

# Distributional Generalized Two Dimensional Fractional Cosine Transform and its operators <br> V. D. Sharma ${ }^{1}$ and S. A. Khapre ${ }^{2}$ <br> ${ }^{1}$ Department of Mathematics, Arts, Commerce and Science College, Amravati- 444606(M.S), India. <br> ${ }^{2}$ Department of Mathematics, P. R. Patil College of Engineering and Technology, Amravati (M.S.), 444604 India. 

## ARTICLE INFO

## Article history:

Received: 29 October 2015;
Received in revised form:
2 December 2015;
Accepted: 7 December 2015;

## Keywords

Fractional Fourier Transform, Fractional Cosine Transform, Fractional Sine Transform.


#### Abstract

Distribution theory is powerful tool for dealing with pseudo-functions including impulsive phenomena, absolutely divergent integrals and their Fourier transforms. A new theory of generalized functions which is an extension of classical theory of distributions has been introduced and various properties have been generalized. The purpose of this paper is two dimensional fractional cosine transform is extended in the distributional generalized sense. Operators on the testing function E and its dual space $\boldsymbol{E}^{*}$ are obtained.


© 2015 Elixir All rights reserved.

## Introduction

Distributions are objects that generalize the classical notion of functions in mathematical analysis. Distributions make it possible to differentiate functions whose derivatives do not exist in the classical sense.

The basic idea in distribution theory is to reinterpret functions as linear functional acting on a space of test functions. Standard functions act by integration against a test function, but many other linear functional do not arise in this way, and these are the "generalized functions". There are different possible choices for the space of test functions, leading to different spaces of distributions. The basic space of test function consists of smooth functions with compact support, leading to standard distributions. Use of the space of smooth, rapidly decreasing test functions gives instead the tempered distributions, which are important because they have a well-defined distributional Fourier transform. Every tempered distribution is a distribution in the normal sense, but the converse is not true: in general the larger the space of test functions, the more restrictive the notion of distribution. On the other hand, the use of spaces of analytic test functions leads to Sato's theory of hyper functions; this theory has a different character .Distributions are also important in physics and engineering where many problems naturally lead to differential equations whose solutions or initial conditions are distributions.

## Generalized two dimensional fractional Cosine transform

Two dimensional fractional Cosine transform with parameter $\boldsymbol{\alpha} \mathrm{f}(\mathrm{x}, \mathrm{y})$ denoted by $\boldsymbol{F}_{\boldsymbol{C}}^{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{y})$ perform a linear operation given by the integral transform.
$F_{C}^{\alpha}\{f(x, y)\}(u, v)=\int_{0}^{\infty} \int_{0}^{\infty} f(x, y) K_{\alpha}(x, y, u, v) d x d y$
Where the kernel,
$K_{c}^{\alpha}(x, y, u, v)=\sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha}{2}} \cos (\operatorname{cosec} \alpha \cdot u x) \cdot \cos (\operatorname{cosec} \alpha . v y)$

## The test function space $E$

An infinitely differentiable complex valued function $\emptyset$ on $\boldsymbol{R}^{\boldsymbol{n}}$ belongs to $\boldsymbol{E}\left(\boldsymbol{R}^{\boldsymbol{n}}\right)$ if for each compact set $\boldsymbol{I} \subset \boldsymbol{S}_{a, b}$,
where,
$S_{a, b}=\left\{x, y: x, y \in R^{n},|x| \leq a,|y| \leq b, a>0, b>0\right\}, I \in R^{n}$

$$
\gamma_{E_{p, q}}(\varnothing)={ }_{x, y}^{s u p}\left|D_{x, y}^{p, q} \varnothing(x, y)\right|<\infty
$$

Where, $\mathrm{p}, \mathrm{q}=1,2,3 \ldots$
Thus $\boldsymbol{E}\left(\boldsymbol{R}^{\boldsymbol{n}}\right)$ will denote the space of all $\varnothing \in \boldsymbol{E}\left(\boldsymbol{R}^{\boldsymbol{n}}\right)$ with support contained in $\boldsymbol{S}_{a, \boldsymbol{b}}$
Note that: the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member of $\boldsymbol{E}^{*}$, the dual space of $E$.
In the present work, two dimensional fractional cosine transform is extended in the distributional generalized sense. Operators on the testing function E and its dual space $\boldsymbol{E}^{*}$ as shifting operator, differential operator, ad joint shifting operator, ad joint differential operator etc. are obtained.

## Tele:

E-mail addresses: vdsharma@hotmail.co.in

## Distributional two-dimensional fractional Cosine transform

The two dimensional distributional fractional Cosine transform of $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{E}^{*}\left(\boldsymbol{R}^{\boldsymbol{n}}\right)$ defined by $F_{c}^{\alpha}\{f(x, y)\}=F^{\alpha}(u, v)=\left\langle f(x, y), K_{\alpha}(x, y, u, v)\right\rangle$
$K_{c}^{\alpha}(x, y, u, v)=\sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha}{2}} \cos (\operatorname{cosec} \alpha \cdot u x) \cdot \cos (\operatorname{cosec} \alpha \cdot v y)$
Where,
RHS of equation (2.1) has a meaning as the application of $\boldsymbol{f} \in \boldsymbol{E}^{*} \operatorname{to} \boldsymbol{K}_{\boldsymbol{\alpha}}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{v}) \in \boldsymbol{E}$
Shifting operator
If $\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y}) \boldsymbol{\epsilon} \boldsymbol{E}$ and a and b are real numbers then $\boldsymbol{\varphi}(\boldsymbol{x}+\boldsymbol{a}, \boldsymbol{y}+\boldsymbol{b}) \boldsymbol{\epsilon} \boldsymbol{E}, \boldsymbol{x}+\boldsymbol{a}>\mathbf{0}, \boldsymbol{y}+\boldsymbol{b}>\mathbf{0}$ where $\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y})=\boldsymbol{K}_{c}^{\alpha}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{v})$
Proof
Consider
$\gamma_{E, p, q}(\varphi(x+a, y+b))={ }_{x, y \in I}^{s u p}\left|D_{x}^{p} D_{y}^{q} \varphi(x+a, y+b)\right|$
$\gamma_{E, p, q}(\varphi(x+a, y+b))={ }_{x, y \in I}^{x, y p}\left|D_{x}^{p} D_{y}^{q} K_{c}^{\alpha}(x+a, y+b, u, v)\right|$
$\gamma_{E, p, q}(\varphi(x+a, y+b))={ }_{x, y \in I}^{s u p}\left|D_{x}^{p} D_{y}^{q} K_{c}^{\alpha}(x+a, u, y+b, v)\right|$
$\gamma_{E, p, q}(\varphi(x+a, y+b))={ }_{x, y \in I}{ }^{\text {sup }}\left|D_{x^{\prime}}^{p} D_{y^{\prime}}^{q} K_{c}^{\alpha}\left(x^{\prime}, u, y^{\prime}, v\right)\right|$
For any fixed $\boldsymbol{x}^{\prime} \boldsymbol{a n d} \boldsymbol{y}^{\prime}, \boldsymbol{u}, \boldsymbol{v} \boldsymbol{\epsilon} \boldsymbol{R}^{\boldsymbol{n}}$ and any fixed p, q, $\mathbf{0}<\boldsymbol{\alpha} \leq \frac{\pi}{2}$
Thus $\boldsymbol{\varphi}(x+a, y+b) \epsilon E, x+a>0, y+b>0$
The translation (shifting) operator
$\boldsymbol{T}: \boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y}) \rightarrow \boldsymbol{\varphi}(\boldsymbol{x}+\boldsymbol{a}, \boldsymbol{y}+\boldsymbol{b})$ is a topological automorphism on E, for $\boldsymbol{x}+\boldsymbol{a}>\mathbf{0}, \boldsymbol{y}+\boldsymbol{b}>\mathbf{0}$
If $\varphi(x, y) \epsilon E$ and $a>0 b>0$ then $\varphi(a x, b y) \epsilon E$
Proof
Consider
$\gamma_{E, p, q}(\varphi(a x, b y))={ }_{x, y \in I}^{s u p}\left|D_{x}^{p} D_{y}^{q} \varphi(a x, b y)\right|$
$\gamma_{E, p, q}(\varphi(a x, b y))={ }_{x, y \in I}^{s u p}\left|D_{x}^{p} D_{y}^{q} K_{c}^{\alpha}(a x, b y, u, v)\right|$
$\gamma_{E, p, q}(\varphi(a x, b y))={ }_{x, y \in I}^{\sup }\left|D_{x}^{p} D_{y}^{q} K_{c}^{\alpha}(a x, u, b y, v)\right|$

$$
\gamma_{E, p, q}(\varphi(a x, b y))={ }_{x, y \in I}^{s u p}\left|D_{x^{\prime}}^{p} D_{y^{\prime}}^{q} K_{c}^{\alpha}\left(x^{\prime}, u, y^{\prime}, v\right)\right|
$$

For any fixed $\boldsymbol{x}^{\prime} \boldsymbol{a n d}_{\boldsymbol{\prime}}{ }^{\prime}, \boldsymbol{u}, \boldsymbol{v} \boldsymbol{\epsilon} \boldsymbol{R}^{\boldsymbol{n}}$ and any fixed p, q, $\mathbf{0}<\boldsymbol{\alpha} \leq \frac{\pi}{2}$
Thus $\boldsymbol{\varphi}(\boldsymbol{a x}, \boldsymbol{b y}) \epsilon \boldsymbol{\epsilon}, \boldsymbol{x}+\boldsymbol{a}>\mathbf{0}, \boldsymbol{y}+\boldsymbol{b}>\mathbf{0}$
If $\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y}) \boldsymbol{\epsilon} \boldsymbol{E}$ and $\boldsymbol{a}>\mathbf{0} \boldsymbol{b}>\mathbf{0}$ then scaling operator $\boldsymbol{R}: \boldsymbol{E} \rightarrow \boldsymbol{E}$ defined $\boldsymbol{R} \boldsymbol{\varphi}=\boldsymbol{\psi}$ where $\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y})=\boldsymbol{\varphi}(\boldsymbol{a x}, \boldsymbol{b y})$ is topological automorphism. Combining result 2 and 3 immediately yields next proposition.
Proposition
If a, b, c, d $\boldsymbol{\epsilon}$ R such that $\boldsymbol{x}+\boldsymbol{a}>\mathbf{0}, \boldsymbol{y}+\boldsymbol{b}>\mathbf{0}$ and $\boldsymbol{c x}>\mathbf{0}, \boldsymbol{d} \boldsymbol{y}>\mathbf{0}$
Then the shifting scaling operator $\boldsymbol{S}: \boldsymbol{E} \rightarrow \boldsymbol{E}$ defined $\boldsymbol{S}(\boldsymbol{\varphi})=\boldsymbol{\psi}$
Where $\boldsymbol{\psi}(\boldsymbol{x}, \boldsymbol{y}) \rightarrow \boldsymbol{\varphi}(\boldsymbol{c}(\boldsymbol{x}+\boldsymbol{a}), \boldsymbol{d}(\boldsymbol{y}+\boldsymbol{b}))$ is a topological automorphism

## Differential operator

The operator $\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y}) \rightarrow \boldsymbol{D}_{x, y} \boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y})$ is defined on the space E and transform this space E into itself, where $\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{y})=$
$K_{c}^{\alpha}(x, y, u, v)$
Proof
Let $D_{x, y} \varphi(x, y)=\varphi(x, y)$
$\gamma_{E, p, q}(\varphi(x, y))={ }_{x, y \in I}^{\text {sup }}\left|D_{x}^{p} D_{y}^{q} \varphi(x, y)\right|$
$\gamma_{E, p, q}(\varphi(x, y))={ }_{x, y \in I}^{\sup }\left|D_{x}^{p} D_{y}^{q} D_{x, y} \varphi(x, y)\right|$
$\gamma_{E, p, q}(\varphi(x, y))={ }_{x, y \in I}^{s u p}\left|D_{x}^{p+1} D_{y}^{q+1} K_{c}^{\alpha}(x, y, u, v)\right|$
$D_{x}^{p+1} K_{c}^{\alpha}(x, y, u, v)=\sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i}{2}\left(u^{2}+v^{2}+x^{2}+y^{2}\right) \cot \alpha} \cos (\csc \alpha . v y)$
$\sum_{n=0}^{p+1} \sum_{r=0}^{k}\binom{p}{n} \frac{n!}{(k-2 r)!r!}(i \cot \alpha)^{k-r}(2 x)^{(k-2 r)}(\csc \alpha . u x)^{p-n}$
$\cos \left(\csc \alpha \cdot u x+\frac{(p+1-n) \pi}{2}\right)$

$$
(\csc \alpha . u x)^{p-n}(\csc \alpha . v y)^{q-m}
$$

$\cos \left(\csc \alpha \cdot u x+\frac{(\mathbf{p}-\mathbf{n}) \pi}{2}\right) \cos \left(\csc \alpha \cdot v y+\frac{(q-m) \pi}{2}\right)$
Put in equation (1)

For any fixed $\boldsymbol{x}, \boldsymbol{y} \boldsymbol{\epsilon} \boldsymbol{R}^{\boldsymbol{n}}$ and fixed p and $\mathrm{q}, \mathbf{0}<\boldsymbol{\alpha} \leq \frac{\pi}{2}$

$$
\begin{aligned}
& \therefore \varphi(x, y) \epsilon E \\
& \therefore D_{x, y} \varphi(x, y) \epsilon E
\end{aligned}
$$

Adjoint shifting operator
The adjoint shifting operator is a continuous function from $\boldsymbol{E}^{*}$ to $\boldsymbol{E}^{*}$.the operator of (2) is $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \rightarrow \boldsymbol{f}(\boldsymbol{x}-\boldsymbol{a}, \boldsymbol{y}-\boldsymbol{b})$ Leads to the operation transform formula.
Solution
$\boldsymbol{F}_{\alpha}^{c}\{f(x+a, y+b)\}=\langle f(x+a, y+b)$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=\left\langle f(x+a, y+b), \sqrt{\frac{1-\boldsymbol{i} \cot \alpha}{2 \pi}} e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha}\right\rangle$
$\cos (\csc \alpha . u x) \cos (\csc \alpha . v y)$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=\left\langle f(x, y), \sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha}\right\rangle$

$$
\cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))
$$

$e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha} \cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))=e^{\frac{i}{2}\left(x^{2}+y^{2}+2 \times a+2 y b+a^{2}+b^{2}+u^{2}+v^{2}\right) \cot \alpha}$ $\cos (\csc \alpha .(u x+u a)) \cos (\csc \alpha .(v y+v b))$
$e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha}$
$\cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))=e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha} e^{\frac{i}{2}\left(2 x a+2 y b+a^{2}+b^{2}\right) \cot \alpha}$
$[\cos (\csc \alpha u x) \cos (\csc \alpha \mathbf{u a})-\sin (\csc \alpha u x) \sin (\csc \alpha u a)]$
$[\cos (\csc \alpha v y) \cos (\csc \alpha v b)-\sin (\csc \alpha v y) \sin (\csc \alpha v b)]$
$e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha}$
$\cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))=e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha} e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha}$
$[\cos (\csc \alpha u x) \cos (\csc \alpha u a) \cos (\csc \alpha v y) \cos (\csc \alpha v b)$
$-\cos (\csc \alpha u x) \cos (\csc \alpha u a) \sin (\csc \alpha v y) \sin (\csc \alpha v b))$
$-\sin (\csc \alpha u x) \sin (\csc \alpha u a) \cos (\csc \alpha v y) \cos (\csc \alpha v b)$
$[+\sin (\csc \alpha u x) \sin (\csc \alpha u a) \sin (\csc \alpha v y) \sin (\csc \alpha v b)]$
Let

$$
\begin{gathered}
P=\cos (\csc \alpha u a) Q=\cos (\csc \alpha v b) \\
R=\sin (\csc \alpha u a) S=\sin (\csc \alpha v b)
\end{gathered}
$$

$e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha}$

$$
\begin{aligned}
& \sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha} \\
& \sum_{m=0}^{q} \sum_{s=0}^{\mathbf{l}} \sum_{n=0}^{p} \sum_{r=0}^{k}\binom{\mathbf{p}}{\mathbf{n}}\binom{\mathbf{q}}{\mathbf{m}} \frac{n!}{(k-2 r)!r!} \\
& \frac{\mathbf{m !}}{(1-2 s)!s!}(i \cot \alpha)^{k+1-r-s}(2 x)^{(k-2 r)}(2 y)^{(1-2 s)} \\
& (\csc \alpha . u x)^{p-n}(\csc \alpha . v y)^{q-m} \\
& \left.x_{x, y \epsilon I} \cos \left(\csc \alpha \cdot u x+\frac{(p-n) \pi}{2}\right) \cos \left(\csc \alpha \cdot v y+\frac{(q-m) \pi}{2}\right) \right\rvert\, \\
& <\infty
\end{aligned}
$$

$$
\begin{aligned}
& D_{x, y}^{p+1, q+1} K_{c}^{\alpha}(x, y, u, v)=\sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha} \\
& \sum_{m=0}^{q} \sum_{s=0}^{1} \sum_{n=0}^{p} \sum_{r=0}^{k}\binom{p}{n}\binom{q}{m} \frac{n!}{(k-2 r)!r!} \\
& \frac{m!}{(1-2 s)!s!}(i \cot \alpha)^{k+1-r-s}(2 x)^{(k-2 r)}(2 y)^{(1-2 s)}
\end{aligned}
$$

$\cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))=e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha} e^{\frac{i}{2}\left(2 \times a+2 y b+a^{2}+b^{2}\right) \cot \alpha}$
$\left[\begin{array}{c}P Q \cos (\csc \alpha u x) \cos (\csc \alpha v y) \\ -P S \cos (\csc \alpha u x) \sin (\csc \alpha v y) \\ -Q R \sin (\csc \alpha u x) \cos (\csc \alpha v y) \\ + \text { RS } \sin (\csc \alpha u x) \sin (\csc \alpha v y)\end{array}\right]$

$e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha}$
$\cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))=$

$$
\mathrm{PQ} \mathrm{e}^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} 2 \mathrm{D} K_{c}^{\alpha}(x, y, u, v)
$$

$$
-\mathrm{PS} e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)}
$$

$$
\sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{c}^{\alpha}(x, u) 1 \mathrm{D} K_{s}^{\alpha}(y, v)
$$

$-\mathbf{Q R} e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)}$
$\sqrt{\frac{2 \pi}{1-\mathrm{icot} \alpha}} 1 \mathrm{D} K_{s}^{\alpha}(x, u) 1 \mathrm{D} K_{c}^{\alpha}(y, v)$
$\left[+\mathrm{RS}^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\right]$
$e^{\frac{i}{2}\left((x+a)^{2}+(y+b)^{2}+u^{2}+v^{2}\right) \cot \alpha}$
$\cos (\csc \alpha \cdot u(x+a)) \cos (\csc \alpha \cdot v(y+b))=e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha}$
$\left[\begin{array}{c}\operatorname{PQ2DK_{c}^{\alpha }(x,y,u,v)} \\ -\mathrm{PS} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{c}^{\alpha}(x, u) 1 \mathrm{D} K_{s}^{\alpha}(y, v) \\ -\mathrm{QRe}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{s}^{\alpha}(x, u) 1 \mathrm{D} K_{c}^{\alpha}(y, v) \\ +\mathrm{RSe}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\end{array}\right]$
Equation (1) is
$f(x, y), e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) c o t \alpha}$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=\left\langle\left[\begin{array}{c}\operatorname{PQ2DK_{c}^{\alpha }}(x, y, u, v) \\ -\mathbf{P S ~ e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{c}^{\alpha}(x, u) 1 \mathrm{D} K_{s}^{\alpha}(y, v) \\ -\mathbf{Q R e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{s}^{\alpha}(x, u) 1 \mathrm{D} K_{c}^{\alpha}(y, v) \\ +\operatorname{RSe}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\end{array}\right]\right\rangle$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=e^{\frac{i}{2}\left(2 \times a+2 y b+a^{2}+b^{2}\right) c o t \alpha}$
$\left\{\begin{array}{c}\left\langle f(x, y), \operatorname{PQ2D} K_{c}^{\alpha}(x, y, u, v)\right\rangle- \\ \sqrt{\frac{2 \pi}{1-\mathrm{icot} \alpha}} \\ \left\langle f(x, y), \mathrm{PS} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \mathbf{1 \mathrm { D } K _ { c } ^ { \alpha } ( x , u ) 1 \mathrm { D } K _ { s } ^ { \alpha } ( y , v ) \rangle}\right. \\ -\sqrt{\frac{2 \pi}{1-\mathrm{icot} \alpha}} \\ \left\langle f(x, y), \mathrm{QRe}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \mathbf{1 \mathrm { D } K _ { s } ^ { \alpha } ( x , u ) 1 \mathrm { D } K _ { c } ^ { \alpha } ( y , v ) \rangle}\right. \\ +\left\langle f(x, y), \mathrm{RSe}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\right\rangle\end{array}\right\}$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=e^{\frac{i}{2}\left(2 \times a+2 y b+a^{2}+b^{2}\right) \cot \alpha}$
$\left\{\begin{array}{c}\operatorname{PQ}\left\langle f(x, y), 2 \mathrm{D} K_{c}^{\alpha}(x, y, u, v)\right\rangle- \\ \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \mathrm{PS} \\ \left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} 1 \mathrm{D} K_{c}^{\alpha}(x, u) 1 \mathrm{D} K_{s}^{\alpha}(y, v)\right\rangle \\ -\mathbf{Q R} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \\ \left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \mathbf{1 \mathrm { D } K _ { s } ^ { \alpha } ( x , u ) 1 \mathrm { D } K _ { c } ^ { \alpha } ( y , v ) \rangle}\right. \\ +\mathrm{RS}\left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\right\rangle\end{array}\right\}$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha}$
$\left\{\begin{array}{c}\cos (\csc \alpha \mathrm{ua}) \cos (\csc \alpha \mathrm{vb})\left\langle f(x, y), 2 \mathrm{D} K_{c}^{\alpha}(x, y, u, v)\right\rangle \\ -\cos (\csc \alpha \mathrm{ua}) \sin (\csc \alpha v \mathrm{~b}) \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \\ \left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} 1 \mathrm{D} K_{c}^{\alpha}(x, u) 1 \mathrm{D} K_{s}^{\alpha}(y, v)\right\rangle \\ -\cos (\csc \alpha v \mathrm{v}) \sin (\csc \alpha \mathrm{ua}) \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \\ \left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} 1 \mathrm{D} K_{s}^{\alpha}(x, u) 1 \mathrm{D} K_{c}^{\alpha}(y, v)\right\rangle \\ +\sin (\csc \alpha u a) \sin (\csc \alpha v \mathrm{v})\left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\right\rangle\end{array}\right\}$
$F_{\alpha}^{c}\{f(x+a, y+b)\}=e^{\frac{i}{2}\left(2 \times a+2 y b+a^{2}+b^{2}\right) \cot \alpha} \cos (\csc \alpha u a) \cos (\csc \alpha v b)$
$\left\langle f(x, y), 2 \mathrm{D} K_{c}^{\alpha}(x, y, u, v)\right\rangle-e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} \cos (\csc \alpha u a) \sin (\csc \alpha v b)$
$\sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}}\left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} 1 \mathrm{D} K_{c}^{\alpha}(x, u) 1 \mathrm{D} K_{s}^{\alpha}(y, v)\right\rangle-e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} \cos (\csc \alpha v b) \sin (\csc \alpha u a)$
$\sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}}\left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} 1 \mathrm{D} K_{s}^{\alpha}(x, u) 1 \mathrm{D} K_{c}^{\alpha}(y, v)\right\rangle+e^{\frac{i}{2}\left(2 \mathrm{xa}+2 \mathrm{yb}+a^{2}+b^{2}\right) \cot \alpha} \sin (\csc \alpha \mathrm{ua}) \sin (\csc \alpha \mathrm{vb})$ $\left\langle f(x, y), \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} K_{s}^{\alpha}(x, y, u, v)\right\rangle$

The Adjoint differential operator
$F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=\left\langle D_{x, y}, K_{c}^{\alpha}(x, y, u, v)\right\rangle$
$\boldsymbol{F}_{\alpha}^{c}\left\{\boldsymbol{D}_{x, y} f(x, y)\right\}=\left\langle\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}),-\boldsymbol{D}_{x, y} K_{c}^{\alpha}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{v})\right\rangle$
$F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=\left\langle f(x, y),-D_{x, y}\binom{\sqrt{\frac{1-i \cot \alpha}{2 \pi}} e^{\frac{i}{2}\left(x^{2}+y^{2}+u^{2}+v^{2}\right) \cot \alpha}}{\cos (\csc \alpha . u x) \cos (\csc \alpha . v y)}\right\rangle$
$\left.F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=\chi^{f(x, y),-\sqrt{\frac{1-i \cot \alpha}{2 \pi}} D_{x}\left(e^{\frac{i}{2}\left(x^{2}+u^{2}\right) \cot \alpha} \cos (\csc \alpha . u x)\right)}\right\rangle$

$$
D_{y}\left(e^{\frac{i}{2}\left(y^{2}+v^{2}\right) \cot \alpha} \cos (\csc \alpha \cdot v y)\right)
$$

Let $A=\sqrt{\frac{1-i \operatorname{cota} \alpha}{2 \pi}} e^{\frac{i}{2}\left(v^{2}+u^{2}\right) \cot \alpha}$
$F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=\left\langle\begin{array}{c}f(x, y)-A D_{x}\left(e^{e^{\frac{i}{(x}\left(x^{2}\right) \cot \alpha} \cos (\csc \alpha . u x)}\right)\end{array}\right\rangle$
$D_{y}\left(e^{\frac{i}{2}\left(y^{2}\right) \cot \alpha} \cos (\csc \alpha . v y)\right)$
$f(x, y)$
$\left.F_{\alpha}^{c}\left\{\boldsymbol{D}_{x, y} f(x, y)\right\}=\int^{-A\binom{-e^{\frac{i}{2}\left(x^{2}\right) \cot \alpha} \sin (\csc \alpha \cdot u x) \csc \alpha \cdot u x}{+\cos (\csc \alpha \cdot u x) e^{\frac{i}{2}\left(x^{2}\right) \cot \alpha} \operatorname{ixcot} \alpha}}\right\rangle$

$$
\binom{-e^{\frac{i}{2}\left(y^{2}\right) \cot \alpha} \sin (\csc \alpha \cdot v y) \csc \alpha \cdot v y}{+\cos (\csc \alpha \cdot v y) e^{\frac{i}{2}\left(y^{2}\right) \cot \alpha_{i y}} \cot \alpha}
$$

$\boldsymbol{F}_{\alpha}^{c}\left\{\boldsymbol{D}_{x, y} f(x, y)\right\}=\left\langle^{\left.-\boldsymbol{A} \boldsymbol{e}^{\frac{i}{2}\left(x^{2}+y^{2}\right) \cot \alpha}\binom{-\sin (\csc \alpha \cdot u x) \csc \alpha \cdot u x}{+\cos (\csc \alpha \cdot u x) \operatorname{ixcot} \alpha}\right\rangle}\right.$

$$
\begin{gathered}
\binom{-\sin (\csc \alpha \cdot v y) \csc \alpha \cdot v y}{+\cos (\csc \alpha \cdot v y) \text { iy } \cot \alpha} \\
f(x, y) \\
-A e^{\frac{i}{2}\left(x^{2}+y^{2}\right) \cot \alpha}
\end{gathered}
$$

$F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=\left\langle\left[\begin{array}{c}\sin (\csc \alpha \cdot u x) \csc \alpha \cdot u \sin (\csc \alpha \cdot v y) \csc \alpha \cdot v \\ -\sin (\csc \alpha \cdot u x) \csc \alpha \cdot u \cos (\csc \alpha \cdot v y) \operatorname{iycot} \alpha \\ -\cos (\csc \alpha \cdot u x) \operatorname{ixcot} \alpha \sin (\csc \alpha \cdot v y) \csc \alpha \cdot v \\ +\cos (\csc \alpha \cdot u x) \operatorname{ixcot} \alpha \cos (\csc \alpha \cdot v y) \operatorname{iycot} \alpha\end{array}\right]\right\rangle$

$$
f(x, y)-A e^{\frac{i}{2}\left(x^{2}+y^{2}\right) \cot \alpha}
$$

$F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=\left\langle\left[\begin{array}{c}u v(\csc \alpha)^{2} \sin (\csc \alpha . u x) \sin (\csc \alpha . v y) \\ -\mathrm{iycot} \alpha \csc \alpha u \sin (\csc \alpha \cdot u x) \cos (\csc \alpha . v y) \\ -\mathrm{ixcot} \alpha \csc \alpha \cdot v \cos (\csc \alpha \cdot u x) \sin (\csc \alpha \cdot v y) \\ -\mathrm{xy}(\cot \alpha)^{2} \cos (\csc \alpha \cdot u x) \cos (\csc \alpha . v y)\end{array}\right]\right\rangle$ $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$,
$F_{\alpha}^{c}\left\{\boldsymbol{D}_{x, y} f(x, y)\right\}=\left\langle\left(\begin{array}{c}-u v(\csc \alpha)^{2} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \boldsymbol{K}_{s}^{\alpha}(x, y, u, v) \\ -\mathrm{iycot} \alpha \csc \alpha u \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \mathbf{1 D} K_{s}^{\alpha}(x, u) \\ \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{c}^{\alpha}(y, v) \\ -\mathrm{ixcot} \alpha \csc \alpha \cdot v \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \mathbf{1 D} K_{c}^{\alpha}(x, u) \\ \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} 1 \mathrm{D} K_{s}^{\alpha}(y, v) \\ -\mathrm{xy}(\cot \alpha)^{2} K_{c}^{\alpha}(x, y, u, v)\end{array}\right)\right\rangle$

$$
F_{\alpha}^{c}\left\{D_{x, y} f(x, y)\right\}=-\boldsymbol{u} v(\csc \alpha)^{2} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)}\left\langle\boldsymbol{f}(x, y), \boldsymbol{K}_{s}^{\alpha}(x, y, u, v)\right\rangle
$$

$$
-\mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \mathrm{i} \cot \alpha \csc \alpha u\left\langle f(x, y), 1 \mathrm{D} K_{s}^{\alpha}(x, u)\right\rangle\left\langle y, 1 \mathrm{D} K_{c}^{\alpha}(y, v)\right\rangle
$$

$$
-\mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \mathrm{i} \cot \alpha \csc \alpha v
$$

$\left\langle f(x, y), 1 \mathrm{D} K_{c}^{\alpha}(x, u)\right\rangle\left\langle x, 1 \mathrm{D} K_{s}^{\alpha}(y, v)\right\rangle-(\cot \alpha)^{2}\left\langle f(x, y) x y, K_{c}^{\alpha}(x, y, u, v)\right\rangle$
$\boldsymbol{F}_{\alpha}^{c}\left\{\boldsymbol{D}_{x, y} f(x, y)\right\}=-\boldsymbol{u v}(\csc \alpha)^{2} \mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \boldsymbol{F}_{\alpha}^{s}\{f(x, y)\}-\mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\operatorname{icot} \alpha}} \operatorname{icot} \alpha \csc \alpha u 1_{D F_{\alpha}}^{s}\{f(x, y)\}(u) 1_{-} F_{\alpha}^{c}\{1 . y\}(v)$

$$
-\mathrm{e}^{-\mathrm{i}\left(\alpha-\frac{\pi}{2}\right)} \sqrt{\frac{2 \pi}{1-\mathrm{i} \cot \alpha}} \mathrm{i} \cot \alpha \csc \alpha v 1_{D F_{\alpha}}^{s}\{x .1\}(u)
$$

$$
\mathbf{1}_{-} \boldsymbol{F}_{\alpha}^{c}\{\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})\}(\boldsymbol{v})-(\cot \boldsymbol{\alpha})^{2} \boldsymbol{F}_{\alpha}^{c}\{\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})\}
$$

## Conclusion

In the present work Operators on the testing function E and its dual space $\boldsymbol{E}^{*}$ for two dimensional fractional cosine transform are obtained

## Reference

[1] R. S. Pathak, A course in distributional theory and application, Narosa publishing house, New Delhi, 2001
[2]A. H. Zemanian, Generalized Integral Transform, Interscience publication, New York, 1968
[3] V. D Sharma., Khapre S. A.; Applications on generalized two dimensional fractional Cosine transforms, International journal of engineering and innovative technology vol.3, issue4 October 2013. ISSN: 2277-3754.
[4] V. D Sharma, "operators on the distributional generalized two-dimensional fractional flourier transform", Int. J. Modern Math. Sci. 2014, 9(1): 39-45
[5] V. D Sharma., Khapre S. A., " Inversion theorem on generalized two dimensional fractional cosine transform ", Int. J. of advances in sci. engg. And tech. Issue-1, June-2015, ISSN: 2321-9009.
[6] V. D Sharma., Khapre S. A., "Modulation and Parseval,s Relation of Generalized Two Dimensional Fractional Cosine transform ", Int. J. of pure and applied research in engg. And tech.vol 3(9): 110-125, ISSN: 2319-507X.
[7] V. D Sharma., Khapre S. A.; "Analyticity of the generalized two dimensional fractional Cosines transforms, "J. Math. Computer Sci. ISSN: 1927-5307.
[8] V. D Sharma., P. B. Dolas.; "Analyticity of distributional generalized Laplace-Finite Mellin transform, "J. Math. Analysis vol 6, 2012, no.9, 44-451.

