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Correlation Measure for Rough Neutrosophic Refined Sets and its Application in Medical Diagnosis

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ABSTRACT

Correlation measure is an important tool in the field of fuzzy, rough and neutrosophic environments. The main aim of this paper is to introduce the correlation measure for rough neutrosophic refined sets. This concept is the extension of correlation measure of neutrosophic sets and intuitionistic fuzzy multi sets. Finally, using the correlation of rough neutrosophic refined set measure, the application of medical diagnosis and pattern recognition are presented.

Introduction

In recent years, several theories have been proposed to deal with uncertainty, imprecision and vagueness. Probability set theory, fuzzy set theory [34], intuitionistic fuzzy set theory[6], interval intuitionistic fuzzy set theory[5] are used to deal with uncertainties and imprecision embedded in a system. But, all these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. F. Smarandache[28] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets. NS can be described by membership degree, indeterminate degree and non-membership degree. After that, Wang et al. [31] introduced an instance of neutrosophic sets known as single valued neutrosophic sets (SVNS), which were motivated from the practical point of view and that can be used in real scientific and engineering application, and provide the set theoretic operators and various properties of SVNSs. This theory is applied and found to be useful in many different fields such as control theory[1], databases[3, 4], medical diagnosis problem[2], decision making problem [11, 18, 19, 33], physics[22] etc.

In 1982, Pawlak [18] introduced the notion of rough set theory as the extension of the Cantor set theory [7]. Broumi et al. [9] comment that the concept of rough set is a formal tool for modeling and processing incomplete information in information systems. Rough set theory [18] is very useful to study of intelligent systems characterized by uncertain or insufficient

information. Main mathematical basis of rough set theory is formed by two basic components namely, crisp set and equivalence relation. Rough set is the approximation of a pair of sets known as the lower approximation and the upper approximation. Here, the lower and upper approximation operators are equivalence relation.

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. In 2014, Broumi et al. [7, 8, 9] introdced the concept of rough neutrosophic set. İt is developed based on the concept of rough set theory [21] and single valued neutrosophic set theory [25] Rough neutrosophic set theory [8, 9] is the generalization of rough fuzzy sets [15, 16, 17], and rough intuitionistic fuzzy sets [30].

The multiset theory was formulated first in [32] by Yager as generalization of the concept of set theory and then the multiset was developed in [10] by Calude et al. Several authors from time to time made a number of generalizations of the multiset theory. Sebastian and Ramakrishnan introduced a new notion called multi fuzzy sets which is a generalization of the multiset. Since then, several researchers [13, 23, 24] discussed more properties on multi fuzzy set. And they made an extension of the concept of fuzzy multisets to an intuitionstic fuzzy set which was called intuitionstic fuzzy multisets (IFMS). Since then in the study on IFMS, a lot of excellent results have been achieved by researchers [25, 26, 27]. An element of a multi fuzzy set can occur more than once with possibly the same or different membership values whereas an element of intuitionistic fuzzy multiset allows the repeated occurrences of membership and non membership values. The concepts of FMS and IFMS fail to deal with indeterminacy. In 2013 Smarandache [29], and Deli et al.14] used the concept of neutrosophic refined sets and studied some of their basic properties. The concept of neutrosophic refined set (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets.

Recently, Broumi and Smarandache[7] defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance such as set theoretic approach and matching function to calculate the similarity degree between neutrosophic sets. In the same year, Broumi and Smarandache [9, 10, 11] also proposed the correlation coefficient between interval neutrosphic sets. Hanafy et al. [16] proposed the correlation coefficients of neutrosophic sets and studied some of their basic properties. Based on centroid method, Hanafy et al. [17], introduced and studied the concepts of correlation and correlation coefficient of neutrosophic sets and studied some of their properties.

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In this paper, we propose correlation measure of rough neutrosophic sets and establish some of its properties. Finally, a numerical example of medical diagnosis is presented to demonstrate the applicability and effectiveness of the proposed approach. This diagnosis method can deal with the medical diagnosis problem with indeterminate and inconsistent information which cannot be handled by the diagnosis method based on other existing methods.

Preliminaries

Definition 2.1[34]

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in a fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples A = $\{(x, \mu_A(x)) : x \in X\}$

Definition 2.2[21]

Suppose (*U*, *R*) is an approximation space, where, *U* is a finite nonempty domain, *R* indicates an equivalence relation on *U*, and $[x]_R$ represents the equivalence classes including $x \in U$ based on the equivalence relation *R*. If *X* is a nonempty subset on U, then *R*-lower approximation and *R*-upper approximation of *X* on *U* are separately defined by

$\underline{R}X = \{x \in U / [x]_R \subseteq X\}$, $\overline{R}X = \{x \in U / [x]_R \cap X \neq \phi\}$

If $\underline{R}X = \overline{R}X$, we say that *X* is definable; otherwise, we say *X* is rough, and the set pair

($\underline{R}X$, $\overline{R}X$) is called the rough set of *X*.

Definition 2.3[1]

Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set (N-set) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0,1^+[$.

It can be written as

A = {< $x,(T_A(x), I_A(x), F_A(x))$ >: x ∈ U, $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq [0, 1]$}.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$, so

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+.$$

Definition 2.4[31]

Let U be a space of points (objects), with a generic element in U denoted by u. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ belongs to [0,1] for each point u in U. Then, an SVNS A can be expressed as

A = {< $x,(T_A(x), I_A(x), F_A(x))$ > : x ∈ U, $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq [0, 1]$}.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$, so

$$0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3.$$

Definition 2.5[31]

Let E be a universe. A neutrosophic refined (multi) set(NRS) A on E can be defined as follows:

A = {< $x, (T_A^1(x), (T_A^2(x), ...(T_A^P(x)), (I_A^1(x), (I_A^2(x), ...(I_A^P(x)), (F_A^1(x), (F_A^2(x), ...(F_A^P(x))$ >: x ∈ E}

where, $T_A^1(x), (T_A^2(x), ...(T_A^P(x) : E \to [0,1]$,

$I_A^1(x), (I_A^2(x), ...(I_A^P(x) : E \to [0,1]$, and

$F_A^1(x), (F_A^2(x), ...(F_A^P(x) : E \to [0,1]$

such that $0 \le \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \le 3$, (i = 1, 2, ……, P) and

$T_A^1(x) \le (T_A^2(x) \le .... \le (T_A^P(x)$ for any x ∈ E.

$(T_A^1(x), (T_A^2(x), ...(T_A^P(x))$, $(I_A^1(x), (I_A^2(x), ...(I_A^P(x))$

and $(F_A^1(x), (F_A^2(x), ...(F_A^P(x))$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension (cardinality) of Nms A, denoted d(A). We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all Neutrosophic refined (multi)sets on E is denoted by NRS(E).

Definition 2.6[14]

Let U be a non-null set and R be an equivalence relation on U. Let A be neutrosophic set in A with the membership function T_A, indeterminacy function I_A and non-membership function F_A. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$\underline{N}(A) = \langle < x, T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x) > / U \in [x]_R,\ x \in U \rangle$,

$\overline{N}(A) = \langle < x, T_{\overline{N}(A)}(x), I_{\overline{N}(A)}(x), F_{\overline{N}(A)}(x) > / U \in [x]_R,\ x \in U \rangle$

Where,

$T_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\, T_A(U)$, $I_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\, I_A(U)$, $F_{\underline{N}(A)}(x) = \wedge_z \in [x]_R\, F_A(U)$,

$T_{\overline{N}(A)}(x) = \vee_z \in [x]_R\, T_A(U)$, $I_{\overline{N}(A)}(x) = \vee_z \in [x]_R\, I_A(U)$, $F_{\overline{N}(A)}(x) = \vee_z \in [x]_R\, F_A(U)$,

So, $0 \le T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \le 3$ and

$0 \le T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \le 3$

Where $\vee$ and $\wedge$ indicate "max" and "min" operators respectively, $T_A(U)$, $I_A(U)$ and $F_A(U)$ are the membership , indeterminacy and non-membership of U with respect to A. It is easy to see that $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic sets in U. Thus NS mappings $\underline{N}$, $\overline{N}$: $N(U) \to N(U)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair ($\underline{N}(A), \overline{N}(A)$) is called the rough neutrosophic set in (U, R).

From the above definition, it is seen that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence classes of R if $\underline{N}(A) = \overline{N}(A)$. That is

$T_{\underline{N}(A)}(x) = T_{\overline{N}(A)}(x)$, $I_{\underline{N}(A)}(x) = I_{\overline{N}(A)}(x)$, $F_{\underline{N}(A)}(x) = F_{\overline{N}(A)}(x)$ for any x belongs to U.

A is said to be a definable neutrosophic set in the approximation (U, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 2.7[16]

Let $X = \{x_1, x_2, x_3, \ldots\ldots\ldots x_n\}$ be the finite universe of discourse and

A = {< $T_A^j(x_i), I_A^j(x_i), F_A^j(x_i) > / x_i \in X$ >}, B = {< $T_B^j(x_i), I_B^j(x_i), F_B^j(x_i) > / x_i \in X$ >} be two neutrosophic refined sets consisting of the membership, indeterminate and non-membership functions. Then the correlation measure between rough neutrosophic sets of A and B is given by

$$\rho_{NRS}(A,B) = \frac{C_{NRS}(A,B)}{\sqrt{C_{NRS}(A,A) * C_{NRS}(B,B)}}$$

Where,

$$C_{NRS}(A,B) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{T_A^j(x_i)T_B^j(x_i) + I_A^j(x_i)I_B^j(x_i) + F_A^j(x_i)F_B^j(x_i)\right\}$$

$$C_{NRS}(A,A) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{T_A^j(x_i)T_A^j(x_i) + I_A^j(x_i)I_A^j(x_i) + F_A^j(x_i)F_A^j(x_i)\right\}$$

and $$C_{NRS}(B,B) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{T_B^j(x_i)T_B^j(x_i) + I_B^j(x_i)I_B^j(x_i) + F_B^j(x_i)F_B^j(x_i)\right\}$$

Definition 2.8[5]

The cardinality of the membership function $M_c(x)$, indeterminate function $I_c(x)$ and the non-membership function $N_c(x)$ is the lenth of an element in a Rough Neutrosophic refined set(RNRS) A and is denoted by η and η = $|M_c(x)| = |I_c(x)| = |N_c(x)|$.

If A, B, C are the RNRS defined on X, then their cardinality η = Max{ η (A), η (B), η (C)}

Correlation Measure of two Rough Neutrosophic refined sets

Definition 3.1

Let $X = \{x_1, x_2, x_3, \ldots\ldots\ldots x_n\}$ be the finite universe of discourse and

A = {< $T_A^j(x_i), I_A^j(x_i), F_A^j(x_i) > / x_i \in X$ >}, B = {< $T_B^j(x_i), I_B^j(x_i), F_B^j(x_i) > / x_i \in X$ >} be two neutrosophic refined sets consisting of the membership, indeterminate and non-membership functions. Then the correlation coefficient of A and B is given by

$$\rho_{RNRS}(A,B) = \frac{C_{RNRS}(A,B)}{\sqrt{C_{RNRS}(A,A) * C_{RNRS}(B,B)}}$$

Where,

$$C_{RNRS}(A,B) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_A^j(x_i)\delta T_B^j(x_i) + \delta I_A^j(x_i)\delta I_B^j(x_i) + \delta F_A^j(x_i)\delta F_B^j(x_i)\right\}$$

$$C_{RNRS}(A,A) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_A^j(x_i)\delta T_A^j(x_i) + \delta I_A^j(x_i)\delta I_A^j(x_i) + \delta F_A^j(x_i)\delta F_A^j(x_i)\right\}$$

and $$C_{RNRS}(B,B) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_B^j(x_i)\delta T_B^j(x_i) + \delta I_B^j(x_i)\delta I_B^j(x_i) + \delta F_B^j(x_i)\delta F_B^j(x_i)\right\}$$

Here $\delta T_A^j(x_i) = \dfrac{\underline{T}_A^j(x_i)+\overline{T}_A^j(x_i)}{2}$, $\delta I_A^j(x_i) = \dfrac{\underline{I}_A^j(x_i)+\overline{I}_A^j(x_i)}{2}$, $\delta F_A^j(x_i) = \dfrac{\underline{F}_A^j(x_i)+\overline{F}_A^j(x_i)}{2}$;

And $\delta T_B^j(x_i) = \dfrac{\underline{T}_B^j(x_i)+\overline{T}_B^j(x_i)}{2}$, $\delta I_B^j(x_i) = \dfrac{\underline{I}_B^j(x_i)+\overline{I}_B^j(x_i)}{2}$, $\delta F_B^j(x_i) = \dfrac{\underline{F}_B^j(x_i)+\overline{F}_B^j(x_i)}{2}$.

Proposition 3.2

Let A and B be two rough neutrosophic sets. Then, the defined correlation measure between A and B satisfies the following properties

1. $0 \le \rho_{RNRS}(A,B) \le 1$
2. $\rho_{RNRS}(A,B) = 1$ if and only if A = B
3. $\rho_{RNRS}(A,B) = \rho_{RNRS}(B,A)$

Proof

1. As the membership, indeterminate and non-membership functions of the RNRS lies between 0 and 1, $\rho_{RNRS}(A,B)$ also leis between 0 and 1.
2. To Prove $\rho_{RNRS}(A,B) = 1$if and only if A = B

(i) Let the two RNRS be equal. Hence for any

$\delta T_A^j(x_i) = \delta T_B^j(x_i)$

$\delta I_A^j(x_i) = \delta I_B^j(x_i)$

$\delta F_A^j(x_i) = \delta F_B^j(x_i)$

That is, $\underline{T}_A^j(x_i) = \underline{T}_B^j(x_i)$, $\underline{I}_A^j(x_i) = \underline{I}_B^j(x_i)$, $\underline{F}_A^j(x_i) = \underline{F}_B^j(x_i)$

$\overline{T}_A^j(x_i) = \overline{T}_B^j(x_i)$, $\overline{I}_A^j(x_i) = \overline{I}_B^j(x_i)$, $\overline{F}_A^j(x_i) = \overline{F}_B^j(x_i)$

Then $C_{RNRS}(A,A) = C_{RNRS}(B,B) =$

$$\frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_A^j(x_i)\delta T_A^j(x_i) + \delta I_A^j(x_i)\delta I_A^j(x_i) + \delta F_A^j(x_i)\delta F_A^j(x_i)\right\}$$

and
$$\begin{aligned} C_{RNRS}(A,B) &= \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_A^j(x_i)\delta T_B^j(x_i) + \delta I_A^j(x_i)\delta I_B^j(x_i) + \delta F_A^j(x_i)\delta F_B^j(x_i)\right\} \\ &= \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_A^j(x_i)\delta T_A^j(x_i) + \delta I_A^j(x_i)\delta I_A^j(x_i) + \delta F_A^j(x_i)\delta F_A^j(x_i)\right\} \\ &= C_{RNRS}(A,A) \end{aligned}$$

Hence $\rho_{RNRS}(A,B) = \dfrac{C_{RNRS}(A,B)}{\sqrt{C_{RNRS}(A,A)*C_{RNRS}(B,B)}} = \dfrac{C_{RNRS}(A,A)}{\sqrt{C_{RNRS}(A,A)*C_{RNRS}(A,A)}} = \mathbf{1.}$

(ii) Let $\rho_{RNRS}(A,B) = 1$. Then, the unit measure is possible only if

$$\frac{C_{RNRS}(A,B)}{\sqrt{C_{RNRS}(A,A)*C_{RNRS}(B,B)}} = 1$$

This refers that $\delta T_A^j(x_i) = \delta T_B^j(x_i)$, $\delta I_A^j(x_i) = \delta I_B^j(x_i)$, $\delta F_A^j(x_i) = \delta F_B^j(x_i)$

That is $\underline{T}_A^j(x_i) = \underline{T}_B^j(x_i)$, $\underline{I}_A^j(x_i) = \underline{I}_B^j(x_i)$, $\underline{F}_A^j(x_i) = \underline{F}_B^j(x_i)$

$\overline{T}_A^j(x_i) = \overline{T}_B^j(x_i)$, $\overline{I}_A^j(x_i) = \overline{I}_B^j(x_i)$, $\overline{F}_A^j(x_i) = \overline{F}_B^j(x_i)$ for all i, j values.

Hence A = B.

3. It is obvious that

$$\rho_{RNRS}(A,B) = \frac{C_{RNRS}(A,B)}{\sqrt{C_{RNRS}(A,A)*C_{RNRS}(B,B)}} = \frac{C_{RNRS}(B,A)}{\sqrt{C_{RNRS}(A,A)*C_{RNRS}(B,B)}} = \rho_{RNRS}(B,A)$$

as
$$C_{RNRS}(A,B) = \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_A^j(x_i)\delta T_B^j(x_i) + \delta I_A^j(x_i)\delta I_B^j(x_i) + \delta F_A^j(x_i)\delta F_B^j(x_i)\right\}$$
$$= \frac{1}{p}\sum_{j=1}^{p}\sum_{i=1}^{n}\left\{\delta T_B^j(x_i)\delta T_A^j(x_i) + \delta I_B^j(x_i)\delta I_A^j(x_i) + \delta F_B^j(x_i)\delta F_A^j(x_i)\right\}$$
$$= C_{RNRS}(B,A)$$

Numerical Evaluation

Example 4.1

Let X = {A_1, A_2, A_3, A_4,………., A_n} with A = {A_1, A_2, A_3, A_4} and B = {A_5, A_6, A_7, A_8} are RNRS defined as

$$A = \left\{ \left\langle A_1 : \left\langle \begin{matrix}(0.6,0,5,0.4)\\(0.8,0.5,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.3,0,3,0.3)\\(0.5,0.3,0.1)\end{matrix} \right\rangle \right\rangle, \left\langle A_2 : \left\langle \begin{matrix}(0.6,0,5,0.4)\\(0.8,0.5,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.4,0,3,0.8)\\(0.6,0.3,0.2)\end{matrix} \right\rangle \right\rangle, \left\langle A_3 : \left\langle \begin{matrix}(0.4,0,5,0.4)\\(0.8,0.3,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.6,0,3,0.3)\\(0.8,0.3,0.1)\end{matrix} \right\rangle \right\rangle, \left\langle A_4 : \left\langle \begin{matrix}(0.7,0,5,0.6)\\(0.9,0.3,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.5,0,3,0.3)\\(0.7,0.3,0.1)\end{matrix} \right\rangle \right\rangle \right\}$$

$$B = \left\{ \left\langle A_5 : \left\langle \begin{matrix}(0.7,0,4,0.4)\\(0.9,0.4,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.6,0,3,0.2)\\(0.4,0.3,0.2)\end{matrix} \right\rangle \right\rangle, \left\langle A_6 : \left\langle \begin{matrix}(0.7,0,7,0.4)\\(0.9,0.5,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.6,0,4,0.3)\\(0.8,0.4,0.1)\end{matrix} \right\rangle \right\rangle, \left\langle A_7 : \left\langle \begin{matrix}(0.3,0,7,0.5)\\(0.5,0.5,0.3)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.3,0,3,0.5)\\(0.5,0.3,0.1)\end{matrix} \right\rangle \right\rangle, \left\langle A_8 : \left\langle \begin{matrix}(0.6,0,6,0.4)\\(0.8,0.6,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.5,0,6,0.5)\\(0.7,0.4,0.1)\end{matrix} \right\rangle \right\rangle \right\}$$

Here the cardinality η = 4 as $\eta(A) = |M_c(A)| = |I_c(A)| = |N_c(A)| = 4$ and

$\eta(B) = |M_c(B)| = |I_c(B)| = |N_c(B)| = 4$ and the RNRS Correlation measure is 0.9673.

Example 4.2

Let X = {A_1, A_2, A_3, A_4, ……, A_n} with A = {A_1, A_2, A_3, A_4} and B = {A_5, A_6, A_7, A_8} are RNRS defined as

$$A = \left\{ \left\langle A_1 : \left\langle \begin{matrix}(0.6,0,5,0.4)\\(0.8,0.5,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.3,0,3,0.3)\\(0.5,0.3,0.1)\end{matrix} \right\rangle \right\rangle, \left\langle A_2 : \left\langle \begin{matrix}(0.6,0,5,0.4)\\(0.8,0.5,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.4,0,3,0.8)\\(0.6,0.3,0.2)\end{matrix} \right\rangle \right\rangle, \left\langle A_3 : \left\langle \begin{matrix}(0.7,0,5,0.6)\\(0.9,0.3,0.2)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.5,0,3,0.3)\\(0.7,0.3,0.1)\end{matrix} \right\rangle \right\rangle \right\}$$

$$B = \left\{ \left\langle A_4 : \left\langle \begin{matrix}(0.7,0.8,0.7)\\(0.9,0.8,0.5)\end{matrix} \right\rangle, \left\langle \begin{matrix}(0.5,0,8,0.7)\\(0.9,0.6,0.7)\end{matrix} \right\rangle \right\rangle \right\}$$

Here $\eta(A) = |M_c(A)| = |I_c(A)| = |N_c(A)| = 3$ and

$\eta(B) = |M_c(B)| = |I_c(B)| = |N_c(B)| = 1$

Therefore the cardinality η = max {$\eta(A), \eta(B)$} = max {3, 1} = 3

and the RNRS Correlation measure is 0.6409.

Application

In this section we give some applications of RNRS in medical diagnosis and pattern recognition problems using the correlation measure.

Medical Diagnosis Using Rnrs Correlation Measure

Realistic practical problems consist of more uncertainty and complexity. So, it is necessary to employ more flexible tool which can deal uncertain situation easily. In this situation, rough neutrosophic set is very useful tool to uncertainty and incompleteness. In real medical diagnosis problems, however, by only taking one time inspection, we cannot come to a conclusion whether a particular person is found with a particular decease or not. Sometimes he/she may also show the symptoms of different diseases. In that case we cannot give a proper solution. So in order to get the right solution the patient has to be examined at different time intervals (e.g. two or three times a day). In this case, a Rough Neutrosophic multi set concept is very suitable for expressing this information at different time intervals, which allows the repeated occurrences of any element. The unique feature of this proposed method is that it considers multi truth membership, indeterminate and false membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis". This can be explained using the following example.

Example 5.1

Let P = {P_1, P_2, P_3} be the set of patients, D = {}be the set of diseases and S = {} be the set of symptoms. Now we have to examine the patients and determine the disease of the patient in rough neutrosophic multi set environment.

Table 1. The relation between the patients and symptoms

A	Temperature	Cough	Throat Pain	Headache	Body Pain
P_1	$\left\langle \begin{matrix}(0.3,0.3,0.4)\\(0.5,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.2,0.6,0.7)\\(0.4,0.2,0.5)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.7,0.6)\\(0.3,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.5,0.4)\\(0.6,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.2,0.4)\\(0.5,0.0,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.2,0.5,0.6)\\(0.4,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.2,0.7,0.6)\\(0.4,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.8,0.6)\\(0.3,0.4,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.0,0.6,0.4)\\(0.2,0.6,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.4,0.6)\\(0.5,0.2,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.4,0.5,0.8)\\(0.6,0.3,0.6)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.3,0.8)\\(0.5,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.3,0.4)\\(0.7,0.1,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.3,0.6)\\(0.3,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.5,0.4)\\(0.1,0.3,0.2)\end{matrix} \right\rangle$
P_2	$\left\langle \begin{matrix}(0.3,0.3,0.6)\\(0.9,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.4,0.5,0.4)\\(0.6,0.5,0.0)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.6,0.6)\\(0.5,0.2,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.3,0.8)\\(0.8,0.3,0.6)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.5,0.2)\\(0.5,0.3,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.5,0.6)\\(0.3,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.5,0.4,0.4)\\(0.7,0.2,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.6,0.4)\\(0.5,0.4,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.0,0.5,0.7)\\(0.2,0.3,0.3)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.3,0.2)\\(0.8,0.3,0.0)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.7,0.9)\\(0.5,0.3,0.7)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.6,0.4)\\(0.3,0.2,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.7,0.6)\\(0.5,0.1,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.3,0.9)\\(0.3,0.1,0.5)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.0,0.7,0.6)\\(0.2,0.3,0.4)\end{matrix} \right\rangle$
P_3	$\left\langle \begin{matrix}(0.7,0.3,0.6)\\(0.9,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.5,0.7,0.4)\\(0.9,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.5,0.5,0.4)\\(0.7,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.6,0.4)\\(0.6,0.4,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.2,0.6,0.4)\\(0.4,0.2,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.0,0.8,0.6)\\(0.2,0.4,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.2,0.3,0.8)\\(0.4,0.3,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.5,0.9)\\(0.3,0.5,0.5)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.0,0.5,0.6)\\(0.2,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.6,0.3,0.6)\\(0.6,0.1,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.5,0.8)\\(0.7,0.1,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.3,0.8)\\(0.3,0.1,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.4,0.5,0.8)\\(0.8,0.3,0.2)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.3,0.4,0.4)\\(0.3,0.2,0.4)\end{matrix} \right\rangle$ $\left\langle \begin{matrix}(0.1,0.3,0.9)\\(0.3,0.1,0.3)\end{matrix} \right\rangle$

Let the samples be taken in three different timings (morning, noon, evening)

Table 2. The relation between the symptoms and the diseases

B	Viral fever	Tuberculosis	Typhoid	Throat disease
Temperature	$\left\langle \begin{matrix}(0.1,0.7,0.6)\\(0.3,0.3,0.6)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.7,0.6)\\(0.5,0.5,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.5,0.8)\\(0.9,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.9,0.9)\\(0.3,0.5,0.7)\end{matrix} \right\rangle$
Cough	$\left\langle \begin{matrix}(0.3,0.6,0.5)\\(0.9,0.2,0.1)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.7,0.3,0.4)\\(0.9,0.1,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.8)\\(0.5,0.1,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.5,0.1)\\(0.3,0.3,0.1)\end{matrix} \right\rangle$
Throat Pain	$\left\langle \begin{matrix}(0.4,0.3,0.4)\\(0.6,0.1,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.5,0.4)\\(0.5,0.5,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.3,0.7,0.6)\\(0.5,0.3,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.9,0.3)\\(0.3,0.3,0.1)\end{matrix} \right\rangle$
Headache	$\left\langle \begin{matrix}(0.3,0.8,0.4)\\(0.9,0.8,0.0)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.7)\\(0.3,0.3,0.5)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.0,0.8,0.4)\\(0.2,0.4,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.0,0.7,0.6)\\(0.4,0.3,0.4)\end{matrix} \right\rangle$
Body Pain	$\left\langle \begin{matrix}(0.5,0.5,0.6)\\(0.9,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.6)\\(0.3,0.3,0.2)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.2,0.5,0.4)\\(0.2,0.1,0.4)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.1,0.3,0.4)\\(0.3,0.1,0.2)\end{matrix} \right\rangle$

Table 3. The correlation measure between RNRS A and B:

	Viral fever	Tuberculosis	Typhoid	Throat disease
P_1	0.846	0.910	0.884	0.880
P_2	0.849	0.868	0.892	0.809
P_3	0.792	0.853	0.872	0.822

The highest correlation measure from table 3 gives the proper medical diagnosis.

Patient P_1 suffers from Viral fever and patients P_2, P_3 suffer from Typhoid.

Pattern Recognition of Rnrs Correlation Similarity Measure

Example 5.2

Let X = {A_1, A_2, A_3, A_4,………., A_n} with A = {A_1, A_2}, B = {A_3, A_4} and C = {A_1, A_4} are RNRS defined as

A =
$$\left\{\left\langle A_1 : \left\langle\begin{matrix}(0.2, 0,4, 0.1)\\(0.6, 0.4, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.2, 0,2, 0.3)\\(0.4, 0.0, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,2, 0.3)\\(0.3, 0.0, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.0, 0.5, 0.5)\\(0.2, 0.3, 0.1)\end{matrix}\right\rangle\right\rangle,\right.$$
$$\left.\left\langle A_2 : \left\langle\begin{matrix}(0.5, 0,4, 0.0)\\(0.7, 0.2, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.2, 0,7, 0.2)\\(0.6, 0.3, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.3, 0,3, 0.3)\\(0.5, 0.3, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,6, 0.3)\\(0.3, 0.6, 0.1)\end{matrix}\right\rangle\right\rangle\right\}$$

B =
$$\left\{\left\langle A_3 : \left\langle\begin{matrix}(0.4, 0,4, 0.4)\\(0.6, 0.4, 0.2)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.2, 0,3, 0.4)\\(0.6, 0.1, 0.2)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.3, 0,1, 0.3)\\(0.5, 0.1, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,2, 0.8)\\(0.1, 0.0, 0.4)\end{matrix}\right\rangle\right\rangle,\right.$$
$$\left.\left\langle A_4 : \left\langle\begin{matrix}(0.2, 0,6, 0.4)\\(0.6, 0.6, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,6, 0.0)\\(0.7, 0.4, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.3, 0,5, 0.3)\\(0.3, 0.3, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,5, 0.1)\\(0.3, 0.3, 0.1)\end{matrix}\right\rangle\right\rangle\right\}$$

C =
$$\left\{\left\langle A_1 : \left\langle\begin{matrix}(0.2, 0,4, 0.1)\\(0.6, 0.4, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.2, 0,2, 0.3)\\(0.4, 0.0, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,2, 0.3)\\(0.3, 0.0, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.0, 0.5, 0.5)\\(0.2, 0.3, 0.1)\end{matrix}\right\rangle\right\rangle,\right.$$
$$\left.\left\langle A_4 : \left\langle\begin{matrix}(0.2, 0,6, 0.4)\\(0.6, 0.6, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,6, 0.0)\\(0.7, 0.4, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.3, 0,5, 0.3)\\(0.3, 0.3, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,5, 0.1)\\(0.3, 0.3, 0.1)\end{matrix}\right\rangle\right\rangle\right\}$$

and the testing RNRS pattern D is given as

D =
$$\left\{\left\langle A_1 : \left\langle\begin{matrix}(0.2, 0,8, 0.3)\\(0.6, 0.4, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.4, 0,8, 0.0)\\(0.4, 0.2, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.3, 0,6, 0.3)\\(0.3, 0.2, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.2, 0.5, 0.1)\\(0.2, 0.3, 0.1)\end{matrix}\right\rangle\right\rangle,\right.$$
$$\left.\left\langle A_2 : \left\langle\begin{matrix}(0.2, 0,4, 0.3)\\(0.6, 0.0, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.4, 0,7, 0.0)\\(0.6, 0.3, 0.0)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,5, 0.3)\\(0.3, 0.3, 0.1)\end{matrix}\right\rangle, \left\langle\begin{matrix}(0.1, 0,6, 0.1)\\(0.3, 0.4, 0.1)\end{matrix}\right\rangle\right\rangle\right\}$$

Here the cardinality η = 2 as $\eta(A) = |M_c(A)| = |I_c(A)| = |N_c(A)| = 2$ and $\eta(B) = |M_c(B)| = |I_c(B)| = |N_c(B)| = 2$, then the proposed correlation similarity measure between the pattern (A, D) is 0.8628, the pattern (B, D) is 0.8175, and the pattern (C, D) is 0.8597. Hence the testing pattern D belongs to pattern A type.

Conclusion

In this paper, we have defined the correlation measure of rough neutrosophic refined sets and proved some of their basic properties. We have presented an application of correlation measure of rough neutrosophic refined sets in medical diagnosis and pattern recognition and found that the correlation measure of RNRS is effective in handling the medical diagnosis problems with indeterminate and inconsistent information. We hope that the proposed concept can be applied in solving realistic multi-attribute decision making problems.

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