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Combined Effect of Surface Roughness and Deformation on the Performance of a Ferrofluid Based Squeeze Film in Rough Porous Circular Plates with Porous Matrix of Variable Thickness

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ABSTRACT

An attempt has been made to evaluate the effect of surface roughness and deformation on the performance of a magnetic fluid based squeeze film in rough porous circular plates considering porous matrix of variable thickness. The stochastic model of Christensen and Tonder has been adopted to characterize the roughness of the bearing system. With appropriate boundary conditions, the associated Reynolds' type equation has been solved by invoking the dilog function, to derive the expression for pressure distribution resulting in the calculation of load carrying capacity. It is observed that the adverse effect of transverse roughness gets aggravated due to the bearing deformation. But the situation remains fairly improved in the case of negatively skewed roughness for a large range of bearing deformation with a suitable strength of the magnetic field. Further, it is appealing to note that this positive effect gets augmented with a proper selection of thickness ratio parameter associated with variable film thickness.

Introduction

During the last decade, the use of magnetic fluid as a lubricant has drawn considerable attentions because the magnetization modifies and improves the performance of the bearing system. There are a good number of research articles available in the literature in this direction [[1], [2], [3], [4], [5]].

The effect of surface roughness on the behaviour of a Ferrofluid based squeeze film has been studied in various investigations [[5], [6], [7], [8]]. [9] analyzed the effect of transverse surface roughness on the performance of a squeeze film in spherical bearing. It was found that among the three roughness parameters the skewness affected the system most and positively skewed roughness caused severe load reduction. The combined effect of surface roughness and deformation on the performance of magnetic fluid based bearing system has been a matter of investigation [[10], [11]].

The effect of variable film thickness on the performance of a magnetic fluid based squeeze film in rough plate has been discussed by many investigators.

[12] studied the effect of the permeability parameter on the pressure profile, load carrying capacity and the squeeze time for a squeeze film porous bearing. It was shown that as the permeability parameter increased the load carrying capacity decreased in the case of pure squeeze motion. Further, for the permeability parameter less than 0.001, the effect of the porous layer on the hydrodynamic lubrication squeeze film porous bearing could be neglected.

[13] presented a calculation for the variable thickness circular plates subjected to the arbitrary rotational symmetric loading. The extremer values of the load carrying capacity were estimated by resorting to upper and lower bound theorems of the limit analysis. The importance of variable thickness was established for boosting the performance.

[14] considered the effect of porous matrix of variable thickness on the behaviour of a squeeze film in porous circular plates under the presence of a Ferrofluid. Here, it was observed that the magnetic fluid based squeeze film registered an enhanced performance with a proper selection of the thickness ratio parameter.

[5] investigated the Ferrofluid based squeeze film between rough porous infinitely long parallel plates considering porous matrix of variable film thickness. It was established that the load carrying capacity increased with increasing values of the parameter associated with the variable film thickness. Further, the variable film thickness provided some scopes in reducing the adverse effect of roughness suitably choosing the magnetic strength.

Here it has been sought to extend and modify the analysis of [14] to incorporate the combined effect of deformation and roughness.

Analysis

The geometry of the bearing system and configuration is shown in the Figure 1.

The axisymmetric flow of an incompressible fluid between two parallel circular plates of radius a is considered. The lower plate is fixed while the upper plate has a porous facing of variable porous matrix thickness backed by a solid wall. The upper plate approaches the lower one perpendicularly with velocity

$$\dot{h} = dh/dt$$

where h is the uniform film thickness.

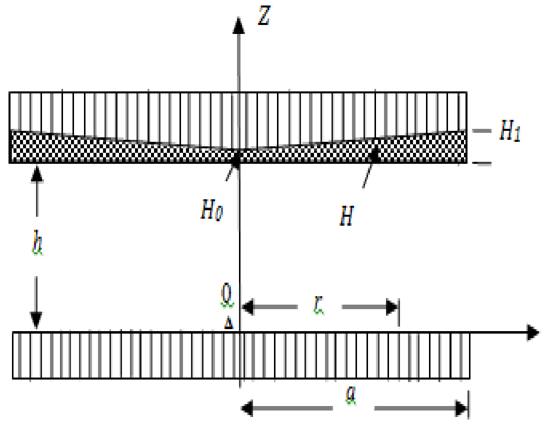

Fig 1. Configuration of bearing system

The porous wall thickness is assumed to vary linearly with its value at $r=0$ as H_0, while this value is H_1 at $r=a$. Accordingly, the porous wall thickness H_0 is given by

$$H = H_0 + \left(H_1 - H_o\right)\left(\frac{r}{a}\right) \tag{1}$$

The $z-$axis is taken normal to the lubricant film. Assuming axially symmetric flow of the magnetic fluid between the plates under an oblique magnetic field

$$M = \left(M(r)\cos\phi,\ \ 0,\ \ M(r)\sin\phi\right) \tag{2}$$

whose magnitude vanishes at $r=0$ and $r=a$, the associated Reynolds equation governing the film pressure is obtained as

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rh^3\frac{d}{dr}\left\{p-0.5\mu_0\bar{\mu}M^2\right\}\right] = 12\mu\left[\dot{h}-\frac{\phi}{\mu}\left(\frac{\partial}{\partial z}\left\{p-0.5\mu_0\bar{\mu}M^2\right\}\right)_{z=h}\right] \tag{3}$$

where $M^2 = Kr(1-r)$, K is a suitably chosen constant so as to produce a desired magnetic strength [15], μ is the fluid viscosity, $\bar{\mu}$ represents the magnetic susceptibility, μ_0 stands for the permeability of the free space and ϕ is the permeability of the porous matrix governed by the Laplace equation [16]

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left\{p-0.5\mu_0\bar{\mu}M^2\right\}\right]+\frac{\partial^2}{\partial z^2}\left\{p-0.5\mu_0\bar{\mu}M^2\right\}=0. \tag{4}$$

The concerned boundary conditions for Eq. (3) and Eq. (4) are

$$\begin{aligned} p&=0 \ \text{ at } r=0\\ p&=0 \ \text{ at } r=a\\ p&=P \ \text{ at } z=h\\ \frac{\partial p}{\partial z}&=0 \ \text{ at } z=h+H \end{aligned} \tag{5}$$

The system (3) – (5) is seen to be coupled both in terms of differential equation as well as boundary conditions. To obtain its solution we first uncouple the system by using the simplifying assumption that H may be considered so small that a Taylor's series representation may be used. As discussed in [17-19] a random variable with nonzero mean, variance and skewness characteristics the roughness of the bearing surfaces. The details regarding the characterization of the film thickness and the associated statistically averaging process can be called from [17-19]. This results in uncoupled modified Reynolds equation, namely

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d}{dr}\left\{p-0.5\mu_0\bar{\mu}M^2\right\}\right]=\frac{12\mu\dot{h}}{g(h)+12\phi H} \tag{6}$$

where,

$$\begin{aligned} g(h) &= \left(h+p_a p'\delta\right)^3+3\left(\sigma^2+\alpha^2\right)\left(h+p_a p'\delta\right)\\ &\quad +3\alpha\left(h+p_a p'\delta\right)^2+3\sigma^2\alpha+\alpha^3+\varepsilon \end{aligned}$$

Introduction of the non-dimensional quantities

$$\begin{aligned} &R=\frac{r}{a},\ \ \bar{\sigma}=\frac{\sigma}{h},\ \bar{\alpha}=\frac{\alpha}{h},\ \bar{\varepsilon}=\frac{\varepsilon}{h^3},\\ &\bar{\delta}=\frac{\delta}{h},\ \bar{p}=p_a p',\ \psi_0=\frac{\phi H_0}{h_0^3},\ \psi=\frac{\psi_0}{\bar{h}^3},\\ &\mu^*=-\frac{\mu_0\bar{\mu}h^3}{\mu\dot{h}},\ \ P=-\frac{h^3 p}{\mu\dot{h}a^2},\ \ \lambda=\frac{H_1}{H_0}-1. \end{aligned} \tag{7}$$

where, h_0 is the initial film thickness; transform Equation (6) into

$$\frac{1}{R}\frac{d}{dR}\left[R\frac{d}{dR}\left\{P-0.5\mu^* R(1-R)\right\}\right]=\frac{-12\mu\dot{h}}{h^3\left[A_1+A_2 R\right]} \tag{8}$$

where,

$$\begin{aligned} &A_1=G(h)+12\psi;\ \ A_2=12\psi\lambda,\\ &G(h)=\left(1+\bar{p}\bar{\delta}\right)^3+3\left(\bar{\sigma}^2+\bar{\alpha}^2\right)\left(1+\bar{p}\bar{\delta}\right)\\ &\qquad +3\bar{\alpha}\left(1+\bar{p}\bar{\delta}\right)^2+3\bar{\sigma}^2\bar{\alpha}+\bar{\alpha}^3+\bar{\varepsilon}. \end{aligned}$$

In view of the boundary conditions

$$\frac{dP}{dR}=-\frac{\mu^*}{2}\quad \text{at}\quad R=0 \text{ and } P=0 \ \text{ at } \ R=1 \tag{9}$$

a solution of equation (8) is obtained as

$$P=0.5\mu^* R(1-R)+\frac{12(1-R)}{A_2}+\frac{12A_1X_1}{A_2^2} \tag{10}$$

where,

$$\begin{aligned} X_1 &= di\log\left(-\frac{A_2R}{A_1}\right)-di\log\left(-\frac{A_2}{A_1}\right)\\ &\quad +\log\left(A_1+A_2R\right)\log\left(-\frac{A_2R}{A_1}\right)-\log\left(A_1+A_2\right)\log\left(-\frac{A_2}{A_1}\right). \end{aligned}$$

Here, for the properties of the Dilog function one is requested to refer [20].

Load carrying capacity of the bearing

$$w=2\pi\int_0^a rp(r)\,dr$$

In dimensionless form is given by

$$W=2\pi\int_0^1 PR\,dR=2\pi\left[\frac{\mu^*}{24}+\frac{X_2}{A_1}-\frac{6A_1X_3}{A_2^2}\right] \tag{11}$$

where,

$$\begin{aligned} X_2 &= \frac{\pi^2A_1^2}{A_2^2}-\frac{2A_1}{A_2}-\frac{4A_2}{15A_1}-\frac{A_2^2}{8A_1^2}-\frac{15}{4};\\ X_3 &= di\log\left(-\frac{A_2}{A_1}\right)+\log\left(1+\frac{A_2}{A_1}\right)\log\left(-\frac{A_2}{A_1}\right). \end{aligned}$$

Results and discussion

It is noticed that the dimensionless pressure distribution is obtained from the Eq. (10) while Eq. (11) presents the variation of non-dimensional load carrying capacity. It is easily observed that while the pressure increases by

$$\frac{\mu^*}{2}R(1-R)$$

and the increase in load carrying capacity is

$$\frac{\mu^*}{2}$$

as compared to the case of a traditional lubricant. When λ is taken to the zero, the results for uniform porous matrix of variable thickness are recovered. In the absence of deformation for a smooth bearing system, this study reduces to the analysis of [14]. The corresponding non magnetic case is obtained by the setting the magnetic parameter μ^* to be zero. One can very well see that the expression for the load carrying capacity is linear with respect to magnetic parameter μ^* . Accordingly, an increase in μ^* would result the increased in load carrying capacity. This is not surprising as the effective viscosity gets increased due to the magnetization.

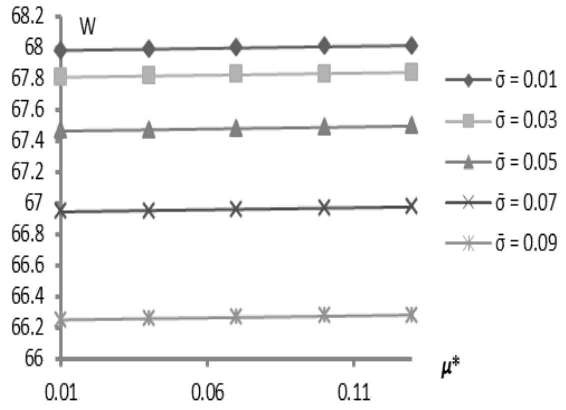

Fig. 2 Variation of Load carrying capacity with respect to μ^* and $\bar{\sigma}$

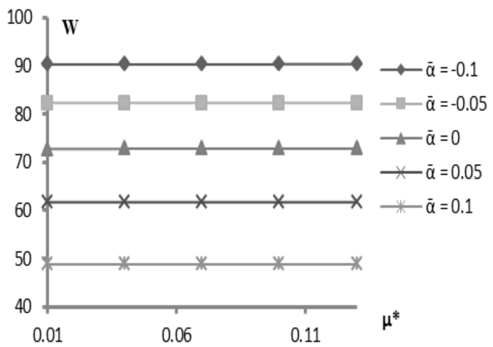

Fig. 3 Variation of Load carrying capacity with respect to μ^* and $\bar{\alpha}$

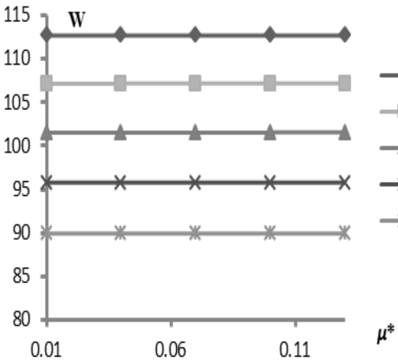

Fig. 4 Variation of Load carrying capacity with respect to μ^* and $\bar{\varepsilon}$

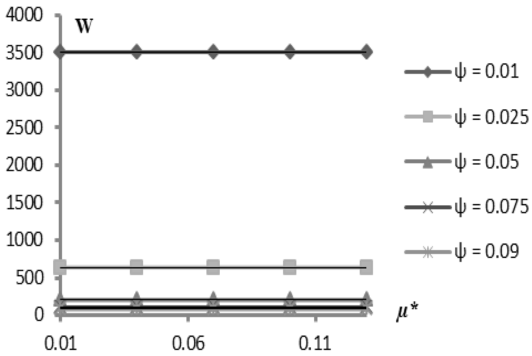

Fig. 5 Variation of Load carrying capacity with respect to μ^* and ψ

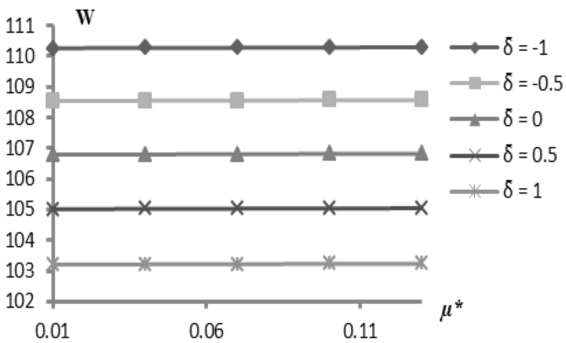

Fig 6. Variation of Load carrying capacity with respect to μ^* and $\bar{\delta}$

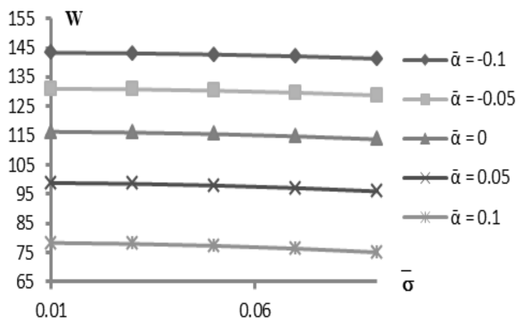

Fig 7. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$

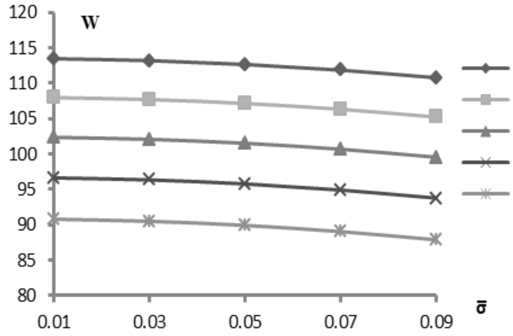

Fig 8. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and $\overline{\varepsilon}$

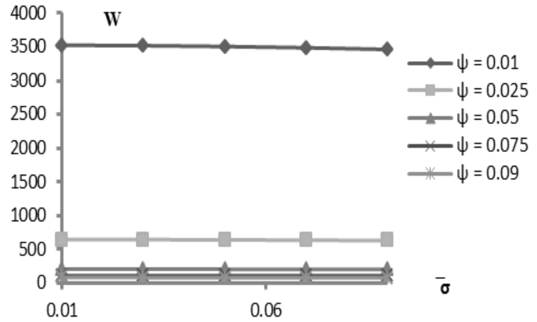

Fig 9. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and ψ

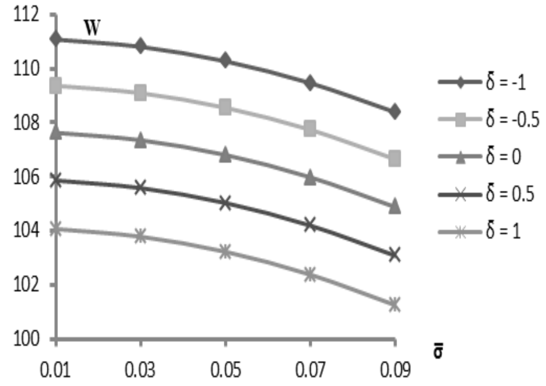

Fig 10. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and $\overline{\delta}$

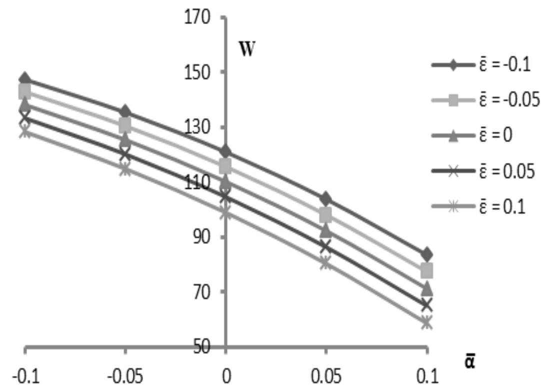

Fig 11. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and $\overline{\varepsilon}$

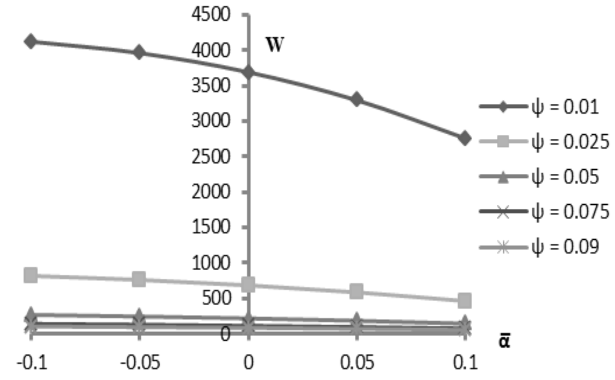

Fig 12. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and ψ

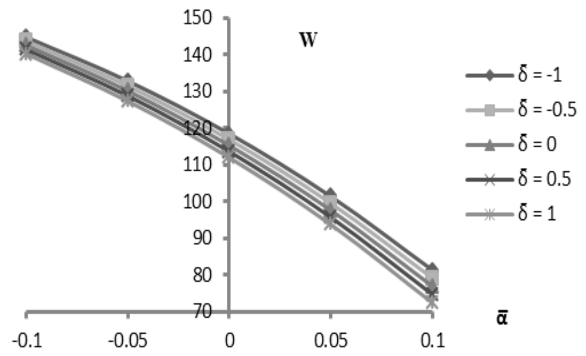

Fig 13. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and $\overline{\delta}$

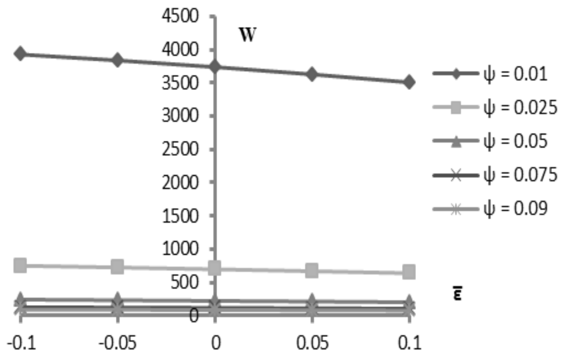

Fig 14. Variation of Load carrying capacity with respect to $\overline{\varepsilon}$ and ψ

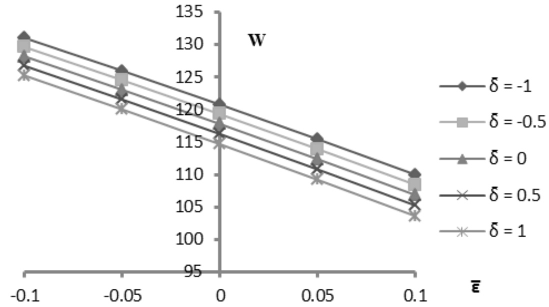

Fig 15. Variation of Load carrying capacity with respect to $\overline{\varepsilon}$ and $\overline{\delta}$

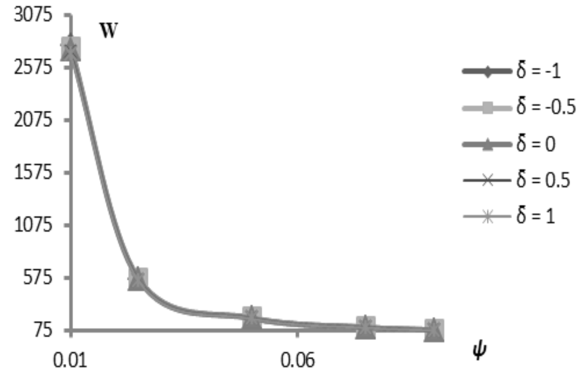

Fig 16. Variation of Load carrying capacity with respect to ψ and $\overline{\delta}$

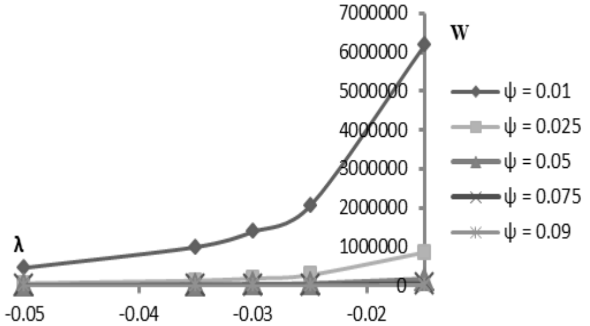

Fig 17. Variation of Load carrying capacity with respect to λ and ψ

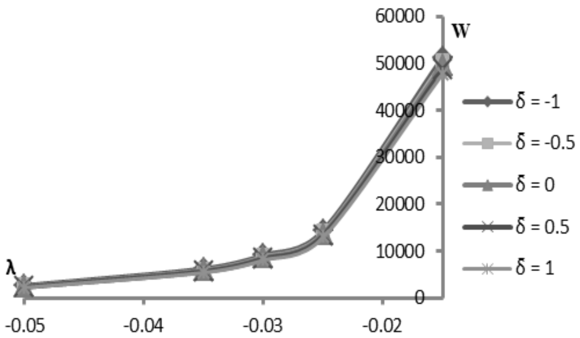

Fig 18. Variation of Load carrying capacity with respect to λ and $\overline{\delta}$

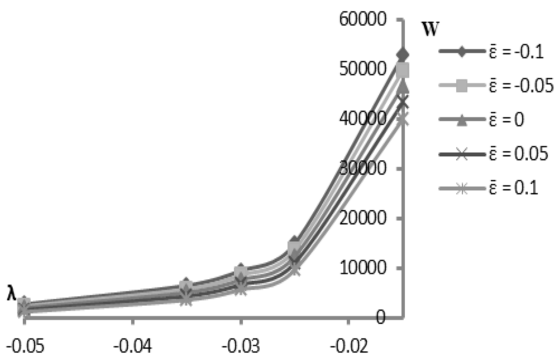

Fig 19. Variation of Load carrying capacity with respect to λ and $\overline{\varepsilon}$

The graphical representations reveal the following:

It is interesting to note that, the magnetization increases the load carrying capacity. However, the increase is not that significant in most cases. Further, the effect of porosity on the variation of load carrying capacity with respect to the magnetization is almost negligible for higher values of porosity.
The standard deviation has a tendency to reduce the load carrying capacity. Further, the effect of higher values of porosity on the variation of load carrying capacity with respect to standard deviation is almost negligible. In addition, the combined effect of standard deviation and deformation decreases the load carrying capacity heavily.

The positively skewed roughness decreases the load carrying capacity while load carrying capacity is enhanced in the case of negatively skewed roughness. Same type of effects of variance is observed. Thus, the combined effect of variance (-ve) and the negatively skewed roughness may play a seminal role in improving the performance of bearing system. However, the combined effect of variance, skewness and porosity remains negligible especially, for higher values of porosity.

It is also noticed that the effect of deformation on the distribution of load carrying capacity with respect to variance is not that significant up to certain values of deformation parameter.

The positive effect induced by $\delta(-ve)$ gets augmented in the case of negatively skewed roughness. Also, the combined effect of positive skewness and $\delta(+ve)$ is quite adverse.

It is interesting to see that for higher values of ψ, the combined effect of porosity and deformation turns out to be not that significant.

The fact that, the parameter λ may provide a decisive role in enhancing the performance of the bearing system for lower values of the porosity.

It is easily observed that the load carrying capacity slowly increases with the increasing values of the parameter λ up to the value $\lambda = -0.0275$ and then rises sharply. The effect of negatively skewed roughness on the variation of load carrying capacity with respect to λ remains quite significant. However, the effect of deformation and porosity on the distribution of load carrying capacity with respect to λ is almost nominal.

A close look at the figures reveals that by suitably choosing the values of λ, the porosity and deformation induced negative effect can be reduced considerably by the positive effect of the magnetization parameter μ^*.

Thus, it is clear that higher values of porosity may not be of that favourable for this type of bearing system. It is found that the increased load carrying capacity due to negatively skewed roughness and variance (–ve) gets further increased when the negative values of deformation are in place.

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