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Work Principle Method for Elastic Buckling and Postbuckling Behaviour of all Edges Simply Supported Thin Rectangular Plate

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ABSTRACT

All the previous studies on buckling and postbuckling loads of plate having all four edges simply supported (SSSS) have been limited to the use of assumed double trigonometric functions of displacement and stress. This has constrained further studies on practical plates' problems, as the buckling and postbuckling load equation derived thereafter by this approach lacked practical interpretation and application because major associated parameters such as the displacement parameter, W_{uv}, stress coefficient, W_{uv}^2 and load factor, K_{cx} were not determined. Hence, this paper obtained the displacement and stress functions of buckling and postbuckling loads of SSSS plate by applying the direct integration theory to the Kirchhoff's linear buckling governing differential equation and Von Karman's non–linear governing differential compatibility equation consecutively. Work principle was applied to the Von Karman's non–linear governing differential equilibrium equation to obtain the buckling and postbuckling loads of the plate. Yield/maximum stress of the plate was also obtained by imposing the bending stress influences of the in-plane loads on their direct stress influences. W_{uv}, W_{uv}^2 and K_{cx} were also defined. Therefore, the buckling/postbuckling load and critical yield stress characteristics of SSSS plate under uniformly distributed uniaxial loads could be analyzed completely.

Introduction

Postbuckling of plates may readily be understood through an analogy to a simple grillage model, as shown in figure 1. In the grillage model, the continuous plate is replaced by vertical columns and horizontal ties. Under loading on the x edges, the vertical columns will buckle. If they were not connected to the ties, they would buckle at the same load and no postbuckling reserve would exist. However, the ties are stretched as the columns buckle outward, thus restraining the motion and providing postbuckling reserve. The columns nearer to the supported edge are restrained more by the ties than those in the middle. This occurs too in a real plate, as more of the longitudinal in-plane compression is carried nearer the edges of the plate than in the center. Thus, the grillage model provides a working analogy for both the source of the postbuckling reserve and its most important result; i.e., re-distribution of longitudinal stresses.

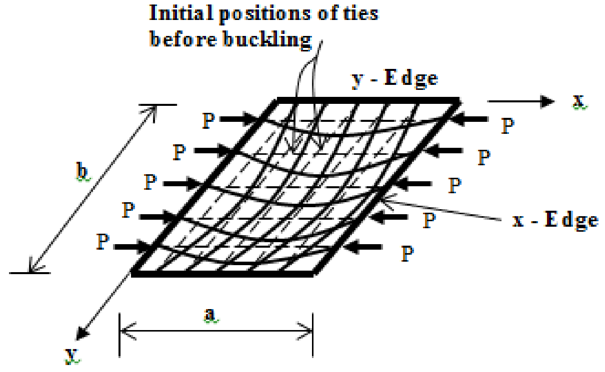

Figure 1. Post-buckling model of a thin plate under in-plane loads

In his context, Chaje [1] defined postbuckling load as the increase in stiffness with increase in deflection characteristic of the plate. This represents possible resistance of axial load by plate at excess of the critical load subsequent to buckling. Hence, the postbuckling response of thin elastic plates is very important in engineering analysis. Therefore, concerted effort to thoroughly studying thin plates postbuckling behaviour becomes imminent.

Postbuckling load analysis of thin plates accounts for the membrane stretching and their corresponding strains and stresses, while buckling analysis accounts also for the membrane stretching but do not consider the corresponding strains and stresses developed by the stretching. Postbuckling load analysis of plate involves nonlinear large-deflection plate bending theory, contrary to buckling load study which is based on classical or Kirchhoff's linear theory of plates. Researchers have not done much on postbuckling behaviour of thin plates as its analysis involves nonlinear large-deflection plate theory, which usually reduces to two indeterminate nonlinear governing differential equations originally derived by Von Karman in 1910 [2, 3]. These equations are written as follows:

$$\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}=E\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \quad (1)$$

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=$$
$$\frac{h}{D}\left[\frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right] \quad (2)$$

$$D\left[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]=$$

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$$N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{dxdy} + N_y \frac{\partial^2 w}{\partial y^2} \quad (3)$$

where, Φ is the stress function, w is deflection function, h is the plate's thickness and D is flexural rigidity. Equation is the "Compatibility Equation". It ensures that in an elastic plate the in-plane and out-of-plane displacements are compatible. Equations 2 and 3 are based on equilibrium principles of stress and in-plane loads respectively. They are termed "Equilibrium equations" [2, 3]. Equations 1 and 2 are usually called Von Karman's coupled equations.

The exact solutions of these equations have been a rigor from the conceptual time to the recent time, in which the coupled solutions would give the buckling/postbuckling load of plates from which the true failure load is determined. This exact solutions of these equation is imminent, as the critical load predicted by buckling analysis is adjudged unsatisfactory [1, 4]. Despite these revelations, very few researchers have made effort to solving these coupled equations to obtain the expressions for the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under compression. Researchers such as: Von Karman [5], Marguerre [6], Levy [7], Timoshenko and Woinowsky – Krieger [8], Volmir [9], Iyengar [10], Ventsel and Krauthammer [11], Chai [12]; and Yoo and Lee [1] have tried to solve these equations to obtain the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under uniaxial compression. They tried to solve the problem by assuming double trigonometric solutions for deflection, w and stress, Φ functions to solve the governing differential equations of thin rectangular plates. In which case, the buckling/postbuckling load as well as the actual failure load of thin rectangular plates under compression they obtained would also be said to be assumed, as the solutions of the governing differential equations of the plate (deflection and stress functions) were assumed abinitio. No researcher has bothered to solve for these parameters by the direct solution of these coupled governing differential equations.

In addition, these researchers restricted themselves to the use of either direct variational or indirect variational energy methods to finally evaluate the buckling/postbuckling load of this simply supported edges thin rectangular plate. None of the researchers considered applying direct work principle to finally evaluate the buckling/postbuckling loads of the SSSS plate or any other plate.

Von Karman evaluated the final buckling/postbuckling loads by solving the equilibrium equation 3, after assuming trigonometric functions for deflection and stress. Marguerre [6], Timoshenko and Woinowsky – Krieger [8] and Volmir [9] also assumed doubled trigonometric functions of deflection and stress; and employed the principle of minimum potential energy, rather than the equilibrium equation to furnish the final solution. Iyengar [10], Ventsel and Krauthammer [11] and Yoo and Lee [13] also assumed doubled trigonometric functions of deflection and stress used Galerkin's energy methods to obtain the final buckling/postbuckling load of SSSS plate.

Researchers in later years very often assumed doubled trigonometric functions of deflection and stress and used a similar type of approach, i.e., combining an exact solution of the compatibility equation with either evaluation and minimization of the potential energy, or an approximate solution (for example, using Galerkin's method, Ritz method or Rayleigh-Ritz method) of the equilibrium equation.

In all these, none of these researchers obtained the displacement parameter, W_{uv}, stress coefficient, W_{uv}^2 and load factor, K_{cx} associated with SSSS plate buckling and postbuckling characteristics. This situation has been the bane of comprehensive solution of the buckling/postbuckling characteristics of plates, as the actual yield/maximum stress of the plate could not be obtained, which this paper intends to address.

The Direct Integration Approach for Exact General Deflection and Stress Profile for Buckling and Postbuckling of SSSS Plate

Oguaghamba [14] used direct integral calculus approach and evaluated equation 3 to obtain the exact general displacement function of a buckled plate. The deflection function, W in its non – dimensional coordinates: R and Q is given as:

$$\mathrm{W(R,Q)} = \Lambda \sum_{m=0}^{4} \sum_{n=0}^{4} \mathrm{U_m R^m V_n Q^n} \quad (4)$$

where non – dimensional coordinates: R and Q in equation 4 relates to the usual independent coordinates x and y by the relation:

$$\mathrm{x = aR}: \ 0 \le R \le 1 \ and \ \mathrm{y = bQ}: \ 0 \le Q \le 1 \quad (5)$$

Coefficients in equation 4 were determined by Oguaghamba [14] using the Benthem's boundary conditions of SSSS plate as follows:

$\mathrm{U_o = 0;\ U_1 = U_4;\ U_2 = 0; U_3 = -2U_4;\ U_4 = U_4}$
$\mathrm{V_o = 0;\ V_1 = V_4;\ V_2 = 0; V_3 = -2V_4;\ V_4 = V_4}$

Hence, the SSSS displacement and stress functions (profiles) in buckling and postbuckling regimes are obtained by substituting these coefficients into equation 4 as:

$$\mathrm{W(R,Q) = W_{uv}(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)}$$
$$\mathrm{W(R,Q) = W_{uv} h_1(R,Q)} \quad (6)$$

where, $W_{uv} = \Lambda U_4 V_4$

$$\mathrm{h_1}(R,Q) = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \quad (7)$$

$\mathrm{U_m}$ and $\mathrm{V_n}$ are coefficients to be determined

He also solved equation 1 by direct integral calculus approach and factored the stress distribution of the plate prior to buckling to obtain the exact general stress function, $\phi(x,y)$ of buckling and postbuckling load of plate. This expression in non-dimensional coordinates, R and Q is given as:

$$\phi(R,Q) = \frac{Ep^2\Lambda^2}{(1+2p^2+\mathrm{p}^4)} \Bigg[\Bigg[\Bigg(\frac{U_1^{\ 2}}{24}R^4 + \frac{U_1U_2}{30}R^5 + \frac{1}{180}[2U_2^{\ 2} + 3U_1U_3]R^6 + \frac{1}{210}[2U_1U_4 + 3U_2U_3]R^7 + \frac{1}{1680}[16U_2U_4 + 9U_3^{\ 2}]R^8 + \frac{U_3U_4}{126}R^9 + \frac{U_4^{\ 2}}{315}R^{10}\Bigg) \times \Bigg(\frac{V_1^{\ 2}}{24}Q^4 + \frac{V_1V_2}{30}Q^5 + \frac{1}{180}[2V_2^{\ 2} + 3V_1V_3]Q^6 + \frac{1}{210}[2V_1V_4 + 3V_2V_3Q^3]Q^7 + \frac{1}{1680}[16V_2V_4 + 9V_3^{\ 2}]Q^8 + \frac{V_3V_4}{126}Q^9 + \frac{V_4^{\ 2}}{315}Q^{10}\Bigg) - \Bigg(\frac{U_0U_2}{12}R^4 + \frac{1}{60}[3U_0U_3 + U_1U_2]R^5 + \frac{1}{180}[6U_0U_4 + 3U_1U_3 + U_2^{\ 2}]R^6 + \frac{1}{210}[3U_1U_4 + 2U_2U_3]R^7 + \frac{1}{840}[3U_3^{\ 2} + 7U_2U_4]R^8 + \frac{U_3U_4}{168}R^9 + \frac{U_4^{\ 2}}{420}R^{10}\Bigg)\Bigg(\frac{V_0V_2}{12}Q^4 + \frac{1}{60}[3V_0V_3 + V_1V_2]Q^5 + \frac{1}{180}[6V_0V_4 + 3V_1V_3 + V_2^{\ 2}]Q^6 + \frac{1}{210}[3V_1V_4 + 2V_2V_3]Q^7 + \frac{1}{840}[3V_3^{\ 2} + 7V_2V_4]Q^8 + \frac{V_3V_4}{168}Q^9$$

$$+\frac{V_4{}^2}{420}Q^{10}\bigg)\bigg] - \frac{N_{cx}b^2}{2h}Q^2 \quad (8)$$

On substitution of the coefficients: U_m and V_n gave:

$$\phi(R,Q) = \frac{\varphi W_{uv}^2}{6350400}[(105R^4 - 84R^6 + 24R^7 + 54R^8 - 40R^9 + 8R^{10})(105Q^4 - 84Q^6 + 24Q^7 + 54Q^8 - 40Q^9 + 8Q^{10}) - 36(-14R^6 + 6R^7 + 6R^8 - 5R^9 + R^{10}) \times (-14Q^6 + 6Q^7 + 6Q^8 - 5Q^9 + Q^{10})] - \frac{N_{cx}b^2}{2h}Q^2 \quad (9)$$

In compact form, the stress function is given as:

$$\phi(R,Q) = \varphi W_{uv}^2 h_2(R,Q) - \frac{N_{cx}b^2}{2h}Q^2$$

$$\phi(R,Q) = \varphi W_{uv}^2 h_2(R,Q) - N_{cx} \in Q^2 \quad (10)$$

where,

$$\varphi = \frac{Ep^2}{(1+2p^2+p^4)} \quad (11)$$

W_{uv}^2 = Stress function coefficient for a plate in postbuckling regime

$$\epsilon = \frac{b^2}{2h} \quad (12)$$

Λ^2 = Consolidated coefficient factor of stress in postbuckling regime

$h_2(R,Q)$ = Non-dimensional stress shape (profile) function of the slightly bent plate, given as:

$$h_2(R,Q) = \frac{1}{6350400}[(105R^4 - 84R^6 + 24R^7 + 54R^8 - 40R^9 + 8R^{10})(105Q^4 - 84Q^6 + 24Q^7 + 54Q^8 - 40Q^9 + 8Q^{10}) - 36(-14R^6 + 6R^7 + 6R^8 - 5R^9 + R^{10}) \times (-14Q^6 + 6Q^7 + 6Q^8 - 5Q^9 + Q^{10})] \quad (13)$$

Expressions for the deflection and stress functions factors, W_{uv} and W_{uv}^2 of the plate behaviour under pre – buckling, buckling and post buckling regimes deduced by Oguaghamba [14] is given as:

$$W_{uv} = \frac{\alpha h}{h_{1max}};\ W_{uv}^2 = \frac{\alpha^2 h^2}{(h_{1max})^2};\ h_{1max} = \frac{25}{256} \quad (14)$$

Work Principle Application for Buckling and Postbuckling Load and Stress of SSSS Plate

With the complete solutions of deflection and stress in equations 6 and 10 respectively, Oguaghamba [14] applied the work principle according to Ibearugbulem *et al.* [15, 16] to equation 2 in non – dimensional coefficient and obtained the exact general buckling and postbuckling load, N_{cx} (R, Q) of thin rectangular plates in non – dimensional coordinates as:

$$N_{cx} = \left(-\frac{49}{484}\beta + \frac{294}{121}\frac{(1-\mu^2)p^2W_{uv}^2}{(1+2p^2+p^4)h^2}\psi\right)\frac{\pi^2 D}{b^2} \quad (15)$$

where,

$$\beta = \frac{\int\int_{0,0}^{1,1}\left(\frac{\partial^4 h_1}{p^2\partial R^4}\cdot h_1 + \frac{2\partial^4 h_1}{\partial R^2\partial Q^2}\cdot h_1 + p^2\frac{\partial^4 h_1}{\partial Q^4}\cdot h_1\right)dRdQ}{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial R^2}\cdot h_1\right)dRdQ} \quad (16)$$

$$\psi = \frac{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial Q^2}\frac{\partial^2 h_2}{\partial R^2} + \frac{\partial^2 h_1}{\partial R^2}\cdot\frac{\partial^2 h_2}{\partial Q^2} - \frac{2\partial^2 h_1}{\partial R\partial Q}\frac{\partial^2 h_2}{\partial R\partial Q}\right)h_1 dRdQ}{\int\int_{0,0}^{1,1}\left(\frac{\partial^2 h_1}{\partial R^2}\cdot h_1\right)dRdQ} \quad (17)$$

where the first and the second terms account for critical buckling load of the plate and the gain in load of the plate at postbuckling regime respectively.

Substituting the expressions of $h_1(R,Q)$ and $h_2(R,Q)$ into equations 16 and 17; solving out the resulting integrand expressions and substituting their results into equation 15 gave the buckling and postbuckling load expression for an SSSS thin rectangular plate as:

$$N_{cx} = \left[\left(\frac{1.00048614}{P^2} + 1.99866702 + 1.00048614P^2\right) + 1.92493709\times 10^{-3}\frac{(1-\mu^2)p^2W_{uv}^2}{(1+2p^2+p^4)h^2}\right]\frac{D\pi^2}{b^2} \quad (18)$$

Introducing the expression of W_{uv}^2 given in equation 14 into equation 18; the buckling and postbuckling load expression for an SSSS thin rectangular plate reduced to:

$$N_{cx} = \left[\left(\frac{1.00048614}{P^2} + 1.99866702 + 1.00048614P^2\right) + 2.018442834\times 10^{-1}\frac{p^2\alpha^2(1-\mu^2)}{(1+2p^2+p^4)}\right]\frac{D\pi^2}{b^2} \quad (19)$$

$$N_{cx} = K_{cx}\frac{D\pi^2}{b^2} \quad (20)$$

$$K_{cx} = \left(\frac{1.00048614}{P^2} + 1.99866702 + 1.00048614P^2\right) + 2.018442834\times 10^{-1}\frac{p^2\alpha^2(1-\mu^2)}{(1+2p^2+p^4)} \quad (21)$$

where, K_{cx} is the buckling and postbuckling load coefficient.

Oguaghamba [14] also evaluated the inplane and bending buckling and postbuckling yield stress developed by the SSSS as:

$$\sigma_{x_{max}} = \frac{N_{cx}}{h} - \frac{72D}{h^2b^2}W_{uv}\left(\frac{1}{p^2}(-R+R^2)(Q-2Q^3+Q^4) + \mu(R-2R^3+R^4)(-Q+Q^2)\right) \quad (22)$$

Critical Yield Stress for SSSS Plate under Buckling and Postbuckling Loads

The SSSS thin rectangular plate under uniaxial loads reaches its critical yield stress at yield coordinates, (R' = 0.5, Q' = 0.5) [14]. Substituting W_{uv} in equation 14 and N_{cx} in equation 19 into equation 22 at non-dimensional yield coordinates, R' and Q' gave the critical yield stress as:

$$\sigma_{cri} = \frac{N_{cx}}{h} + 5.625\frac{D}{hb^2}\frac{\alpha}{h_{1max}}\left(\frac{1}{P^2} + \mu\right)$$

$$\sigma_{cri} = \left[\left(\frac{1.00048614}{P^2} + 1.99866702 + 1.00048614P^2\right) + 2.018442834\times 10^{-1}\frac{p^2\alpha^2(1-\mu^2)}{(1+2p^2+p^4)}\right]\frac{D\pi^2}{hb^2} + 57.6\alpha\left(\frac{1}{P^2} + \mu\right)\frac{D}{hb^2} \quad (23)$$

Results and Discussions

Figure 2 shows an SSSS thin rectangular plate subjected to uniaxial compression loads on the R - edges. The interest is to evaluate the buckling and postbuckling load of the plate.

Iyengar [10]; Ventsel and Krauthammer [11]; Szilard [4]; and Yoo and Lee [13] in their separate works obtained the buckling and postbuckling load of SSSS – thin rectangular plate as:

$$N_{cx} = \left[\left(\frac{1}{P^2} + 2 + P^2\right) + 3\left(\frac{1}{p^2} + p^2\right)\frac{W_{11}^2}{h^2}\frac{(1-\mu^2)}{4}\right]\frac{D\pi^2}{b^2} \quad (24)$$

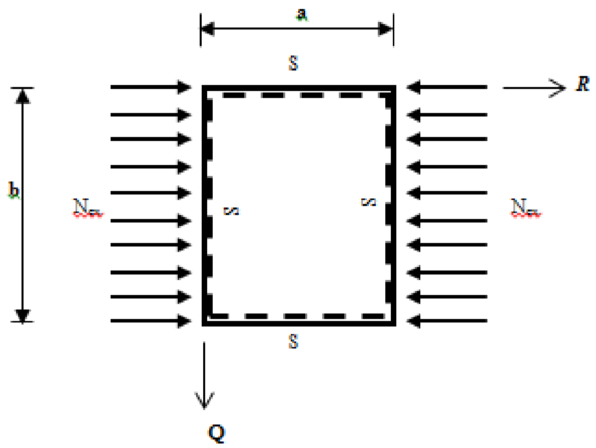

Figure 2. SSSS – Thin Rectangular Plate under Uniaxial Load on R - Edges

Equation 19 looks somewhat similar to equation 24. The first term (pre-buckling range load) of these expressions 19 and 24 are approximately the same, despite that the approximate expressions used for the buckled middle surface of the plate are not the same. In the former, the expression for the buckled middle surface of the plate was obtained from the direct solution of the buckling governing differential equation of plate, whereas, the latter assumed doubled trigonometic function as the approximate buckled middle surface of the plate. However, this trend of conformity of coefficients in the pre-buckling range is not the same for the postbuckling range for the former and latter formulations (i.e.; present study formulation and the literature formulation). The stress function coefficient, W_{uv}^2 in the present study is well defined in equation 14. This is not the case for the stress function coefficient, W_{11}^2 in the literature formulation.

Hence, the stress function coefficient, W_{11}^2 in the literature formulation has no empirical interpretation. This leaves the literature formulation as a mere theoretical exercise rather than real life adventure.

The aspect ratio amplifying factor for the study and literature formulations do not also agree. In the study, the aspect ratio amplifying factor is given as:

$$1.92493709 \times 10^{-3} \frac{p^2}{(1 + 2p^2 + p^4)} \tag{25}$$

Whereas, the literature formulation is given as:

$$\frac{3}{4}\left(\frac{1}{p^2} + p^2\right) \tag{26}$$

The variation on the aspect ratio amplifying factor may be said to be quite understandable, as both expressions were obtained from different backgrounds for buckled middle surface of the plate. Nevertheless, a close consideration of these aspect ratio amplifying factors suggests that the double trigonometric function as used in the earlier studies might not have been a good approximation for the buckled middle surface of the plate in the postbuckling range. For instance, at a low aspect ratio, p (i.e. p = a/b less than unity), the plate would relatively behave more as strips of short columns. Failure in this context would be by buckling along the R – edge and crushing along the Q – edge. Hence, there would be no postbuckling reserved load because there is no significant deformation; but there would be high material stiffness displayed by the plate due to material in compression. This scenario is demonstrated in equation 25, where the aspect ratio amplying factor is approximately equal to the square of inverse of p (where $1/p^2 \geq 1$). Similarly, at a high aspect ratio, p (i.e. p = a/b greater than unity), the plate would relatively behave more as strips of slender columns. Failure in this context would be by buckling along the R and Q edges.

Prior to failure, the plate's strength would be majorly due to postbuckling reserve increase with corresponding increase in deformation; and slight slender material stiffness. Equation 25 depicts also this scenario as the stiffness is approximately the inverse of square of p (where $1/p^2 \leq 1$).

In contrast, the literature formulation for the aspect ratio amplifying factor do not make these distinctions. At a low aspect ratio, p (i.e. p = a/b less than unity) and high aspect ratio, p (i.e. p = a/b greater than unity), equation 26 depicts that the plate possesses high postbuckling reserve and low postbuckling reserve respectively. This is utterly erroneous.

In addition, the inability of the previous researchers to define the stress function coefficient, W_{11}^2 of the SSSS plate under buckling and postbuckling loads constrained their effort towards obtaining the actual critical yield stress of such a plate. The present study clearly defined these parameters: the displacement parameter, W_{uv}, stress coefficient, $W_{uv}{}^2$ and load factor, K_{cx}. With this parameters, the present study obtained critical yield stress of the SSSS plate under buckling and postbuckling loads as given in equation 23. Therefore, equations 19 and 23 can be used to obtain the actual value of the buckling and postbuckling load and critical yield stress of an SSSS plate, knowing other parameters: deflection coefficient, α; Poisson ration, μ; breadth, b; aspect ratio, p and thickness, h of the plate.

For instance, an ASTM grade A36 thin rectangular steel plate possessing SSSS edge conditions; subjected to uniformly distributed in-plane load on its R – edge, b and having the following physical and geometric properties as: breadth, b = 4000mm; thickness of plate, h = 20mm; yield load, σ_{ys} = 250 MPa; Ultimate Stress, σ_u = 400 – 550 MPa; Poisson's ratio, μ = 0.30; Modulus of elasticity, E = 200 GPa; density of plate, ρ = 7,800 kg/m^3 would have its buckling and postbuckling load coefficient and critical yield stress through aspect ratios range: $0.5 \leq p \leq 1.0$; and deflection coefficients range: $0 \leq \alpha \leq 5.0$ as in figures 3 and 4.

Figures 3 and 4 show the graphs of postbuckling load coefficients, K_{cx} and buckling and postbuckling load critical yield/maximum stress, σ_{cri} variations with deflection coefficients, α (where, $0 \leq \alpha \leq 5.0$) at aspect ratio of unity i.e., p = 1.

In Figure 3, the graph shows that the buckling and postbuckling load parameter, K_{cx} increases quadratically as the out of plane deflection factor, α increases. The buckling and postbuckling load parameters, K_{cx} are higher at other aspect ratios lower than 1.0. Thus, the behaviour of buckling and postbuckling load parameter which is a function of the buckling and postbuckling load means that the buckling and postbuckling load would continue to increase as the out of plane deflection increases. This is contrary to the literature's hypothesis that the axial stiffness reduces, as the plate as a whole sustains increase in load after buckling or deflection.

However, this hypothesis is clarified in figure. The linear relationship in the yield stress behaviour against out of plane deflection explained that the plate would resist extra in-plane load after buckling, while reduces in material stiffness. That is, the plate resists further in-plane load due to postbuckling reserve but loses stiffness due to in-plane bending stress developed. Where the in-plane load bending stress is not considered, the plate would behave as if it had higher yield stress. Though, it does not.

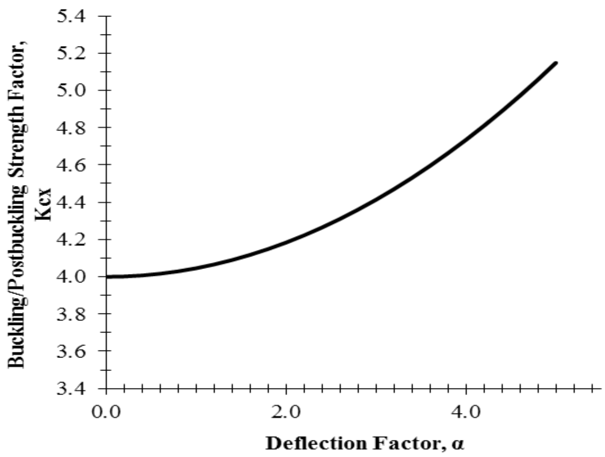

Figure 3. Buckling and Postbuckling Load Coefficient, K_{cx} and Deflection Factor, α at aspect ratio of unity for SSSS – Plate

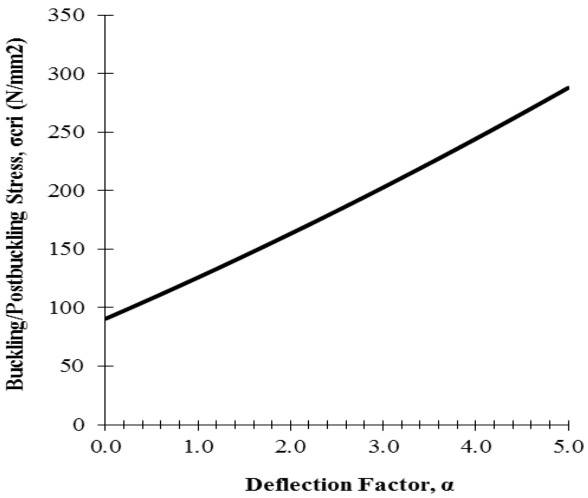

Figure 4. Buckling and postbuckling load critical yield/maximum stress, σ_{cri} and deflection Factor, α at aspect ratio of unity for SSSS – Plate

For an SSSS plate material having yield stress of 250MPa, failure of would not occur until the out of plane deflection of the plate is about four times the thickness of the plate (4h). It is at this point that the induced stress in the plate would reach the failure stress for the plate material, of which failure may occur.

Conclusion

SSSS plate possesses increased load resistance beyond buckling because the supports which are hinges and rollers allow stretching of the longitudinal fibers of the plate on deformation of the transverse fibers. In this way, the longitudinal fibers of the plate would undergo stress redistribution under load, as well as develop transverse tensile stresses after buckling. These tensile stresses provide the postbuckling reserve load. Thus, additional load may often be applied without structural damage.

However, as the structural requirements for plates are that the structure should not be so flexible that the behaviour would cause alarm or discomfort to the users; other structural criteria may be applied to select the applicable load below this yield stress, which is also far above the buckling load.

The results of the buckling and postbuckling loads characteristics of this study are novel. The comparative analysis of the study's results and the literature results show that double trigonometric functions as were used in the past for the deflection and stress functions of the SSSS plate are not satisfactory for predicting postbuckling load characteristics of SSSS plate or any other plate. The corresponding particular deflection, w and stress, ϕ functions established in this study are satisfactory for predicting the buckling and postbuckling load characteristics of SSSS plate. Therefore, the expressions obtained in this study for the buckling/postbuckling loads and critical yield/ maximum stresses of thin rectangular plates should be used for practical analysis and design of the thin rectangular plates under uniformly distributed uniaxial in – plane loads.

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