\int_0^\infty \alpha^{n-a}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha$$

We will solve it numerically assuming a=2 because it is not in close form.

Bayesian Estimation under different loss functions

We have used four loss functions for estimation. The Bayes estimators (BEs) and their Bayes posterior risks (PRs) are as follow

**Square error Loss function (SELF)**

The Bayes estimator and their cross ponding risks under SELF using uniform prior is

$$\alpha^*=\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha$$

$$\rho\left(\alpha^*\right)=\int_0^\infty \frac{1}{k_1}\alpha^{n+2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha-$$
$$=\left(\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha\right)^2$$

The BEs and PRs under Jeffreys prior for SELF is

$$\alpha^*=\int_0^\infty \frac{1}{k_2}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha$$

$$\rho\left(\alpha^*\right)=\int_0^\infty \frac{1}{k_2}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha-$$
$$=\left(\int_0^\infty \frac{1}{k_2}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha\right)^2$$

The BEs and PRs under GNIP under SELF is

$$\alpha^*=\int_0^\infty \frac{1}{k_3}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha$$

$$\rho\left(\alpha^*\right)=\int_0^\infty \frac{1}{k_1}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha-$$
$$=\left(\int_0^\infty \frac{1}{k_1}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha\right)^2$$

**Precautionary Loss Function (PLF)**

The BEs and PRs under PLF using uniform prior is

$$\alpha^*=\sqrt{\int_0^\infty \frac{1}{k_3}\alpha^{n+2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}$$

$$\rho\left(\alpha^*\right)=2*\left[\sqrt{\int_0^\infty \frac{1}{k_1}\alpha^{n+2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}-\left(\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha\right)^2\right]$$

The BEs and PRs under PLF using Jeffreys prior is

$$\alpha^*=\sqrt{\int_0^\infty \frac{1}{k_2}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}$$

$$\rho\left(\alpha^*\right)=2*\left[\sqrt{\int_0^\infty \frac{1}{k_2}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}-\left(\int_0^\infty \frac{1}{k_2}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha\right)^2\right]$$

The BEs and PRs under PLF using GNIP prior is

$$\alpha^*=\sqrt{\int_0^\infty \frac{1}{k_3}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}$$

$$\rho\left(\alpha^*\right)=2*\left[\sqrt{\int_0^\infty \frac{1}{k_3}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}-\left(\int_0^\infty \frac{1}{k_3}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha\right)^2\right]$$

**Simple Precautionary Loss Function (SPLF)**

The BEs and PRs under SPLF using uniform prior is

$$\alpha^*=\sqrt{\frac{\int_0^\infty \frac{\alpha^{n+1}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}{\int_0^\infty \frac{\alpha^{n-1}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}}$$

$$\rho\left(\alpha^*\right)=2*\left[\sqrt{\int_0^\infty \frac{1}{k_1}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}*\sqrt{\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}-1\right]$$

The BEs and PRs under SPLF using Jeffreys prior is

$$\alpha^*=\sqrt{\frac{\int_0^\infty \frac{1}{k_2}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}{\int_0^\infty \frac{1}{k_1}\alpha^{n-2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1}e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}}d\alpha}}$$

$$\rho\left(\alpha^*\right)=2*\left[\sqrt{\int_0^\infty \frac{1}{k_2}\alpha^{n-2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha} * \sqrt{\int_0^\infty \frac{1}{k_2}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha} -1\right]$$

The BEs and PRs under SPLF using GNIP prior is

$$\alpha^* = \sqrt{\frac{\int_0^\infty \frac{1}{k_3}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}{\int_0^\infty \frac{1}{k_3}\alpha^{n-3}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}}$$

$$\rho\left(\alpha^*\right)=2*\left[\sqrt{\int_0^\infty \frac{1}{k_2}\alpha^{n-1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha} * \sqrt{\int_0^\infty \frac{1}{k_2}\alpha^{n-3}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha} -1\right]$$

**DeGroot Loss Function (1970) (DeLF)**

The BEs and PRs under DeLF using uniform prior is

$$\alpha^* = \frac{\int_0^\infty \frac{\alpha^{n+2}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}{\int_0^\infty \frac{\alpha^{n+1}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}$$

$$\rho\left(\alpha^*\right)=\frac{\int_0^\infty \frac{1}{k_1}\alpha^{n+2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha-\left(\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha\right)^2}{\int_0^\infty \frac{1}{k_1}\alpha^{n+2}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}$$

The BEs and PRs under DeLF using Jeffreys prior is

$$\alpha^* = \frac{\int_0^\infty \frac{\alpha^{n+1}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}{\int_0^\infty \frac{\alpha^{n}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}$$

$$\rho\left(\alpha^*\right)=\frac{\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha-\left(\int_0^\infty \frac{1}{k_1}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha\right)^2}{\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}$$

The BEs and PRs under DeLF using GNIP prior is

$$\alpha^* = \frac{\int_0^\infty \frac{\alpha^{n}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}{\int_0^\infty \frac{\alpha^{n-1}}{k_1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}$$

$$\rho\left(\alpha^*\right)=\frac{\int_0^\infty \frac{1}{k_1}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha-\left(\int_0^\infty \frac{1}{k_1}\alpha^{n+1}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha\right)^2}{\int_0^\infty \frac{1}{k_1}\alpha^{n}\beta^{n\alpha}\prod_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\alpha+1} e^{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^\alpha} d\alpha}$$

**Simulation Study**

Simulation has many properties that whether the data are discrete or continuous. By simulation, Bayesian problems of updating estimates can be handled easily. By this process the results tend to be very clear. When the analysis solution of the problems may be difficult or impossible, simulation can provide an effective way to handle. So by taking different values of parameters along different sample size the properties of posterior distribution are verified through simulation study. Also take different values of parameter α = {2, 4, 6} for three cases the properties of posterior distribution are verified. This section presents simulation study of Bayes estimators and posterior risks.

**Table 1. Simulation result for PLF Assuming β =3**

| n | Prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.0831 (0.0372) | 6.2852 (0.1128) |
| | 50 | 2.0612 (0.0222) | 6.1781 (0.0666) |
| | 70 | 2.0315 (0.0157) | 6.1253 ( 0.0477) |
| | 100 | 2.0346 (0.0111) | 6.1240 ( 0.0334) |
| JP | 30 | 2.0599 (0.0382) | 6.1657 (0.1149) |
| | 50 | 2.0320 (0.0226) | 6.0972 (0.0673) |
| | 70 | 2.0221 (0.0159) | 6.0606 (0.0481) |
| | 100 | 2.0079 (0.0110) | 6.0339 (0.0331) |
| GNIP | 30 | 2.0027 (0.0796) | 6.0094 (0.7228) |
| | 50 | 1.9991 (0.0459) | 5.9753 (0.4143) |
| | 70 | 2.0006 (0.0326) | 6.0078 (0.2932) |
| | 100 | 1.9954 (0.0224) | 6.0878 (0.0054) |

**Table 2. Simulation Result for PLF Assuming β =5**

| n | Prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.0848 (0.0376) | 6.2770 (0.1130) |
| | 50 | 2.0650 (0.0224) | 6.1507 (0.0667) |
| | 70 | 2.0382 (0.0158) | 6.1004 (0.0475) |
| | 100 | 2.0273 (0.0110) | 6.0794 (0.0330) |
| JP | 30 | 2.0505 (0.0383) | 6.1500 (0.1138) |
| | 50 | 2.0305 (0.0224) | 6.1092 (0.0675) |
| | 70 | 2.0291 (0.0160) | 6.0484 (0.0478) |
| | 100 | 2.0142 (0.0111) | 6.0460 (0.0333) |
| GNIP | 30 | 2.1001 (0.0381) | 6.2587 (0.0380) |
| | 50 | 2.0578 (0.0209) | 6.1849 (0.0208) |
| | 70 | 2.0493 (0.0178) | 6.1513 (0.0178) |
| | 100 | 2.0317 (0.0154) | 6.0342 (0.0333) |

**Table 3. Simulation Result for SELF Assuming β =3**

| n | Prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.0908 (0.0373) | 6.2273 (0.1111) |
| | 50 | 2.0608 (0.0224) | 6.1642 (0.0672) |
| | 70 | 2.0455 (0.0158) | 6.1093 (0.0473) |
| | 100 | 2.0253 (0.0110) | 6.0899 (0.0331) |
| JP | 30 | 2.0502 (0.0816) | 6.0963 (0.7144) |
| | 50 | 2.0253 (0.0462) | 6.1064 (0.4245) |
| | 70 | 2.0215 (0.0326) | 6.0687 (0.2946) |
| | 100 | 2.0148 (0.0225) | 6.0331 (0.2024) |
| GNIP | 30 | 1.9930 (0.0796) | 6.0094 (0.7228) |
| | 50 | 2.0014 (0.0464) | 5.9753 (0.4143) |
| | 70 | 2.0064 (0.0328) | 6.0078 (0.2932) |
| | 100 | 1.9976 (0.0224) | 5.9999 (0.2028) |

**Table 4. Simulation Result for SELF Assuming β =5**

| n | prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.0997 (0.0376) | 6.2672 (0.1122) |
| | 50 | 2.0512 (0.0222) | 6.1762 (0.0669) |
| | 70 | 2.0467 (0.0159) | 6.1127 (0.0473) |
| | 100 | 2.0277 (0.0111) | 6.0840 (0.0330) |
| JP | 30 | 2.0509 ( 0.0811) | 6.1154 0.7223 |
| | 50 | 2.0185 ( 0.0460) | 6.0523 (0.4106) |
| | 70 | 2.0113 ( 0.0323) | 6.0320 (0.2920) |
| | 100 | 2.0179 ( 0.0226) | 6.0291 (0.2017) |
| GNIP | 30 | 1.9947 (0.0791) | 5.972 (0.7167) |
| | 50 | 2.0028 ( (0.0465)) | 5.9994 (0.4186) |
| | 70 | 1.9944 (0.0323) | 6.0135 (0.2962) |
| | 100 | 2.0083 (0.0226) | 6.0134 (0.2036) |

**Table 5. Simulation Result for DeLF Assuming β =3**

| n | Prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.1156 (0.0177) | 6.3414 (0.0177) |
| | 50 | 2.0687 (0.0108) | 6.2304 (0.0108) |
| | 70 | 2.0454 (0.0076) | 6.1351 (0.0077) |
| | 100 | 2.0314 (0.0054) | 6.1045 (0.0054) |
| JP | 30 | 2.0869 (0.0185) | 6.2685 (0.0186) |
| | 50 | 2.0419 (0.0110) | 6.0853 (0.0111) |
| | 70 | 2.0289 (0.0079) | 6.1223 (0.0078) |
| | 100 | 2.0195 (0.0054) | 6.0662 (0.0055) |
| GNIP | 30 | 2.0027 (0.0803) | 6.0094 (0.7228) |
| | 50 | 1.9991 (0.0459) | 5.9753 (0.4143) |
| | 70 | 2.0006 (0.0326) | 6.0078 (0.2932) |
| | 100 | 1.9954 (0.0224) | 6.0508 (0.0334) |

**Table 6. Simulation Result for DeLF Assuming β =5**

| n | prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.1277 (0.0178) | 6.3937 (0.0077) |
| | 50 | 2.0742 (0.0107) | 6.2219 (0.0107) |
| | 70 | 2.0506 (0.0077) | 6.1400 (0.0077) |
| | 100 | 2.0277 (0.0054) | 6.0937 (0.0054) |
| JP | 30 | 2.0868 (0.0184) | 6.2555 (0.0186) |
| | 50 | 2.0492 (0.0110) | 6.1396 (0.0110) |
| | 70 | 2.0353 (0.0079) | 6.1163 (0.0078) |
| | 100 | 2.0241 (0.0055) | 6.0629 (0.0055) |
| GNIP | 30 | 1.9987 (0.0799) | 6.0038 (0.7201) |
| | 50 | 2.0053 0.0469 | 5.9762 (0.4147) |
| | 70 | 2.0045 (0.0326) | 6.0112 (0.2943) |
| | 100 | 1.9941 (0.0223) | 5.9999 (0.2028) |

**Table 7. Simulation Result for SPLF Assuming β =3**

| n | Prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.0588 (0.0189) | 6.2336 (0.0186) |
| | 50 | 2.0330 (0.0111) | 6.1150 (0.0111) |
| | 70 | 2.0227 (0.0079) | 6.0718 (0.0079) |
| | 100 | 2.0103 (0.0054) | 6.0643 (0.0055) |
| JP | 30 | 2.0121 (0.0194) | 6.1003 (0.0192) |
| | 50 | 2.0059 (0.0114) | 6.0236 (0.0114) |
| | 70 | 2.0133 (0.0080) | 5.9981 (0.0080) |
| | 100 | 2.0047 (0.0056) | 6.0336 (0.0056) |
| GNIP | 30 | 1.9900 (0.0202) | 6.1304 (0.1136) |
| | 50 | 1.9955 (0.0117) | 6.0727 (0.0677) |
| | 70 | 1.9975 (0.0082) | 6.0744 (0.0479) |
| | 100 | 1.9974 (0.0056) | 6.0348 (0.0332) |

**Table 8. Simulation Result For SPLF Assuming β =5**

| n | prior | α= 2 | α = 6 |
|---|---|---|---|
| UP | 30 | 2.0623 (0.0186) | 6.2343 (0.0188) |
| | 50 | 2.0403 0.0110 | 6.1024 (0.0111) |
| | 70 | 2.0207 (0.0079) | 6.0850 (0.0079) |
| | 100 | 2.0214 (0.0055) | 6.0454 (0.0055) |
| JP | 30 | 2.0122 (0.0195) | 6.0736 (0.0195) |
| | 50 | 2.0105 (0.0114) | 6.0264 (0.0114) |
| | 70 | 2.0123 0.0080 | 6.0345 ( 0.0080) |
| | 100 | 2.0023 (0.0055) | 6.0288 (0.0056) |
| GNIP | 30 | 2.0461 (0.0380) | 6.1532 (0.1147) |
| | 50 | 2.0346 (0.0225) | 6.1271 (0.0682) |
| | 70 | 2.0242 (0.0161) | 6.0754 (0.0480) |
| | 100 | 2.0144 (0.0110) | 6.0559 (0.0333) |

## Conclusion

From the above simulation study we conclude that by increasing sample size Bayes estimators approaches to its true value and by increasing sample size Bayes posterior risks decreases. DeGroot loss function with uniform prior is performing better as Bayes posterior risks is minimum among all loss functions.

This study can be further extended by using other non-informative priors and other loss functions.

## References


[1] Abbas, K. and Tang, Y. (2013). Estimation of parameters for Frechet distribution based on type-ii censored samples. Caspian Journal of Applied Sciences Research, 2(7).

[2] Berger, J. O. (1985). Statistical decision theory and Bayesian analysis. Springer.

[3] DeGroot, M. H. (2005). Optimal statistical decisions, volume 82. John Wiley & Sons.

[4] Engelund, S., & Rackwitz, R. (1992). On predictive distribution functions for the three asymptotic extreme value distributions. *Structural Safety*, *11*(3), 255-258.

[5] Fréchet, M. (1927). Sur la loi de probabilité de l'écart maximum. In *Annales de la societe Polonaise de Mathematique* (Vol. 6, pp. 93-116).

[6] Gumbel, E. J. (1965). A quick estimation of the parameters in Fréchet's distribution. *Revue de l'Institut International de Statistique*, 349-363.

[7] Hooda, B. K., Singh, N. P., & Singh, U. (1993). On estimation in fre'chet distribution of m-th maxima. *Communications in Statistics-Theory and Methods*, *22*(3), 907-919.

[8] Hooda, B. K., Singh, N. P., & SINGH, U. (1990). Estimates of the parameters of type-II extreme value distribution from censored samples. *Communications in statistics. Theory and methods*, *19*(8), 3093-3110.

[9] Jeffreys, H. (1998). The theory of probability. Oxford University Press.

[7] Kotz, S. and Nadarajah, S. (2000). Extreme value distributions, volume 31. World Scientific.

[10] Nasir, W., & Aslam, M. (2015). Bayes approach to study shape parameter of Frechet distribution. *International Journal of Basic and Applied Sciences*, *4*(3), 246-254.