



Tubular Steel Truss Optimization Using Genetic Algorithms and Stiffness Method

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ABSTRACT

In this paper, an attempt is made to develop an optimization model for design of a steel truss using Genetic Algorithms (GAs) and 2D truss analysis using stiffness method program of NewCivil.com in the MATLAB. The objective function is to minimize the total weight of the truss, subjected to satisfaction of the constraints of axial stresses in the members of the truss and nodal displacements. The 2D truss analysis program is validated with STADD.Pro V8i. The constraints violation is handled by using an adaptive penalty function. GA tool kit functions and the 2D analysis program are combined and implemented in MATLAB. The proposed model is tested on a 3-bar steel tubular truss. After several runs and out of many optimal solutions obtained, the solution with a total weight of 150.15 Kg is considered as the best and the corresponding results are presented.

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Introduction

One of the most important and challenging problems in modern structural design is how to achieve the optimal dimensions for structures under general multiple constraints, such as stress, displacement, and bulking limits which can be easily formulated mathematically since it is essentially a problem of optimization. However, in practice it is very challenging to accomplish due to the large number of design variables, the highly irregular feasible region, and the large number of design constraints [6].

Trusses, as an efficient structural form, have been widely used in structural engineering. Different configurations of trusses are pitched roof trusses, Parallel Chord and Trapezoidal Trusses. Design of the truss plays a vital role in structure (like bridge/building) and, in turn, its optimal design is the aim of any agency dealing with structure. A lot of research has been done and is still in progress on various aspects of structural elements like optimization of trusses, frames, etc.

Optimal design of truss-structure has always been an active area of research in the field of optimization. Various techniques based on classical optimization methods have been developed to find optimal truss-structures [1]. The optimal design of structural system can be classified as sizing optimal design, shape optimal design or topology optimal design. The nature of the design variable determines the type of optimal design problem [1]. In the sizing optimization of trusses, cross-sectional areas of members are considered as decision variables and the coordinates of the nodes and connectivity among various members are considered to be fixed [3]. In most attempts, multi-level optimization methods have been used [2] for truss design. It is obvious that such a multi-level optimization technique may not always provide the globally best design, since these problems are not linearly separable. A genetic algorithm is presented here, which is a modified simple genetic algorithm (SGA) proposed by Goldberg (1989)[3], based on natural genetics. It combines Darwin's principle of survival of the fittest and a structured information exchange using randomized operators to evolve an efficient search mechanism. Genetic

algorithms (GAs) efficiently exploit useful information contained in a population of solutions to generate new solutions with better performance [8]. Where as Hejela, Lee, and Lin (1993)[4] have used a multi-level optimization scheme of first finding multiple optimal topologies and then finding the optimal member areas for each of the truss topologies. The genetic algorithm (GA) has been receiving increasing use as a global search and optimization methodology [3]. GA is most directly suited to unconstrained optimization. Traditionally, external penalty functions have been used to convert a constrained optimization problem into an unconstrained problem for GA-based optimization [13]. The GA is well suited to unconstrained optimization, yet most "real world" engineering design problems involve constrained optimization. To remedy this, it has been common practice to use external penalty functions to transform a constrained objective function into an unconstrained fitness function [9].

In the present study, an attempt is made to minimize the weight of truss as objective function subjected to stresses and displacements as constraints. The combination of genetic algorithms and 2D truss analysis using stiffness method program of NewCivil.com in the MATLAB environment is utilized to develop a model for the optimal design of plane truss structures. The analysis of the tubular truss by the NewCivil.com in the MATLAB environment is validated with STAAD.Pro V8i. The proposed methodology is tested on a 3-bar truss. The adaptive penalty function is used as the external penalty function to penalize the violation of constraints while optimization process is running.

Design problem formulation

In the present study, the sizing optimal design of a tubular truss is attempted. The decision variables considered are the cross sectional areas of the truss elements. The cross sectional areas are assumed to be available as discrete values.

The main objective of this optimization problem is to minimize the weight of truss structure subjected to the constraints of axial stresses in members (either compression or tension) and displacements at the nodes of the truss.

The objective function and constraints are written as follows

$$f(X) = \sum_{j=1}^{NE} \rho X_j L_j$$

Where $f(X)$ = objective function expressed in terms of decision variables (cross sectional areas of the truss members); ρ = weight density of truss material; X_j = cross sectional area of j^{th} element; L_j = length of j^{th} element; NE = number of members in the truss.

The axial stress constraint is taken as

$$\sigma_j \leq \sigma_a$$

The displacement constraint is expressed as

$$u_i \leq u_a \quad \text{for } i=1,2,3,4,\dots,N$$

$$\text{and } X_j > 0$$

Where N = number of nodes in the truss; σ_j = axial stress in j^{th} member; σ_a = allowable stress in all members; u_i = displacement at i^{th} node; u_a = allowable displacement at all the nodes.

Methodology

The combination of genetic algorithms tool kit functions of Matlab (for design) and newcivil.com (program developed in Matlab for analysis of plane trusses using stiffness method by Sobhan Rostami and Ali Moeinadini(2009)) is used to develop a code in MATLAB environment for optimal design of the given truss is adopted in the present study. The generalized step by step procedure is presented below.

1. Objective function is defined in the Matlab editor file keeping the same name to the objective function as that of the file name by taking the cross sectional areas of the members as decision variables.

Eg: function f=threebartruss(x)

Here, 'f' is fitness function and threebartruss is file name and also name of the objective function, x is a vector of decision variables (cross sectional areas of the truss members).

It is required to write the ga tool kit functions in another Matlab file to invoke the objective function mentioned above to randomly assign the cross sectional areas of the truss members within the range specified by GA. The syntax to accomplish this task is given below.

options=gaoptimset('generations',8,'populationtype','doublevector'),.....)

[x,fval,nvars,population] =ga(@threebartruss,2,options)

Here, options indicate the ga parameters that are to be defined.

2. The input file required for truss analysis by stiffness method is prepared in MS-Excel work sheet (2003) and is linked to the Matlab.

Eg: xlswrite('threebartruss.xls',x¹, 'input', 'p11:p12')

Here, threebartruss.xls is the input file for analysis, x¹ is the transpose of the vector of decision variables randomly generated by ga, input is the worksheet name, p11:p12 is the range of decision variables to be stored in excel work sheet

3. The coding developed for analysis (program developed in Matlab for analysis of plane trusses using stiffness method by Sobhan Rostami and Ali Moeinadini(2009)) is linked to the MATLAB.

4. The output file is generated to store the information of axial stresses and nodal displacements that are computed for the assigned areas.

Eg: xlswrite('stressesanddisp.xls',stress¹, 'sheet1', 'A2:A4')

xlswrite('stressesanddisp.xls',d, 'sheet1', 'A2:A4')

here, stressanddisp.xls is the file name to store the information of stresses and displacements, stress¹ is the transpose of stresses, d is the vector of nodal displacements.

5. The axial stresses and nodal displacements are extracted from output file into Matlab.

Eg: xlsread('stressesanddisp.xls',stress¹, 'sheet1', 'A2:A4')

xlsread('stressesanddisp.xls',d, 'sheet1', 'A2:A4')

6. The axial stresses and nodal displacements are compared with the permissible values and if there is any violation, the penalty is calculated using adaptive penalty function.

If (actual stress/allowable stress -1) is less than zero, the penalty is zero.

If (actual stress/allowable stress -1) is greater than zero, then the penalty is $P_{\text{stress}} = c * [\max(0, \text{actual stress/allowable stress} - 1)]^2$

If (actual displacement/allowable displacement -1) is less than zero, the penalty is zero.

If (actual displacement/allowable displacement -1) is greater than zero, then the penalty is $P_{\text{disp}} = c * [\max(0, \text{actual displacement/allowable displacement} - 1)]^2$

Where c is a constant. Here, we tried with different constant and observed that the optimal solution is obtained for $c=5000$ for stresses and $c=50$ for threebartruss for displacements.

7. The fitness function is calculated including penalty as given below.

$f = \text{weight density} * (l(1) * x(1) + l(2) * x(2) \dots \dots \dots) + P_{\text{stress}} + P_{\text{disp}}$

8. The process is repeated until the minimum weight of the truss is obtained.

The Genetic Algorithm

John Holland at the University of Michigan (Goldberg 1989) originally proposed genetic algorithms which describes the nature of GAs of choice by combining a Darwinian survival of the fittest procedure with a structured, but randomized, information exchange to form a canonical search procedure that is capable of addressing a broad spectrum of problems. GAs are part of the group of Evolutionary Algorithms (EA). The evolutionary algorithms use the three main principles of the natural evolution: reproduction, natural selection and diversity of the species, maintained by the differences of each generation with the previous. GAs works with a set of individuals, representing possible solutions of the task. The selection principle is applied by using a criterion, giving an evaluation for the individual with respect to the desired solution. The best-suited individuals create the next generation. The outline of the basic Genetic Algorithm is given below.

1. Generate random population of n chromosomes (suitable solutions for the problem)

2. Evaluate the fitness $f(x)$ of each chromosome x in the population

3. Create a new population by repeating following steps until the new population is complete

a. Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected)

b. With a crossover probability cross over the parents to form a new offspring (children). If no crossover was performed, offspring is an exact copy of parents.

c. With a mutation probability mutate new offspring at each locus (position in chromosome).

d. Place new offspring in a new population

4. Use new generated population for a further run of algorithm

5. If the end condition is satisfied, stop, and return the best solution in current population

6. Go to step 2

Details of Case Study

In order to apply the proposed methodology, a standard trusses (three bar truss) is taken from the literature. The details of this truss are given below.

The three bar truss is shown in fig 1. This truss has three members and four nodes. The lengths of the members in cm are 254, 359.21, 254; the external forces are applied at nodes 1 magnitudes of 44537.4 kg in vertical direction and 22268.7 kg in horizontal direction; the density of the material is 7850 kg/m³; Young's modulus of the material is 2.008 x10⁵ kg/cm²; allowable axial stresses (both in compression and tension) are 1500 kg/cm²; allowable nodal displacements is 5.08cm. The design variables are a1, a2 and a3, i.e., the cross-sectional areas of the three members. A design member linking is done here by assuming the same areas for members 1 and 3, i.e., a1= a3 As a result of design member linking, there are only two design variables for the optimization problem corresponding to two groups, denoted by A1 and A2. Since GAs work on coded design variables, now it is necessary to code the design variables

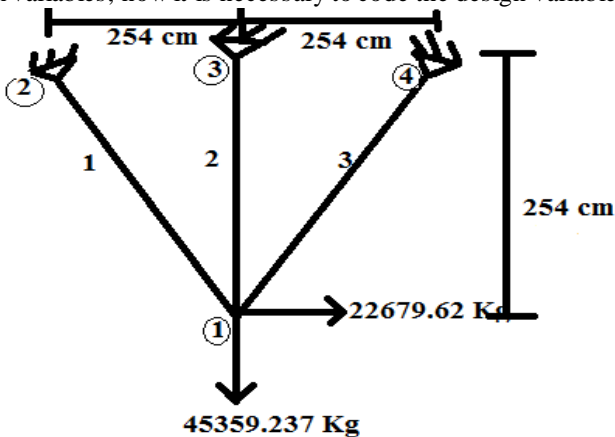


Fig 1. Three Bar Truss with details

Results and Discussions

The proposed methodology is applied on a standard benchmark truss (i.e. three bar truss). The truss analysis program developed by Sobhan Rostami and Ali Moeinadini (2009) is validated with Staad.Pro v8i. The various results obtained are presented in this section. The advantage of GA is that assigning different sets of areas each time while it runs, is captured in the present study by running the program several times until the best optimal solution is obtained. The parameters of GA considered in the present study are given below.

Validation of Analysis of 2D Truss program with Staad.Pro V8i

In the present study, the MATLAB program developed by Sobhan Rostami and Ali Moeinadini(2009) to analyze all types of 2-D trusses (with all degree of freedom) using stiffness method (matrix analysis) is used. General feature of this program includes one "m-file" and an "Excel" input file which to run this program both of them (truss.m and Truss.xls) must be saved in MATLAB directory. The input file (Truss.xls) must be saved as "Excel 2003". Then using MATLAB software run "m-file", later results can be seen in MATLAB's command window.

In order to ensure the analysis results are correct based on this program, an attempt is made to analyze the same truss using STAAD.Pro V8i.

The validation details are given in the following sections.

Input Data for 2D truss analysis program in Matlab and STAAD.Pro V8i

Fig. 2,4,6 and 7 describe the input data required for analysis using NewCivil.com program where as Fig.3,5, 8 and 9 describe the input details for STAAD.Pro V8i.

	A	B	C	D	E	F
4						
5	"Input data in Excel"					
6						
7						
8	% Enter number of nodes :					4
9	% Enter number of elements :					3
10						
11	% please insert coordinates of the nodes, respect to nodes ID :					
12						
13			ID	X	Y	
14			1	254	0	
15			2	0	254	
16			3	254	254	
17			4	508	254	
18						

Fig 2. Co-ordinate details of truss in Excel

Node	X cm	Y cm	Z cm
1	254.000	0.000	0.000
2	0.000	254.000	0.000
3	254.000	254.000	0.000
4	508.000	254.000	0.000
5			

Fig 3. Co-ordinate details of truss in STAAD.Pro V8i

	M	N	O	P	Q	R	S
4							
5	"Input data in Excel"						
6							
7	% please insert properties of the elements, respect to elements ID:						
8							
9							
10		ID	E	A	start	End	
11		1	2.10E+06	21.94161	1	2	
12		2	2.10E+06	13.62637	1	3	
13		3	2.10E+06	21.94161	1	4	
14							

Fig 4. Truss elements details, areas and young's modulus

Beam	Node A	Node B	Property Refn.	Material	Beta	Length cm
1	1	2	1	STEEL	0.0	359.210
2	1	3	1	STEEL	0.0	254.000
3	1	4	1	STEEL	0.0	359.210
4						

Fig 5. Beam details of truss

	X	Y	Z	AA	AB
26					
27	% Enter number of nodes that loaded:				1
28					
29		ID	X-force	Y-force	
30		1	22679.62	-45359.2	
31					

Fig 6. External Loads of truss

	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	
8	%Enter number of constrained nodes(number of supports):							3			
9	Attention:										
10	<the code for constrained in X direction is '1' and in y is '2' and for constrained in both directions is '0'>										
11											
12											
13		ID	Relative code								
14		2	0								
15		3	0								
16		4	0								

Fig 7. Supports Details of truss

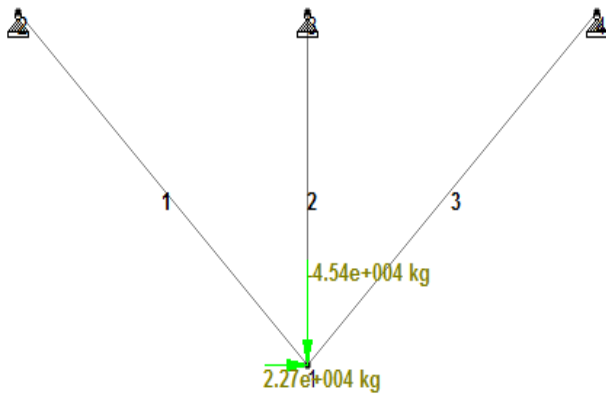


Fig 8. Three bar truss from STAAD.Pro V8i

```

Structure1 - STAAD Editor
File Edit View Tools Help
STAAD PLANE
START JOB INFORMATION
ENGINEER DATE 25-Aug-15
END JOB INFORMATION
INPUT WIDTH 79
UNIT CM KG
JOINT COORDINATES
1 254 0 0; 2 0 254 0; 3 254 254 0; 4 508 254 0;
MEMBER INCIDENCES
1 1 2; 2 1 3; 3 1 4;
DEFINE MATERIAL START
ISOTROPIC STEEL
E 2.09042e+006
POISSON 0.3
DENSITY 0.00783341
ALPHA 1.2e-005
DAMP 0.03
TYPE STEEL
STRENGTH FY 2581.92 FU 4158.4 RY 1.5 RT 1.2
END DEFINE MATERIAL
MEMBER PROPERTY INDIAN
1 TO 3 TABLE ST TUB1131134.5
CONSTANTS
MATERIAL STEEL ALL
SUPPORTS
2 TO 4 PINNED
LOAD 1 LOADTYPE None TITLE LOAD CASE 1
JOINT LOAD
1 FX 22679.6 FY -45359.2
PERFORM ANALYSIS PRINT ALL
FINISH
    
```

Fig 9. STAAD.Pro V8i input editor

Output Data obtained from 2D truss analysis program in Matlab and STAAD.Pro V8i:

The nodal displacements, support reactions, stresses and strains obtained from 2D truss analysis program in Matlab are given below in tables-1to5.

Table 1. Nodal displacements obtained from MATLAB

Node displacement: (cm)	
dx1	= 0.204179
dy1	= -0.169148
dx2	= 0
dy2	= 0
dx3	= 0
dy3	= 0
dx4	= 0
dy4	= 0

Table 2. Support reactions obtained from MATLAB

Support reactions: (kg)	
Rx2	= -20734
Ry2	= 20734
Rx3	= 0
Ry3	= 26570.8
Rx4	= -1945.6
Ry4	= -1945.6

Table 3. Elements force obtained from MATLAB

Elements force: (kg)	
p1	= 29322.3
p2	= 26570.8
p3	= -2751.5

Table 4. Elements stress obtained from MATLAB

Elements stress: (kg/cm ²)	
stress1	= 1543.28
stress2	= 1398.46
stress3	= -144.816

Table 5. Elements strain obtained from MATLAB

Elements strain:	
strain1	= 0.000734895
strain2	= 0.000665936
strain3	= -6.89599E-005

The nodal displacements, support reactions, stresses and strains obtained from STAAD.Pro V8i are given below in figures 10-13.

Node	L/C	Horizontal			Resultant	Rotational		
		X mm	Y mm	Z mm		rX rad	rY rad	rZ rad
1	1 LOAD CAS	2.051	-1.699	0.000	2.663	0.000	0.000	0.001
2	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	-0.000
3	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.001
4	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000	0.001

Fig 10. Generated Nodal Displacements by STAAD.Pro V8i

Table 6. The best 19 optimal solutions obtained for three bar truss

Areas(cm ²)		Stresses(kg/cm ²)			Displacements(cm)		Weight(kg)
A1	A2	1	2	3	1	2	
19.61	22.32	1444.35	1253.35	-191.01	0.19	-0.15	155.12
23.77	22.29	1254.83	1160.11	-94.72	0.16	-0.14	178.48
19.67	19.66	1490.70	1350.99	-139.71	0.20	-0.16	150.15
19.59	22.38	1444.26	1251.67	-192.59	0.20	-0.15	155.14
21.03	27.30	1300.22	1075.47	-224.75	0.18	-0.13	173.06
17.12	29.28	1484.94	1096.11	-388.83	0.23	-0.13	154.91
21.89	25.47	1286.39	1107.65	-178.74	0.18	-0.13	174.25
19.71	28.80	1344.51	1061.41	-283.10	0.20	-0.13	168.56
27.32	13.31	1282.07	1390.21	108.15	0.14	-0.17	180.62
19.01	28.77	1380.94	1074.50	-306.44	0.20	-0.13	164.57
17.65	27.16	1480.60	1144.28	-336.32	0.22	-0.14	153.70
21.04	25.98	1317.27	1110.20	-207.07	0.18	-0.13	170.46
21.58	24.45	1314.42	1142.27	-172.15	0.18	-0.14	170.44
21.84	19.34	1386.24	1303.94	-82.31	0.18	-0.16	161.74
17.36	28.20	1484.15	1120.71	-363.44	0.22	-0.14	154.13
22.25	20.66	1344.15	1246.55	-97.60	0.17	-0.15	166.65
21.17	17.55	1454.99	1395.01	-59.98	0.18	-0.17	154.38
22.19	14.55	1472.60	1499.74	27.14	0.17	-0.18	154.16
19.71	18.87	1505.36	1383.11	-122.25	0.20	0.17	149.38

(Note: - in stress column indicates compression and in displacement column shortening)

Node	L/C	Horizontal	Vertical	Horizontal	Moment		
		Fx kg	Fy kg	Fz kg	Mx kg-cm	My kg-cm	Mz kg-cm
2	1 LOAD CAS	-20723.817	20736.471	0.000	0.000	0.000	0.000
3	1 LOAD CAS	-8.430	26565.863	0.000	0.000	0.000	0.000
4	1 LOAD CAS	-1947.356	-1943.130	0.000	0.000	0.000	0.000

Fig 11. Generated Reactions by STAAD.Pro V8i

Beam	L/C	Node	Fx kg	Fy kg	Fz kg	Mx kg-cm	My kg-cm	Mz kg-cm
1	1 LOAD CAS	1	-29316.850	-8.949	0.000	0.000	0.000	-3214.535
		2	29316.850	8.949	0.000	0.000	0.000	-0.000
2	1 LOAD CAS	1	-26565.863	-8.430	0.000	0.000	0.000	-2141.199
		3	26565.863	8.430	0.000	0.000	0.000	0.000
3	1 LOAD CAS	1	2750.989	-2.988	0.000	0.000	0.000	-1073.336
		4	-2750.989	2.988	0.000	0.000	0.000	-0.000

Fig 12. Generated forces by STAAD.Pro V8i

Beam	L/C	Location	Location		Fx kg	My kg-cm	Mz kg-cm	Stress kg/cm ²
			Dist cm	Stress Pt(Y)				
1	1	1	359.210	0.000	0.000	-29316.850	-0.000	-1542.992
2	2	1	254.000	0.000	0.000	-26565.863	-0.000	-1398.203
3	3	1	359.210	0.000	0.000	2750.989	-0.000	144.789

Fig 13. Generated stresses by STAAD.Pro V8i

It is observed that the results obtained are almost matching 100%, so the 2D analysis program can be used to include in the optimization process and hence it is adopted in the present study.

GA parameters adopted in the present study and the results obtained

Generations

5 to 100; Population Size: 4 to 10; Crossover fraction: 0.6 to 0.8; Mutation function: mutationuniform (mutation fraction is 0.01); selection function: selectiontournament; fitness scaling function: fitnessscalingrank; and crossover function: crossovergathered.

By using the above GA parameters, the program is made run several times. Out of many optimal solutions obtained after running the program for several times, the best 19 optimal solutions are presented in Table 1. Each time while the program is running the function evaluation number (FEN) is varying between 180 to 200 which is very low and hence the solution obtained is very quick but the probability of guarantee of getting optimal solution is less. The fig . 14 represents the convergence of the solution to reach optimal values. However, after many optimal solutions, the least weight of all the solutions obtained is 150.15 kg. The stresses, displacements and weight are presented in the table for all 10 optimal solutions.

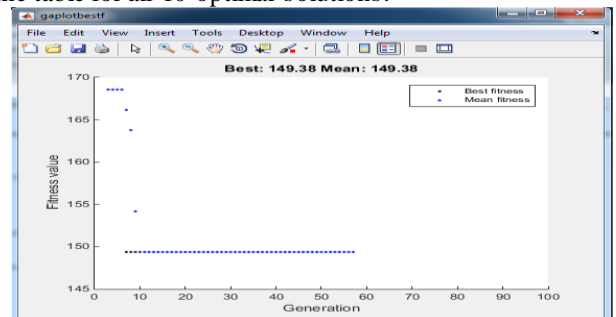


Fig 14. Graph between generation number and fitness value by plot function in MATLAB

Conclusions

In this paper, a detailed methodology on the development of the optimization model for design of 2D-steel truss using Genetic Algorithms (GAs) and 2D truss analysis using stiffness method program of NewCivil.com in the MATLAB is presented. This method is flexible and easy to use. One representative benchmark truss structure (3 bar truss) is optimized by the proposed method.

- This method gives quick results as the function evaluation number (FEN) is much less when compared to other methods; however it requires more number of runs.
- The optimal solution obtained is feasible with zero percent violation for stresses and displacements.
- The presented method is found to produce the optimal designs as good as or better than other stochastic optimization algorithms.
- The penalty function built for the research showed great capabilities to express the constraint violations of the solutions, leading to a much faster algorithm and much better solutions than MATLAB's built-in augmented lagrangian barrier algorithm.
- The least weight obtained for the truss is 150.15 kgs.
- In order to improve the solutions and to accelerate the convergence, more sophisticated GA operators might be helpful. This can be the target of a future research, as well as transforming the problem into a multi-objective optimization one.
- Future work will involve the application of the proposed method to other types of structural optimization design and/or constraints, such as under dynamic loads, involving both discrete design variables and continuous ones.

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