



Resonance Response of Afm Micro-Cantilever using Wave Propagation Method

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ABSTRACT

Wave propagation method (WPM) is utilized, for the first time, to study the resonance response of Atomic Force Microscopy (AFM). The damping coefficient with in the interaction of tip-sample, which is assumed as a linear visco-elastic force, is considered in this paper as it has been neglected in the WPM studies of beam in previous investigations. Experiment and analytical results are provided in order to show the reliability and correctness of WPM method. The results are in good agreements with the experiments and show that the WPM is more accurate than analytical method.

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Introduction

All Atomic Force Microscopy is widely used for probing surface properties on atomic scale. AFM has been utilized to topography the surface of sample in nano-scale, study the properties of materials like adhesion [1] Young modulus, elasticity and friction [2], scanning the unknown potential function of materials, and study the behavior of materials in their natural environments [3]. AFM is growing in such a great pace that in near future scientists will find some applications of it in their fields.

Generally AFM functions in three different modes, non-contact mode [4], tapping mode, and contact mode. In tapping mode, tip contacts the sample surface for a short period of time during the scanning of the surface while micro-cantilever vibrates at a frequency near to the resonant frequency of micro-cantilever [5]. The dynamics of AFM is affected by the tip and sample interaction forces.

By examining the variation in amplitude and phase of the micro-cantilever vibration it is possible to study the properties of sample such as topography of scanning surface. Various sets of parameters affect the dynamics of AFM like contact stiffness between tip and sample [6], probe mass [7], micro-cantilever's features [8], non-linear nature of the tip and sample interaction forces [9]. Cumulative effects of all of these parameters complicate the recognition of dynamic behavior of AFM.

Mass-spring system [10] and continuous beam model [11] are two general methods of approaching dynamic behavior of micro-cantilever vibration in studying the AFM. Mass-spring model is a useful method to find the dynamic behavior of AFM especially when the nonlinear behavior of system is desired [9,10].

However, Burnham et al [12] proved that lumped mass model is impractical when AFM works in fluid environment as the hydrodynamic force exerted on a cantilever may not be well presented by drag force on a sphere. However it has been shown that linear continuous beam model is more reliable [13].

Semi analytical solution is a useful method to study the dynamics of AFM. Mokhtari-Nezhad et al [14] used semi-

analytical solution to study the influence of tip mass on resonant frequency of AFM.

Approximate methods like mode superposition method [15, 16], Rayleigh-Ritz method and finite-element method (FEM) [17] are useful especially when analytical or semi analytical solutions are complicated or impossible. But each of these methods has some inadequacies like limitations in obtaining the number of resonance frequencies of the system and low accuracy [18].

However, Wave Propagation Method (WPM) which offers the privileges of finding the exact value of all resonant frequencies, the number of more frequencies as well as obtaining the resonance frequencies of beam with non-constant thickness makes it as a noble method for investigating the vibration of the AFM micro-beam system. In this paper WPM method is developed and utilized to find the first two resonant frequencies of the system. The results are then compared with experiment.

AFM vibration analysis

When micro cantilever becomes close to the sample, interaction forces between tip and sample affect the dynamics of AFM. Two general regimes are distinguished: short range repulsive and wide range attractive forces [12]. The nature of these forces are nonlinear but if the beam vibrates near to equilibrium position with small amplitude then these forces can be considered as linearized visco-elastic forces with constant coefficients [14].

Based on Euler Bernoulli equation of beams the governing equation of the motion is [12]:

$$EI_z \frac{\partial^4 y(x,t)}{\partial x^4} + \rho A \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad (1)$$

Where ρ is the density of cantilever, A is cross-section area, E is Young modulus, I is area moment of inertia around Z axis, and $y(x,t)$ is the displacement of beam in Y direction as it is shown in Fig 1.

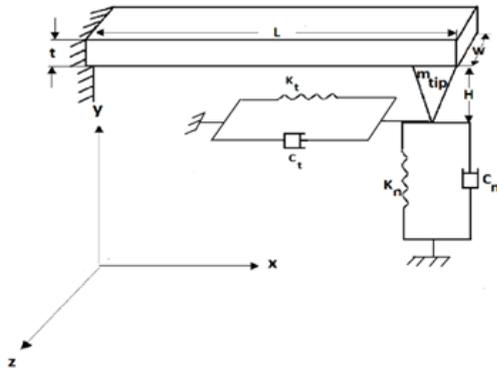


Fig 1. Schematic diagram of AFM micro-cantilever and tip-sample interaction

By assuming harmonic motion of micro-cantilever holder defined as $h_y(t) = y_0 \cdot e^{i\Omega t}$, the solution of Eq. (1) can be declared by $y(x,t) = Y(x) \cdot e^{i\Omega t}$. Due to assuming linear dynamic and harmonic oscillation of micro-cantilever holder the relative displacement at the tip head is also assumed in harmonic. The corresponding boundary conditions of micro-cantilever are:

$$Y(0) = y_0 \quad (2(a))$$

$$\frac{dY(x=0)}{dx} = 0 \quad (2(b))$$

$$EI \cdot \frac{d^2Y(x=L)}{dx^2} = f_x \cdot H \quad (2(c))$$

$$EI \cdot \frac{d^3Y(x=L)}{dx^3} = f_y + m_{tip} \cdot \frac{d^2Y(L)}{dt^2} \quad (2(d))$$

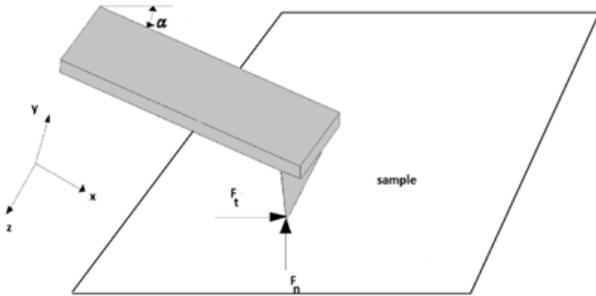


Fig 2. Tangential and normal forces exerted at micro-cantilever and its probe

f_x and f_y are resultant forces which are applied at end of micro-cantilever at $x=0$ in x and y directions respectively, H and m_{tip} are the length and the mass of probe which is considered as a pointed mass as shown in the Fig 1.

With respect to Fig.2, f_x and f_y are equal to:

$$f_x = F_t \cos(\alpha) - F_n \sin(\alpha) \quad (3(a))$$

$$f_x = F_n \cos(\alpha) + F_t \sin(\alpha) \quad (3(b))$$

Where F_n and F_t are normal and lateral forces and α is tilting angle of micro-cantilever. Lateral and normal forces can be obtained as following:

$$F_n = k_n \cdot \Delta_n + c_n \cdot \dot{\Delta}_n \quad (4(a))$$

$$F_t = k_t \cdot \Delta_t + c_t \cdot \dot{\Delta}_t \quad (4(b))$$

In the above equations, t and n are tangential and normal directions and Δ is the displacement of the free end of the

micro-cantilever. The displacement of end of micro-cantilever are obtained by following relations:

$$\Delta_n = Y(L) \cos(\alpha) - \theta(L) H \sin(\alpha) \quad (5(a))$$

$$\Delta_t = Y(L) \sin(\alpha) + \theta(L) H \cos(\alpha) \quad (5(b))$$

Where $\theta(L)$ is the tilting angle of cantilever at $x=L$. Solving governing equation of the motion Eq. 1 with respect to boundary conditions Equations 2.a, 2.b, 2.c, and 2.d will lead us to find the response function of micro-cantilever motion ($Y(x)$).

Wave Propagation Method

Assume that the general solution of the Eq-1 is:

$$w(x,t) = (c_1 e^{-ikx} + c_2 e^{ikx} + c_3 e^{-kx} + c_4 e^{kx}) e^{i\omega t} \quad (6)$$

Where c_1 , c_2 , c_3 and c_4 are constants and k is wave number and is equal to:

$$k = \sqrt[4]{\frac{\rho A \omega^2}{EI}} \quad (7)$$

Based on WPM method [1], Eq-6 can be rewritten as positive (a^+) and negative (a^-) waves:

$$a^+ = \begin{bmatrix} c_1 e^{-ikx} \\ c_1 e^{-kx} \end{bmatrix} \quad (8(a))$$

$$a^- = \begin{bmatrix} c_1 e^{ikx} \\ c_1 e^{kx} \end{bmatrix} \quad (8(b))$$

Then the relation between positive and negative wave matrices is:

$$\begin{Bmatrix} a^+(x+l) \\ a^-(x+l) \end{Bmatrix} = \begin{pmatrix} F^+ & 0 \\ 0 & F^- \end{pmatrix} \begin{Bmatrix} a^+(x) \\ a^-(x) \end{Bmatrix} \quad (9)$$

Where F^+ and F^- are propagation matrices which are as following:

$$F^+ = F^- = \begin{pmatrix} e^{-ikl} & 0 \\ 0 & e^{-kl} \end{pmatrix} \quad (10)$$

Where Eq. 2(a) and 2(b) present the zero displacement and zero slip at $x=0$ respectively. Eq. 2(c) and 2(d) present applied vertical momentum and shear force at end of micro-beam respectively.

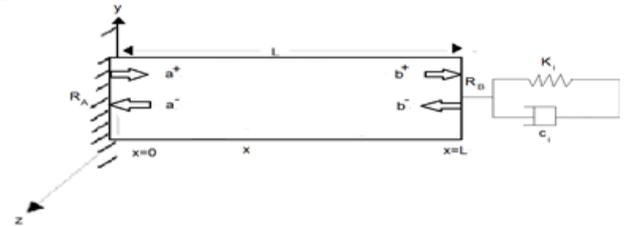


Fig 3. Schematic figure of positive and negative waves at ends of micro-cantilever. External forces are equalized in the forms of springs and dampers

Displacement and force matrices are defined as follows:

$$\bar{F} = \left[-EI \frac{\partial^3 u(x,t)}{\partial x^3}, EI \frac{\partial^2 u(x,t)}{\partial x^2} \right] \quad (11a)$$

$$\bar{W} = \left[w, \frac{\partial w}{\partial x} \right] \quad (11b)$$

Where the relationship between the state vector in physical domain and in the wave domain is obtained as follow [19]:

$$\begin{Bmatrix} \bar{W} \\ \bar{F} \end{Bmatrix} = \begin{pmatrix} \psi^+ & \psi^- \\ \phi^+ & \phi^- \end{pmatrix} \begin{Bmatrix} a^+ \\ a^- \end{Bmatrix} \quad (12)$$

ψ^+ , ψ^- , ϕ^+ , ϕ^- are 2×2 matrices that are as follows:

$$\psi^+ = \begin{pmatrix} 1 & 1 \\ -ik & -k \end{pmatrix} \quad (13a)$$

$$\psi^- = \begin{pmatrix} 1 & 1 \\ ik & k \end{pmatrix} \quad (13b)$$

$$\phi^+ = EI \begin{pmatrix} ik^3 & k^3 \\ -k^2 & k^2 \end{pmatrix} \quad (13c)$$

$$\phi^- = EI \begin{pmatrix} ik^3 & -k^3 \\ -k^2 & k^2 \end{pmatrix} \quad (13d)$$

Regarding to Fig 3 the relations between positive and negative waves at the ends of micro-beam are:

$$a^+ = R_A a^- \quad (14a)$$

$$b^- = R_B b^+ \quad (14b)$$

Where R_A and R_B are the reflection matrices at points A and B. Boundary conditions for the clamped of the cantilever include: Zero movement and zero slope. Based on above boundary conditions, reflection matrix at the clamped side of the cantilever can be calculated:

$$R_A = \begin{pmatrix} -i & -(i+1) \\ -(1-i) & i \end{pmatrix} \quad (15)$$

Transition matrix at the open end of the beam depends on the boundary conditions. Considering the impact of external forces in the form of the spring and damper, this reflection matrix is given by:

$$R_B = ([k]\psi^- - \phi^-)^{-1}(\phi^+ - [k]\psi^+) \quad (16)$$

K is the stiffness matrix and depends on the boundary conditions, For micro-cantilever of the AFM device in pure bending vibration mode and considering the surface reaction forces as a linear spring and damper, the stiffness matrix is as follows:

$$[k] = \begin{pmatrix} \omega^2 - m_t - k_n - i\omega c_n & 0 \\ 0 & H^2(k_t + i\omega c_t) \end{pmatrix} \quad (17)$$

Where m_t is the tip-mass, k_n and k_t are spring coefficient of tip-sample interaction in normal and lateral directions respectively, and ω is the frequency of micro-cantilever. By obtaining the wave equations, the natural frequency of the system can be found. These equations are as the following:

$$a^+ = R_A a^- \quad (18)$$

$$b^- = R_B b^+ \quad (19)$$

$$b^+ = F^+ a^+ \quad (20)$$

$$a^- = F^- b^- \quad (21)$$

These equations can be written in matrix form, therefore, uniform beam equation with linear boundary conditions and considering the applied forces on the beam as a linear spring and damper are:

$$\begin{bmatrix} -I & R_A & 0 & 0 \\ F^+ & 0 & -I & 0 \\ 0 & -I & 0 & F^- \\ 0 & 0 & R_B & -I \end{bmatrix} \begin{bmatrix} a^+ \\ a^- \\ b^+ \\ b^- \end{bmatrix} = 0 \quad (22)$$

Where b^+ and b^- are the wave matrices at $x=L$ as it is shown in Fig 3. To have a unique answer, the determinant of the coefficient matrix of the above matrix should be equal to zero. By solving this equation, we can obtain the resonant frequency of the micro-cantilever.

Results and discussion

In this paper the frequency response of the micro-cantilever of AFM is studied using the Wave Propagation Method. Three types of micro-cantilever are used in the experiments of the current study which was done in ARA-AFM laboratory [21]. The three types of micro-cantilever, NSG01/Tin, NSG01/Co and NSG11, are utilized which the properties of these cantilevers are listed in table 1. The same properties of micro-cantilevers were used in both analytical and experimental analysis in order to make comparison.

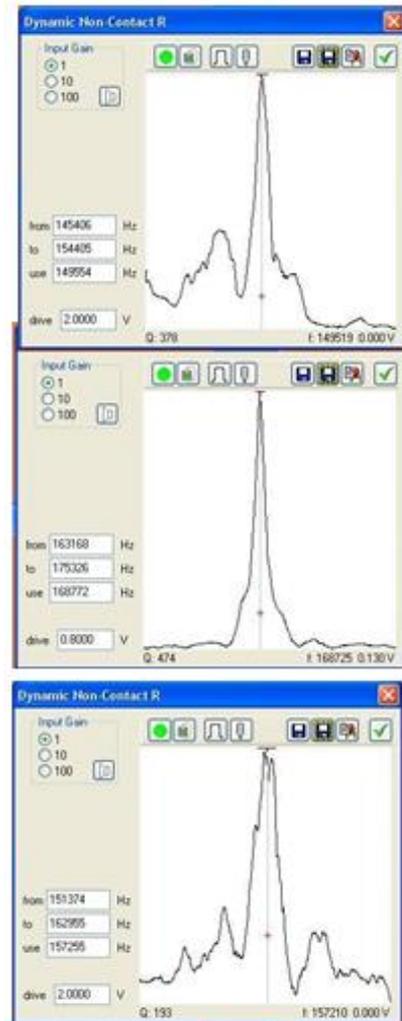


Fig 4. Obtained resonant frequency curve from experiment with different micro-cantilever and its tip, a: NSG01/Co, b: NSG01/Tin, c: NSG11

In order to find the resonance frequency by the AFM device, operator specifies an approximate domain of frequency. Actually, the frequency domain, which should be specified for the device, is already provided by the manufacturer and is given to the operator.

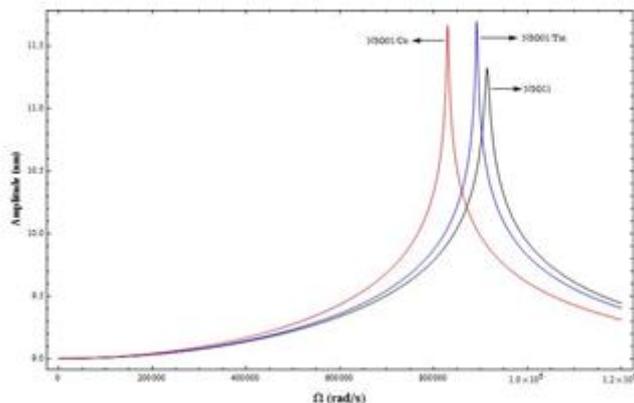
Table 1. Parameters and characteristics of micro-cantilever and its probe according to the catalogue of producer

Micro-cantilever	NSG01/Tin	NSG01/Co	NSG11
length $\pm 5 \mu$	125	125	130
width $\pm 5 \mu$	30	30	35
thickness (μ m)	1.5-2.5	2	1.7-2.3
Domain of RF (KH)	87-230	90-220	115-190
Damping coeff. (Ns/m)	10^{-3}	10^{-3}	8.3×10^{-3}
Young's modulus (pa)	1.3×10^{11}	1.3×10^{11}	1.5×10^{11}
Probe mass (kg)	1.036×10^{-19}	3.027×10^{-17}	2.021×10^{-20}
Micro-cantilever	NSG01/Tin	NSG01/Co	NSG11
length $\pm 5 \mu$	125	125	130
width $\pm 5 \mu$	30	30	35
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Young's modulus (pa)	1.3×10^{11}	1.3×10^{11}	1.5×10^{11}
Probe mass (kg)	1.036×10^{-19}	3.027×10^{-17}	2.021×10^{-20}

Table 2. Comparison of first resonant frequencies obtained by WPM method with experiment and analytical method

Tip model	Obtained RF from WPM (rad/s)	Experiment R.F Result (rad/s)	Obtained RF from Analytical (rad/s)
NSG01/Tin	1122489	1060000	≈ 1184600
NSG01/Co	1015157	939680	≈ 1106700
NSG11	1098459	988000	≈ 1139200

The device begins to increase the vibration frequency of the micro-cantilever from minimum to maximum along the specified frequency domain. Figure 4 illustrates the frequency response of the system examined by the operator.

**Fig 5. Frequency response of three different MC obtained from analytical study**

Moreover, resonance frequencies of micro-cantilevers are obtained using analytical method. Response function of all three types of micro-cantilever ($Y(L)$) are illustrated in Fig. 5.

By solving the characterization equation of equation 17, the resonance frequencies of micro-cantilever can be obtained using

WPM. The 1st resonant frequency of all micro-cantilever types which were obtained from the experiment, WPM, and analytical method are listed in table 2.

Some issues such as approximations for modeling the micro-cantilever system like using the linear beam theory and the linear visco-elastic of tip-sample interaction as well as the errors in parameters' measurement like cantilever features, have some effects on the obtaining RFs of system, experimentally and theoretically, and it would lead to some deviation from exact resonant frequency.

However, Based on above mentioned, obtained RFs from WPM method are in good agreement with the experiments. It is shown that WPM is reliable method to find the resonance frequency of AFM micro-cantilever. Results show that this method is more accurate than the analytical method.

Conclusion

In this paper modified wave propagation method (WPM) , for the first time, is utilized to find the resonance frequencies of Atomic Force Microscope micro-cantilever by considering the damping coefficients at boundary condition and tip mass. It is shown that Wave Propagation Method (WPM) has a great ability to find the resonance frequencies of the AFM.

In this study, the RF of micro-cantilever is obtained using three methods including analytical method, WPM, and experimental method. To verify the accuracy of the WPM

method, the obtained results are compared against the experimental and analytical results and a good agreement is observed. The results show that this method is more accurate rather than the analytical method. Modified Wave Propagation Method, therefore, can be used for further studies of AFM with non-constant parameters like v-shaped AFM or Fork AFM.

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