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Batch Arrival Retrial Queue with Positive and Negative Customers, Priority or Collisions, Delayed Repair and Orbital Search

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ABSTRACT

Batch arrival retrial queue with positive and negative customers is considered. Positive customers arrive in batches according to Poisson process. If the server is idle upon the arrival of a batch, one of the customers in the batch receives service immediately and others join the orbit. If the server is busy, the arriving batch joins the orbit or collides with the customer in service resulting in all being shifted to the orbit or one of the customers in the batch interrupts the customer in service to get his own service. The arrival of a negative customer brings the server down and makes the interrupted customer to leave the system. The repair of the failed server starts after a random amount of time. During the repair time and delay time, customers may balk the system. After each service completion, the server searches for customers in the orbit with certain probability. Using supplementary variable technique various performance measures are derived. Stochastic decomposition property is established. Special cases are discussed and numerical results are presented.

Introduction

Queueing systems with repeated attempts are found suitable for modelling the processes in telephone switching systems, digital cellular mobile networks, packet switching networks, local area networks, stock and flow etc. Review of retrial queueing literature can be found in the survey papers of Yang and Templeton (1987) and Falin (1990), the bibliographies of Artalejo (1999a, 1999b) and the books by Falin and Templeton (1997) and Artalejo and Gomez Corral (2008).

Preemptive resume priority queues have important uses in modelling and analysing computer system and communication network. Artalejo et al. (2001) considered a retrial queueing system where customers at the retrial group have preemptive priority over customers at the waiting line. Krishna Kumar et al. (2002) analysed an M/G/1 retrial queueing system with additional phase of service and preemptive resume service discipline. Ayyappan et al. (2010) studied a retrial queueing system with single working vacation under preemptive priority service using matrix geometric technique. Senthil Kumar et al. (2013) analysed preemptive resume priority retrial queue with two classes of MAP arrivals.

Retrial queues with collisions arise from the medium access control protocols for wireless LANs. Choi et al. (1992) discussed a retrial queueing system with constant retrial rate and collision in the specific communication protocol CSMA-CD. Using generating function technique, Krishna Kumar et al. (2010) studied a Markovian single server feedback retrial queue with linear retrial rate and collision of customers. Kim (2010) considered an M/M/1 retrial queue with collision and impatience. Wu et al. (2011) considered a discrete time Geo/G/1 retrial queue with preemptive resume and collisions.

In recent years, a variety of industrial applications have created interest in the modelling of reliability in queues with negative arrivals called G queues. Arrival of a negative customer not only removes the customer in service from the system but also causes the server breakdown. Liu et al. (2009) analysed an M/G/1 retrial G queue with preemptive resume and feedback under N policy vacation. Wang and Zhang (2009) considered a discrete time retrial queue with negative arrivals. Aissani (2010) obtained the generating function of the number of primary customers in the stationary regime of an M/G/1 retrial queue with negative arrivals and unreliable server. Wu and Lian (2013) discussed an M/G/1 retrial G queue with priority resume, Bernoulli vacation and server breakdown. Peng et al. (2013) suggested an M/G/1 retrial G queue with preemptive resume priority and collisions under linear retrial policy subject to server breakdowns and delayed repairs.

In this paper we have analysed unreliable batch arrival retrial queue with positive and negative customers, priority or collisions, delayed repair and orbital search

Model Description

Single server queueing system with two types of arrivals positive and negative is considered. Positive customers arrive in batches according to Poisson process with rate λ^+. At every arrival epoch, a batch of k customers arrives with probability C_k. The generating function of the sequence $\{C_k\}$ is C(z) with first two moments m_1 and m_2. There is no waiting space in front of the server and therefore if the arriving batch of positive customers finds the server idle, then one of the customers receives his service and the others join the orbit. If the server is busy, then the arriving batch proceeds to the server with probability δ or enters the orbit with probability $\bar{\delta}$ (= 1 – δ). In the first case, with probability α one of the customers interrupts the customer in service to commence his own service and the interrupted customer along with remaining customers join the orbit. Otherwise, with probability $\bar{\alpha}$ (= 1 – α) the arriving batch collides with the customer in service resulting in all being shifted to the orbit and the server becomes idle.

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Negative customers arrive independently according to Poisson process with rate λ^-. The arrival of negative customer removes the positive customer in service from the system and causes the server breakdown. The repair of the failed server commences after a random amount of time. If the server is waiting for repair (under repair), the arriving batch enters the orbit with probability p(q) or leaves the system with complementary probability $\bar{p}$ ($\bar{q}$). As soon as the service of the positive customer is completed, the server goes for search of customers in the orbit with probability θ or remains idle with probability $\bar{\theta}$ (= 1– θ). The search time is assumed to be negligible.

Distribution function, density function, Laplace Stieltjes transform and the first two moments of retrial time, service time, delay time and repair time which are generally distributed are given below.

Time	Distribution function	Density function	Laplace Stieltjes transform	First two moments	Hazard rate function
Retrial	A(x)	a(x)	$A^*(s)$	-	$\eta(x)$
Service	B(x)	b(x)	$B^*(s)$	μ_1, μ_2	$\mu(x)$
Delay	D(x)	d(x)	$D^*(s)$	γ_1, γ_2	$\gamma(x)$
Repair	R(x)	r(x)	$R^*(s)$	β_1, β_2	$\beta(x)$

The stochastic behaviour of the retrial queueing system can be described by the Markov process $\{X(t), t \geq 0\} = \{S(t), N(t), \xi(t), t \geq 0\}$ where S(t) denotes the server state 0, 1, 2 or 3 according as the server being idle, providing service, waiting for repair or under repair. N(t) corresponds to the number of customers in the orbit. If S(t) = 0, then $\xi(t)$ represents the elapsed retrial time. If S(t) = 1, then $\xi(t)$ represents the elapsed service time. If S(t) = 2, then $\xi(t)$ represents the elapsed delay time. If S(t) = 3, then $\xi(t)$ represents the elapsed repair time.

Stability Condition

Let $N(t_n^+)$ be the number of customers in the orbit just after time t_n. Then the sequence of random variables $Y_n = N(t_n^+)$ form a Markov Chain, which is the embedded Markov Chain of the system.

Theorem 1

The embedded Markov chain $\{Y_n, n \in N\}$ is ergodic if and only if

$$[1 - B^*(\lambda^+ \delta + \lambda^-)]\ [\lambda^+ (\delta + m_1) + \lambda^- m_1 (p\lambda^+ \gamma_1 + q\lambda^+ \beta_1) + m_1(1 - A^*(\lambda))(\lambda^- + \lambda^+ \delta\ \bar{\alpha})] <$$
$$(\lambda^+ \delta + \lambda^-)\ [1 - m_1(1 - A^*(\lambda^+))\ \bar{\theta}\ B^*(\lambda^+ \delta + \lambda^-)].$$

The theorem can be proved along similar lines as in Gomez-Corral (1999).

Steady State Distribution

For the process $\{X(t), t \geq 0\}$, define the probability densities

$$I_0(t) = P\{S(t) = 0, N(t) = n\}$$
$$I_n(x, t)\,dx = P\{S(t) = 0, N(t) = n, x \leq \xi(t) < x + dx\},\ x \geq 0, n \geq 1$$
$$P_n(x, t)\,dx = P\{S(t) = 1, N(t) = n, x \leq \xi(t) < x + dx\},\ x \geq 0, n \geq 0$$
$$F_{1,n}(x, t)\,dx = P\{S(t) = 2, N(t) = n, x \leq \xi(t) < x + dx\},\ x \geq 0, n \geq 0$$
$$F_{2,n}(x, t)\,dx = P\{S(t) = 3, N(t) = n, x \leq \xi(t) < x + dx\},\ x \geq 0, n \geq 0$$

The steady state equations of the model under study are

$$\lambda^+ I_0 = \int_0^\infty P_0(x)\,\mu(x)\,dx + \int_0^\infty F_{2,0}(x)\,\beta(x)\,dx \quad (1)$$

$$\frac{d}{dx} I_n(x) = -(\lambda^+ + \eta(x))\ I_n(x), \quad n \geq 1 \quad (2)$$

$$\frac{d}{dx} P_n(x) = -(\lambda^+ + \lambda^- + \mu(x))\ P_n(x) + \lambda^+\ \bar{\delta} \sum_{k=1}^{n} C_k\ P_{n-k}(x), \quad n \geq 0 \quad (3)$$

$$\frac{d}{dx} F_{1,n}(x) = -(p\lambda^+ + \gamma(x))\ F_{1,n}(x) + \lambda^+ p \sum_{k=1}^{n} C_k\ F_{1,n-k}(x), \quad n \geq 0 \quad (4)$$

$$\frac{d}{dx} F_{2,n}(x) = -(q\lambda^+ + \beta(x))\ F_{2,n}(x) + \lambda^+ q \sum_{k=1}^{n} C_k\ F_{2,n-k}(x), \quad n \geq 0 \quad (5)$$

with boundary conditions

$$I_1(0) = \bar{\theta} \int_0^\infty P_1(x)\,\mu(x)\,dx + \int_0^\infty F_{2,1}(x)\,\beta(x)\,dx \quad (6)$$

$$I_n(0) = \bar{\theta} \int_0^\infty P_n(x)\,\mu(x)\,dx + \int_0^\infty F_{2,n}(x)\,\beta(x)dx + \lambda^+ \delta\ \bar{\alpha} \sum_{k=1}^{n} C_k \int_0^\infty P_{n-(k+1)}(x)\,dx,\ n \geq 2 \quad (7)$$

$$P_0(0) = \lambda^+ C_1 I_0 + \int_0^\infty I_1(x)\,\eta(x)\,dx + \theta \int_0^\infty P_1(x)\,\mu(x)\,dx \quad (8)$$

$$P_n(0) = \lambda^+ C_{n+1} I_0 + \int_0^\infty I_{n+1}(x)\,\eta(x)\,dx + \lambda^+ \delta\,\alpha \sum_{k=1}^{n} C_k \int_0^\infty P_{n-k}(x)\,dx$$

$$+ \lambda^+ \sum_{k=1}^{n} C_k \int_0^\infty I_{n-k+1}(x)\,dx + \theta \int_0^\infty P_{n+1}(x)\,\mu(x)\,dx, \quad n \ge 1 \tag{9}$$

$$F_{1,n}(0) = \lambda^- \int_0^\infty P_n(x)\,dx, \quad n \ge 0 \tag{10}$$

$$F_{2,n}(0) = \int_0^\infty F_{1,n}(x)\,\gamma(x)\,dx, \quad n \ge 0 \tag{11}$$

The normalising equation is

$$I_0 + \sum_{n=1}^{\infty}\int_0^\infty I_n(x)\,dx + \sum_{n=0}^{\infty}\int_0^\infty P_n(x)\,dx + \sum_{n=0}^{\infty}\int_0^\infty F_{1,n}(x)\,dx + \sum_{n=0}^{\infty}\int_0^\infty F_{2,n}(x)\,dx = 1 \tag{12}$$

To solve the above equations, define the probability generating functions

$$I(x,z) = \sum_{n=1}^{\infty} I_n(x)\,z^n; \qquad P(x,z) = \sum_{n=0}^{\infty} P_n(x)\,z^n;$$

$$F_1(x,z) = \sum_{n=0}^{\infty} F_{1,n}(x)\,z^n \quad \text{and} \quad F_2(x,z) = \sum_{n=0}^{\infty} F_{2,n}(x)\,z^n$$

Multiplying equations (2) to (5) by z^n, summing over n and solving the corresponding partial differential equations, we get

$$I(x,z) = I(0,z)\,e^{-\lambda^+ x}(1 - A(x)) \tag{13}$$

$$P(x,z) = P(0,z)\,e^{-(\lambda^+ + \lambda^- + \lambda^+\bar{\delta}C(z))x}(1 - B(x)) \tag{14}$$

$$F_1(x,z) = F_1(0,z)\,e^{-p\lambda^+(1-C(z))x}(1 - D(x)) \tag{15}$$

$$F_2(x,z) = F_2(0,z)\,e^{-q\lambda^+(1-C(z))x}(1 - R(x)) \tag{16}$$

Multiplying equations (6) to (11) by z^n and summing over n, we get

$$I(0,z) = \bar{\theta}\int_0^\infty P(x,z)\,\mu(x)\,dx + \int_0^\infty F_2(x,z)\,\beta(x)\,dx + \lambda^+\delta\,\bar{\alpha}\,z\,C(z)\int_0^\infty P(x,z)\,dx - \lambda^+ I_0 \tag{17}$$

$$P(0,z) = \frac{\lambda^+ C(z)}{z} I_0 + \frac{1}{z}\int_0^\infty I(x,z)\,\eta(x)\,dx + \frac{\lambda^+ C(z)}{z}\int_0^\infty I(x,z)\,dx$$
$$+ \frac{\theta}{z}\int_0^\infty P(x,z)\,\mu(x)\,dx + \lambda^+\delta\,\alpha\,C(z)\int_0^\infty P(x,z)\,dx \tag{18}$$

$$F_1(0,z) = \lambda^- \int_0^\infty P(x,z)\,dx \tag{19}$$

$$F_2(0,z) = \int_0^\infty F_1(x,z)\,\gamma(x)\,dx \tag{20}$$

Using equations (13) and (16) in equations (17) to (20) and simplifying, we have

$$I(0,z) = [\lambda^+ I_0\,[C(z)\,B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-) - z + \theta(1 - C(z))$$
$$B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-) + K(z)\,(C(z)\,(\lambda^-\,D^*(p\,\lambda^+(1-C(z)))$$
$$R^*(q\,\lambda^+(1-C(z))) + \lambda^+\delta\,\bar{\alpha}\,z\,C(z) + \lambda^+\delta\,\alpha\,z))]]\;/\;T(z) \tag{21}$$

$$P(0,z) = A^*(\lambda^+)\,(C(z)-1)\,\lambda^+ I_0\,/\,T(z) \tag{22}$$

$$F_1(0,z) = \lambda^+ I_0\,A^*(\lambda^+)\,\lambda^-(C(z)-1)\,K(z)\,/\,T(z) \tag{23}$$

$$F_2(0,z) = \lambda^+ I_0\,A^*(\lambda^+)\,\lambda^-(C(z)-1)\,D^*(p\lambda^+(1-C(z)))\,K(z)/T(z) \tag{24}$$

where

$$K(z) = \frac{1 - B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-)}{\lambda^+(1-\bar{\delta}C(z)) + \lambda^-}$$

$$T(z) = z - [C(z) + A^*(\lambda^+)(1 - C(z))]\;\bar{\theta}\;B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-)$$
$$- \theta\,B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-) - K(z)\,[(A^*(\lambda^+) + C(z)\,(1 - A^*(\lambda^+)))$$
$$(\lambda^-\,D^*(p\,\lambda^+(1-C(z)))\;R^*(q\,\lambda^+(1-C(z))) + \lambda^+\delta\,\bar{\alpha}\,z\,C(z)) + \lambda^+\delta\,\alpha\,z\,C(z)]$$

Substituting the expressions of $I(0,z)$, $P(0,z)$, $F_1(0,z)$ and $F_2(0,z)$ in (13), (14), (15) and (16), we get

$$I(x,z) = [\lambda^+ I_0\,[C(z)\,B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-) - z + \theta(1 - C(z))$$
$$B^*(\lambda^+(1-\bar{\delta}C(z)) + \lambda^-) + K(z)\,(C(z)\,(\lambda^-\,D^*(p\,\lambda^+(1-C(z)))$$
$$R^*(q\,\lambda^+(1-C(z))) + \lambda^+\delta\,\bar{\alpha}\,z\,C(z) + \lambda^+\delta\,\alpha\,z))]]\;e^{-\lambda^+ x}(1-A(x))/\,T(z) \tag{25}$$

$$P(x,z) = \lambda^+ I_0\,A^*(\lambda^+)\,(C(z)-1)\,e^{-(\lambda^+ + \lambda^- + \lambda^+\bar{\delta}C(z))x}(1 - B(x))\;/\;T(z) \tag{26}$$

$$F_1(x,z) = \lambda^+\lambda^- I_0\,A^*(\lambda^+)\,(C(z)-1)\,K(z)\,e^{-p\lambda^+(1-C(z))x}(1 - D(x))\;/\;T(z) \tag{27}$$

$$F_2(x, z) = \lambda^+ \lambda^- I_0 \, A^*(\lambda^+) (C(z) - 1) \, D^*(p \lambda^+ (1 - C(z))) \, K(z) \, e^{-q\lambda^+(1-C(z))x} (1 - R(x)) \, / \, T(z) \quad (28)$$

The partial probability generating function of the orbit size when the server is idle is

$$I(z) = \int_0^\infty I(x,z)\,dx$$

$$= [I_0 (1 - A^*(\lambda^+)) [(C(z) B^*(G(z)) - z + \theta B^*(G(z))(1 - C(z)))(\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) + C(z)(1 - B^*(G(z)))$$
$$(\lambda^- D^*(p\lambda^+(1 - C(z))) R^*(q\lambda^+(1 - C(z))) + \lambda^+ \delta \bar{\alpha} \, z C(z) + \lambda^+ \delta \alpha z)]] \, / \, T_1(z) \quad (29)$$

where

$$G(z) = \lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-$$

$$T_1(z) = [z - (A^*(\lambda^+) + C(z)(1 - A^*(\lambda^+))) \bar{\theta} B^*(G(z)) - \theta B^*(G(z))] \; (\lambda^+ (1 - \bar{\delta} C(z)) + \lambda^-) - (1 - B^*(G(z))) \; [(A^*(\lambda^+)$$
$$+ C(z) (1 - A^*(\lambda^+)))(\lambda^- D^*(p\lambda^+(1 - C(z))) R^*(q\lambda^+(1 - C(z))) + \lambda^+ \delta \bar{\alpha} \, z C(z)) + \lambda^+ \delta \alpha z C(z)]$$

The partial probability generating function of the orbit size when the server is busy is

$$P(z) = \int_0^\infty P(x,z)\,dx$$

$$= I_0 \lambda^+ A^*(\lambda^+) (C(z) - 1) (1 - B^*(G(z))) / T_1(z) \quad (30)$$

The partial probability generating function of the orbit size when the server is waiting for repair is

$$F_1(z) = \int_0^\infty F_1(x,z)\,dx$$

$$= I_0 \lambda^- A^*(\lambda^+) (1 - B^*(G(z))) \, (D^*(p\lambda^+(1 - C(z))) - 1) / (p T_1(z)) \quad (31)$$

The partial probability generating function of the orbit size when the server is under repair is

$$F_2(z) = \int_0^\infty F_2(x,z)\,dx$$

$$= [I_0 \lambda^+ A^*(\lambda^+)(1 - B^*(G(z))) D^*(p\lambda^+(1 - C(z))) \; (R^*(q \lambda^+ (1 - C(z))) - 1)] / (q \; T_1(z)) \quad (32)$$

Using the normalising equation (12), I_0 can be obtained as

$$I_0 = [(B^*(\lambda^+ \delta + \lambda^-) - 1) (\lambda^+ (\delta + m_1) + \lambda^- \lambda^+ m_1 (p \gamma_1 + q \beta_1) + m_1 (1 - A^*(\lambda^+))$$
$$(\lambda^- + \lambda^+ \delta \bar{\alpha})) + (\lambda^+ \delta + \lambda^-) (1 - m_1 (1 - A^*(\lambda)) \bar{\theta} \; B^*(\lambda^+ \delta + \lambda^-))] \; / \; [A^*(\lambda^+)$$
$$((\lambda^+ \delta + \lambda^-) + (B^*(\lambda^+ \delta + \lambda^-) - 1) (\lambda^+ (\delta - \lambda^- m_1 (\bar{p} \gamma_1 + \bar{q} \beta_1))))] \quad (33)$$

The probability generating function of the orbit size is

$$P_q(z) = I_0 + I(z) + P(z) + F_1(z) + F_2(z)$$
$$= I_0 A^*(\lambda^+) [\lambda^+ ((z - 1) (1 - C(z) + \delta C(z) \, B^*(G(z)))) + T_2(z)] / T_1(z) \quad (34)$$

where

$$T_2(z) = \lambda^-[(z - B^*(G(z))) - (1 - B^*(G(z))) \, (D^*(p\lambda^+(1 - C(z))) \, R^*(q\lambda^+(1 - C(z))) + ((1 - D^*(p\lambda^+(1 - C(z))))/p) +$$
$$(D^*(p\lambda^+(1 - C(z)))(1 - R^*(q\lambda^+(1 - C(z))))/q))]$$

The probability generating function of the system size is

$$P_S(z) = I_0 + I(z) + z \, P(z) + F_1(z) + F_2(z)$$
$$= I_0 A^*(\lambda^+) [\lambda^+ (z - 1) B^*(G(z)) \, (1 - C(z) + \delta C(z)) + T_2(z)] / T_1(z) \quad (35)$$

Performance Measures

- If the system is in steady state, then the probability that the system is empty is given by I_0
- The probability that the server is idle during retrial time is given by

$$I = \lim_{z \to 1} I(z)$$
$$= I_0 (1 - A^*(\lambda^+)) [(1 - B^*(\lambda^+ \delta + \lambda^-)) \; (\lambda^- m_1 \; (1 + \lambda^+ p \gamma_1 + \lambda^+ q \beta_1)$$
$$+ \lambda^+ (m_1 + \delta + \delta m_1 \bar{\alpha})) + (\lambda^+ \delta + \lambda^-) \, (\bar{\theta} m_1 B^*(\lambda^+ \delta + \lambda^-) - 1)] \, / \, T_1'(1) \quad (36)$$

where

$$T_1'(1) = (B^*(\lambda^+ \delta + \lambda^-) - 1) \; [\lambda^+ \delta + \lambda^+ m_1 + \lambda^- \lambda^+ m_1 (p \gamma_1 + q \beta_1) + m_1(1 - A^*(\lambda^+)) \, (\lambda^- + \lambda^+ \delta \bar{\alpha})] +$$
$$(\lambda^+ \delta + \lambda^-) \, (1 - m_1 (1 - A^*(\lambda^+)) \; \bar{\theta} \; B^*(\lambda^+ \delta + \lambda^-))$$

- The probability that the server is busy is given by

$$P = \lim_{z \to 1} P(z)$$
$$= I_0 A^*(\lambda^+) \lambda^+ m_1 (1 - B^*(\lambda^+ \delta + \lambda^-)) / T_1'(1) \quad (37)$$

- The probability that the server is in failure mode is given by

$$F = \lim_{z \to 1} (F_1(z) + F_2(z))$$

$$= \quad I_0\, A^*(\lambda^+)\, \lambda^-\, \lambda^+\, m_1(\gamma_1+\beta_1)\,(1-B^*(\lambda^+\,\delta+\lambda^-))\;/T_1'(1) \tag{38}$$

- The probability of unsuccessful retrials made by a customer is given by

$$V_R = \frac{\lambda^+\, m_1\,(1+\lambda^-\,\gamma_1+\lambda^-\,\beta_1)\,A^*(\lambda^+)\,(1-B^*(\lambda^+\,\delta+\lambda^-))}{\begin{gathered}(B^*(\lambda^+\,\delta+\lambda^-)-1)\,[A^*(\lambda^+)\,(\lambda^+(\delta-\lambda^-\,m_1\,(\bar{p}\,\gamma_1+\bar{q}\,\beta_1)))\\ -\lambda^+(\delta+m_1)-\lambda^-\,\lambda^+\,m_1(p\,\gamma_1+q\,\beta_1)-m_1(1-A^*(\lambda^+))\\ (\lambda^-+\lambda^+\,\delta\,\bar{\alpha})]-(\lambda^+\,\delta+\lambda^-)\,(1-A^*(\lambda^+))\,(1-m_1\,\bar{\theta}\,B^*(\lambda^+\,\delta+\lambda^-))\end{gathered}} \tag{39}$$

Let Nr(z) and Dr(z) represent the numerator and denominator of $P_q(z)$.

- The mean number of customers in the orbit L_q is given by

$$L_q = \lim_{z\to 1} \frac{d}{dz} P_q(z)$$

$$= \frac{Dr'(1)\;Nr''(1) - Nr'(1)\,Dr''(1)}{2Dr'(1)^2} \tag{40}$$

where

$$Nr'(1) = I_0\, A^*(\lambda^+)\,[\lambda^+\,\delta\, B^*(\lambda^+\,\delta+\lambda^-) + \lambda^-(1 + (1-B^*(\lambda^+\,\delta+\lambda^-))\,\lambda^+\, m_1\,(\bar{p}\,\gamma_1 + \bar{q}\,\beta_1))]$$

$$Nr''(1) = I_0\, A^*(\lambda^+)\,[2\lambda^+(\delta\, h_3 + m_1\,\delta\, B^*(\lambda^+\,\delta+\lambda^-) - m_1) - \lambda^-(2B^*(\lambda^+\,\delta+\lambda^-)\,\lambda^+\, m_1 (\bar{p}\,\gamma_1 + \bar{q}\,\beta_1)) + \lambda^+\,(\bar{p}\, h_1 + \bar{q}\, h_2 + 2\bar{q}\, p\, \lambda^+\, m_1^2\,\gamma_1\,\beta_1)\,(1 - B^*(\lambda^+\,\delta+\lambda^-))]$$

$$Dr'(1) = T_1'(1)$$

$$Dr''(1) = (B^*(\lambda^+\,\delta+\lambda^-) - 1)\,[\lambda^+\,(2\,m_1 + m_2) + \lambda^-(p\,\lambda^+\, h_1 + q\,\lambda^+\, h_2) + 2\lambda^-\,{\lambda^+}^2\, m_1^2\, p\,\gamma_1\, q\,\beta_1 + 2\,m_1 (\lambda^-\,\lambda^+\, m_1\,(p\,\gamma_1 + q\,\beta_1)\,(1 - A^*(\lambda^+)) + \lambda^+\,\delta\,\bar{\alpha}\,(1 + m_1)) + m_2\,(\lambda^- + \lambda^+\,\delta\,\bar{\alpha})\,(1 - A^*(\lambda^+)) + 2\,\lambda^+\,\bar{\delta}\,\bar{\theta}\, m_1^2\,(1-A^*(\lambda^+))\, B^*(\lambda^+\,\delta+\lambda^-) - (\lambda^+\,\delta+\lambda^-)\,\bar{\theta}\, h_3\,(2\,m_1(1-A^*(\lambda^+)) + m_2\,\bar{\theta}\,(1 - A^*(\lambda^+)) B^*(\lambda^+\,\delta+\lambda^-)) + 2\,h_3\,(\lambda^-\,\lambda^+\, m_1\,(p\,\gamma_1 + q\,\beta_1) + \lambda^+\,\delta\,(1 + m_1) + m_1(1 - A^*(\lambda^+))\,(\lambda^- + \lambda^+\,\delta\,\bar{\alpha}))]$$

$$h_1 = m_2\,\gamma_1 + p\,\lambda^+\, m_1^2\,\gamma_2$$

$$h_2 = m_2\,\beta_1 + q\,\lambda^+\, m_1^2\,\beta_2 \text{ and}$$

$$h_3 = \lambda^+\,\bar{\delta}\, m_1 \int_0^\infty e^{-(\lambda^+\,\delta+\lambda^-)\,x}\, x\, b_1(x)\, dx$$

- The mean number of customers in the system is given by

$$L_S = \lim_{z\to 1} \frac{d}{dz} P_S(z)$$

$$= L_q + P \tag{41}$$

Reliability Indices

Let A(t) be the system availability at time t, that is the probability that the server is idle or working for a customer. Then under steady state condition, the availability of the server is given by

$$A = I_0 + \lim_{z\to 1}\left[\int_0^\infty I(x,z)\,dx + \int_0^\infty P(x,z)\,dx\right]$$

$$= I_0 + I + P$$

$$= \frac{(\lambda^+\,\delta+\lambda^-)+\lambda^+\,(\delta+\lambda^-\,m_1(p\,\gamma_1+q\,\beta_1))\,(B^*(\lambda^+\,\delta+\lambda^-)-1)}{(\lambda^+\,\delta+\lambda^-)+\lambda^+\,(\delta-\lambda^-\,m_1(\bar{p}\,\gamma_1+\bar{q}\,\beta_1))\,(B^*(\lambda^+\,\delta+\lambda^-)-1)} \tag{42}$$

The steady state failure frequency of the server is

$$Ғ = \lambda^-\, P$$

$$= \frac{\lambda^-\,\lambda^+\, m_1\,(1-B^*(\lambda^+\,\delta+\lambda^-))}{(\lambda^+\,\delta+\lambda^-)+(\lambda^+(\delta-\lambda^-\,m_1(\bar{p}\,\gamma_1+\bar{q}\,\beta_1)))\,(B^*(\lambda^+\,\delta+\lambda^-)-1)} \tag{43}$$

Stochastic Decomposition

Theorem 2

The expected number of customers in the system (L_S) can be expressed as the sum of two independent random variables, one of which is the expected number of customers in unreliable batch arrival classical queueing system with positive and negative customers, priority or collisions and delayed repairs (L) and the other is the expected number of customers in the orbit given that the server is idle (L_I).

Proof

The probability generating function Φ(z) of the number of customers in unreliable batch arrival queue with positive and negative customers, priority or collisions and delayed repairs is given by

$$\Phi(z) = \frac{(B^*(\lambda^+\delta+\lambda^-)-1)(\lambda^+(\delta+m_1)+\lambda^-\lambda^+ m_1(p\gamma_1+q\beta_1))+(\lambda^+\delta+\lambda^-)}{(\lambda^+\delta+\lambda^-)+(B^*(\lambda^+\delta+\lambda^-)-1)[\lambda^+(\delta-\lambda^- m_1(\bar{p}\gamma_1+\bar{q}\beta_1))]} \text{ x}$$

$$[\lambda^+(z-1)B^*(G(z))(1-C(z)+\delta C(z))+T_2(z)] \Big/ [(z-B^*(G(z)))(\lambda^+(1-\bar{\delta}C(z))+\lambda^-)-(1-B^*(G(z)))(\lambda^- D^*(p\lambda^+(1-C(z)))R^*(q\lambda^+(1-C(z)))+\lambda^+\delta z C(z)] \quad (44)$$

The probability generating function $\psi(z)$ of the number of customers in the orbit given that the server is idle is given by

$$\psi(z) = \frac{I_0+I(z)}{I_0+I(1)}$$

$$= [[(\lambda^-\lambda^+ m_1(p\gamma_1+q\beta_1)+\lambda^+(m_1+\delta))+(\lambda^+\delta+\lambda^-)]\ (B^*(\lambda^+\delta+\lambda^-)-1)/[(\lambda^+\delta+\lambda^-)+(\lambda^+(\delta-\lambda^- m_1(\bar{p}\gamma_1+\bar{q}\beta_1)(B^*(\lambda^+\delta+\lambda^-)-1)))]]\ \text{x}[[I_0\ A^*(\lambda^+)((\lambda^+(1-\bar{\delta}C(z))+\lambda^-)(z-B^*(G(z)))-(1-B^*(G(z)))(\lambda^- D^*(p\lambda^+(1-C(z)))\ R^*(q\lambda^+(1-C(z))))+\lambda^+\delta z C(z))]\ /\ T_1(z) \quad (45)$$

From equations (35), (44) and (45), we get

$P_S(z) = \Phi(z)\,\psi(z)$

Consequently, in terms of convolution we can state that

$L_S = L + L_I$

Special Cases

Case (i)

If $\lambda^- = 0$ (no negative customers), then our model reduces to $M^X/G/1$ retrial queue with priority or collisions and orbital search. In this case

$$P_S(z) = I_0\,A^*(\lambda^+)[\lambda^+(z-1)B^*(\lambda^+(1-\bar{\delta}C(z)))(1-C(z)+\delta C(z))]\ /\ T_3(z)$$

where

$$T_3(z) = z-(A^*(\lambda^+)+C(z)(1-A^*(\lambda^+)))\ \bar{\theta}\ B^*(\lambda^+(1-\bar{\delta}C(z))) - \theta B^*(\lambda^+(1-\bar{\delta}C(z)))\ (\lambda^+(1-\bar{\delta}C(z))) - (1-B^*(\lambda^+(1-\bar{\delta}C(z)))) [(A^*(\lambda^+)+C(z)(1-A^*(\lambda^+)))+\lambda^+\delta\,\bar{\alpha}\ z\,C(z)] + \lambda^+\delta\alpha z C(z)$$

$$I_0 = [(B^*(\lambda^+\delta)-1)(\lambda^+(\delta+m_1)+m_1(1-A^*(\lambda^+))(\lambda^+\delta\bar{\alpha})) + \lambda^+\delta(1-m_1(1-A^*(\lambda^+))\bar{\theta}B^*(\lambda^+\delta))]\ /\ (\lambda^+\delta(A^*(\lambda^+)+(B^*(\lambda^+\delta)-1)))$$

Case (ii)

$A^*(\lambda) \to 1$ (no retrials), in this case our model becomes $M^X/G/1$ queue with positive and negative customers, priority or collisions, server breakdown and delayed repairs. For this model

$$P_s(z) = \frac{I_0[\lambda^+(z-1)B^*(G(z))\ (1-C(z)+\delta C(z))+T_2(z)]}{(z-B^*(G(z)))(\lambda^+(1-\bar{\delta}C(z))+\lambda^-)-(1-B^*(G(z)))\lambda^- D^* p\lambda^+(1-C(z)))R^* p\lambda^+(1-C(z)))+\lambda^+\delta z C(z)}$$

$$I_0 = \frac{(B^*(\lambda^+\delta+\lambda^-)-1)[\lambda^+(\delta+m_1)+\lambda^-\lambda^+ m_1(p\gamma_1+q\beta_1)]+(\lambda^+\delta+\lambda^-)}{(\lambda^+\delta+\lambda^-)+(B^*(\lambda^+\delta+\lambda^-)-1)[\lambda^+(\delta-\lambda^- m_1(\bar{p}\gamma_1+\bar{q}\beta_1))]}$$

When the computer system experiences failure it has to undergo repair. The repair process begins after an initial delay. During this down time, the programs that come in for execution may be withdrawn (balking) without being executed.

Numerical Results

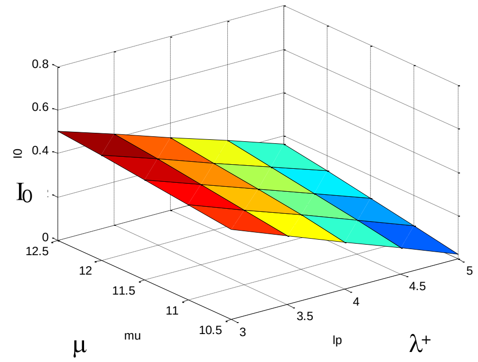
Performance measures are calculated numerically by assuming that the retrial time, service time, delay time and repair time follow exponential distribution with respective rates η, μ, γ and β. For the parameters $\lambda^+ = 2$, $\lambda^- = 0.3$, $\delta = 0.6$, $\alpha = 0.7$, $p = 0.5$, $q = 0.5$, $\theta = 0.5$, $\mu = 4$, $\gamma = 2$, $\beta = 1$, $\eta = 30$, $C_1 = C_2 = 0.5$, the performance measures I_0 - the probability that the system is empty, I - the probability that the server is idle in non-empty system, P - the probability that the server is busy and L_S - the mean number of customers in the system are calculated by varying the rates λ^+, μ, η and θ and presented in Fig. 1 and Fig. 2.

The effect of I_0, I and L_S against the parameters λ^+ and μ are plotted in Fig. 1 (a) to (d). From the figures it is observed that

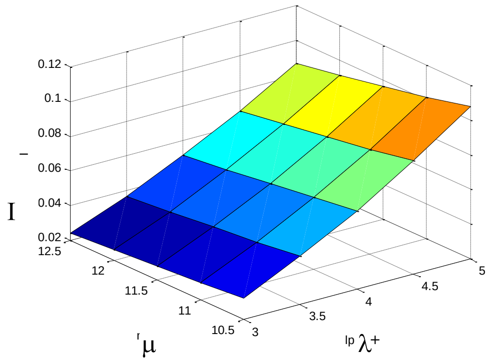
- I_0 decreases with λ^+ and increases with μ.
- I, P and L_S increase for increasing values of λ^+ and decrease for increasing μ.

The variation of I_0, I and L_S with respect to the parameters η and θ are given in Fig. 2 (a) to (d). Figures reveal that

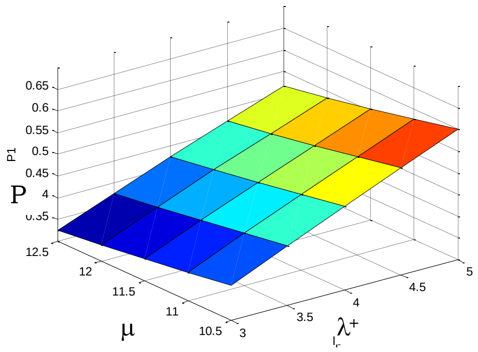
- I_0 increases with increase in η and θ.
- I and L_S decrease with increase in η and θ.
- P is independent of η and θ.



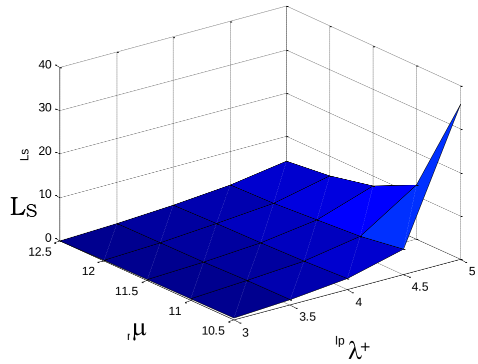
(a) I_0 versus (μ, λ^+)



(b) I versus (μ, λ^+)

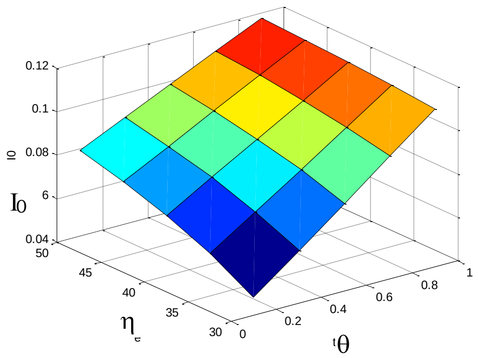


(c) P versus (μ, λ^+)

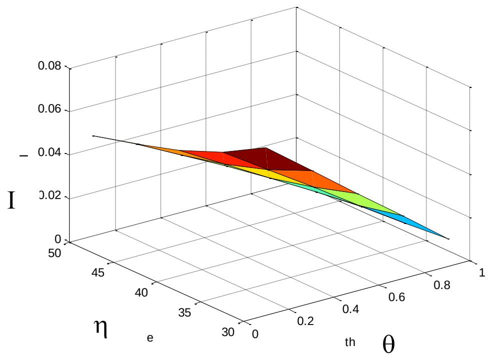


(d) Ls versus (μ, λ^+)

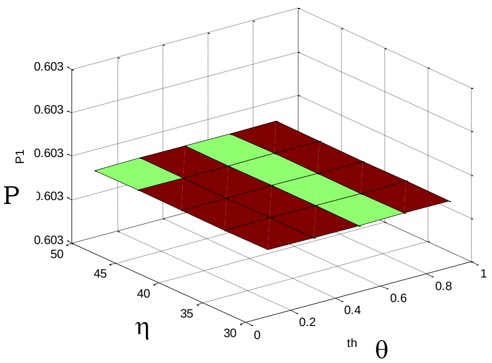
Fig 1. Performance Measures Versus (μ, λ^+)



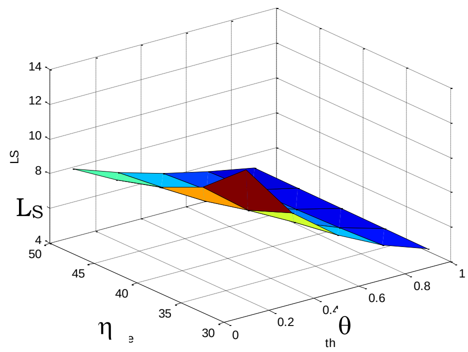
(a) I_0 versus (η, θ)



(b) I versus (η, θ)



(c) P versus (η, θ)



(d) Ls versus (η, θ)

Fig 2. Performance Measures versus (η, θ)

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