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Vague Congruence Relations on Residuated Lattices

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ABSTRACT

The aim of this paper is to establish the concept of vague congruence relation on a residuated Lattice. We discuss the relationship between vague Filters and vague congruence relations. Further we define the vague congruence relation corresponding to a given vague filter and some of its properties are obtain. Finally, we determine the quotient algebra induced by this relation and discuss some properties.

Introduction

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Nowadays, it is generally accepted that in fuzzy logic the algebraic structure should be a residuated lattice which was introduced by Ward and Dilworth [22]. Some other logical algebras such as MTI-algebras [3], BL-algebras [5], MV-algebras [2], G-algebras, □-algebras, and NM-algebras [3], which are also called R_0 -algebras [23], are all able to be considered particular classes of residuated lattices. Filters are an important tool to study these logical algebras and the completeness of the corresponding nonclassical logics. On the one hand, filters are closely related to congruence relations with which one can associate quotient algebras [21]. Since Rosenfeld [16] applied the notion of fuzzy sets [25] to abstract algebra and introduced the notion of fuzzy subgroups, the literature of various fuzzy algebraic concepts has been growing very rapidly [18]. The notion of fuzzy filters was introduced, and some properties of them were obtained [10]. Moreover, based on the notion of intuitionistic fuzzy sets proposed by Atanassov [1], the concept of the intuitionistic fuzzy filter in BL-algebras was introduced in [24]. In this paper, we apply the notion of intuitionistic fuzzy sets to a residuated lattice. Further, we define the notion of intuitionistic fuzzy congruence relation on a residuated lattice and study its properties. Then we prove that the quotient algebra induced by a vague filter is a residuated lattice and investigate some related results.

Preliminaries

Definition 2.1 [5]

A residuated lattice is an algebraic structure $L = (L, \vee, \wedge, *, \to, 0, 1)$ satisfying the following axioms:

1. $(L, \vee, \wedge, 0, 1)$ is a bounded lattice
2. $(L, *, 1)$ is a commutative semigroup (with the unit element 1).
3. $(*, 1)$ is an adjoint pair, i.e., for any $x, y, z, w \in L$,

i. if $x \le y$ and $z \le w$, then $x * z \le y * w$.
ii. if $x \le y$ and $y \to z \le x \to z$ and $z \to x \le z \to y$.
iii. (adjointness condition) $x * y \le z$ if and only if $x \le y \to z$.

In this paper, denote L as residuation lattice unless otherwise specified.

Theorem 2.2 [5]

In each residuated lattice L, the following properties hold for all $x, y, z \in L$:

1. $(x * y) \to z = x \to (y \to z)$.
2. $z \le x \to y \Leftrightarrow z * x \le y$.
3. $x \le y \Leftrightarrow z * x \le z * y$.
4. $x \to (y \to z) = y \to (x \to z)$.
5. $x \le y \Rightarrow z \to x \le z \to y$.
6. $x \le y \Rightarrow y \to z \le x \to z$, $y' \le x'$.
7. $y \to z \le (x \to y) \to (x \to z)$.
8. $y \to x \le (x \to z) \to (y \to z)$.
9. $1 \to x = x$, $x \to x = 1$.
10. $x^m \le x^n$, $m, n \in N$, $m \ge n$.
11. $x \le y \Leftrightarrow x \to y = 1$.

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12. $0^{'} = 1$, $1^{'} = 0$, $x^{'} = x^{m}$, $x \leq x^{n}$.

13. $x \vee y \to z = (x \to z) \wedge (y \to z)$.

14. $x * x^{'} = 0$.

15. $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$.

Definition 2.6: [26]

A fuzzy set A of a residuated lattice L is called a fuzzy filter, if it satisfies, for any $x, y \in L$

1. $A(1) \geq A(x)$.
2. $A(x * y) \geq \min\{A(x), A(y)\}$.

Theorem 2.7: [26]

A fuzzy set A of a residuated lattice L is a fuzzy filter, if and only if it satisfies, for any $x, y \in L$,

1. $A(1) \geq A(x)$.
2. $A(y) \geq \min\{A(x \to y), A(x)\}$

Definition 2.8 [4]

A Vague set A in the universe of discourse S is a Pair (t_A, f_A) where t_A:S $\to$[0,1] and f_A:S$\to$ [0,1] are mappings (called truth membership function and false membership function respectively) where t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and f_A(x) is a lower bound on the negation of x derived from the evidence against x and **t_A(x) + f_A(x) $\leq$ 1 $\forall$x$\in$ S.**

Vague congruence relation

Definition 3.1

Let X be a set and R $\in$ VR(X). Then R is called a vague equivalence relation (in short, VE) on X if it satisfies the following conditions

1. R is vague reflexive, i.e., R(x, x) = 1 for each $x \in X$.
2. R is vague symmetric, i.e., R(x, y) = R(y, x)
3. R is vague transitive, i.e., R o R $\subseteq$ R

Definition 3.2

Let R = [t_R, 1- f_R] be a VE on a residuated lattice L. Then R is called a vague congruence relation (in short VC) if it satisfies the following conditions: for any $x, y, z, w \in L$

1. $V_R(x * z, y * w) \geq V_R(x, y) \wedge V_R(z, w)$
2. $V_R(x \to z, y \to w) \geq V_R(x, y) \wedge V_R(z, w)$
3. $V_R(x \wedge z, y \wedge w) \geq V_R(x, y) \wedge V_R(z, w)$.
4. $V_R(x \vee z, y \vee w) \geq V_R(x, y) \wedge V_R(z, w)$.

Theorem 3.3

Let R = [t_R, 1- f_R] be a VE on a residuated lattice L. Then R is a VC if and only if it satisfies the following conditions

1. $V_R(x * z, y * z) \geq V_R(x, y)$
2. $V_R(x \to z, y \to z) \geq V_R(x, y)$
3. $V_R(z \to x, z \to y) \geq V_R(x, y)$
4. $V_R(x \wedge z, y \wedge z) \geq V_R(x, y)$
5. $V_R(x \vee z, y \vee z) \geq V_R(x, y)$ for all $x, y, z \in L$.

Proof

Let R = [t_R, 1- f_R] be a VC on a residuated lattice L. We have $V_R(z, z) = 1$. Suppose that $* \in \{*, \to, \wedge, \vee\}$. By Definition 3.2, $V_R(x * z, y * z) \geq V_R(x, y) \wedge V_R(z, z) = V_R(x, y)$. Hence it satisfies conditions (1- 5). Conversely, since R = [t_R, 1- f_R] is a VE , then $V_R(x * z, y * w) \geq \bigvee_{u \in L} [V_R(x * z, u) \wedge V_R(u, y * w)] \geq [V_R(x * z, y * z) \wedge V_R(y * z, y * w)] \geq V_R(x, y) \wedge V_R(z, w)$. Therefore R is a vague congruence relation.

Theorem 3.4

Let R = [t_R, 1- f_R] be a VE on a residuated lattice L. Then R = [t_R, 1- f_R] be a VC on L if and only if for all $\alpha, \beta \in [0, 1]$, the sets $U(t_R, \alpha) = \{x \in X : t_R(x) \geq \alpha\}$ and $L(1 - f_R, \beta) = \{x \in X : 1 - f_R(x) \geq \beta\}$ are vague congruence relations on L.

Proof

Suppose that R = [t_R, 1- f_R] be a VC on a residuated lattice L. and $\alpha, \beta \in [0, 1]$.

First, we will show that $U(t_R, \alpha)$ is a equivalence relation on L.

Since $t_R(x, x) = 1 \geq \alpha$, then $(x, x) \in U(t_R, \alpha)$. Hence $U(t_R, \alpha)$ is reflexive. It is clear that $U(t_R, \alpha)$ is symmetric. Let (x, y), (y, z) $\in U(t_R, \alpha)$. Then $t_R(x, y), t_R(y, z) \geq \alpha$. Since R is a vague equivalence relation on L, we obtain that $\alpha \leq t_R(x,z) \wedge t_R(z,y) \leq \bigvee_{u \in L} [t_R(x, u) \wedge t_R(u, y)] = t_{RoR}(x, y) \leq t_R(x, y)$. Therefore $U(t_R, \alpha)$ is transitive. Hence $U(t_R, \alpha)$ is a vague equivalence relation on L. Suppose that $* \in \{*, \to, \wedge, \vee\}$ and (x, y), (z, w) $\in U(t_R, \alpha)$. Then $t_R(x, y), t_R(z, w) \geq \alpha$. Therefore by Definition

3.2, we have $\alpha \le t_R(x,y) \wedge t_R(z,w) \le t_R(x* y, z* w)$, that is $(x * z, y * w) \in U(t_R, \alpha)$. Hence $U(t_R, \alpha)$ is a vague congruence relation on L. Similarly we can prove that $L(1 - f_R, \beta)$ is a vague congruence relation on L. Conversely, suppose that for all $\alpha, \beta \in [0, 1]$, the sets $U(t_R, \alpha)$ and $L(1 - f_R, \beta)$ are congruence relations on L. We will prove that $R=[t_R, 1- f_R]$ is a vague equivalence relation on L. Since $U(t_R, 1)$ and $L(1 - f_R, 1)$ are reflexive, then $R(x, x) = [1, 1]$ for each $x \in L$. It is clear that R is vague symmetric. Suppose that $x, y, z \in L$. Let $t_R(x, z) = p$ and $t_R(z, y) = q$. Put $\alpha = p \wedge q$. Then $t_R(x, z)$, $t_R(z, y) \ge \alpha$. Hence $(x, z), (z, y) \in U(t_R, \alpha)$. Since $U(t_R, \alpha)$ is transitive, we obtain that $(x, y) \in U(t_R, \alpha)$, that is $t_R(x, y) \ge \alpha = p \wedge q = t_R(x, z) \wedge t_R(z, y)$. Since $z \in L$ is arbitrary, we get that $t_R(x,y) \ge \bigvee_{z \in L} [t_R(x,z) \wedge t_R(z, y)]$.

Similarly we prove that $1- f_R(x,y) \ge \bigvee_{z \in L} [1- f_R(x, z) \wedge 1- f_R(z, y)]$. Therefore $R \circ R \subseteq R$ and then R is vague equivalence relation. Let $* \in \{*, \to, \wedge, \vee\}$. Suppose that $t_R(x, y) = r$ and $t_R(z, w)=s$. Put $\alpha=r\wedge s$. Then $t_R(x, y)$, $t_R(z, w) \ge \alpha$. Hence $(x, y), (z, w) \in U(t_R, \alpha)$. Since $U(t_R, \alpha)$ is a vague congruence relation, we obtain that $(x * z, y * w) \in U(t_R, \alpha)$, that is $t_R(x * z, y * w) \ge \alpha = r\wedge s = t_R(x, y) \wedge t_R(z, w)$. Similarly, we can show that $1-f_R(x * z, y * w) \ge 1 - f_R(x, y) \wedge 1- f_R(z, w)$. Hence $R=[t_R, 1- f_R]$ is a vague congruence relation on L.

Vague congruences induced by vague filter

Definition 4.1

Let $R = [t_R, 1- f_R]$ be a VC on a residuated lattice L. Then the vague subset $A_R = [t_{A_R}, 1- f_{A_R}]$ which is defined by $t_{A_R}(x) = t_R(x, 1)$ and $1- f_{A_R}(x) = 1- f_{A_R}(x, 0)$, is called a vague subset induced by R.

Theorem 4.2

Let $R = [t_R, 1- f_R]$ be a VC on a residuated lattice L. Then A_R is a vague filter of L.

Proof:

Let $x, y \in L$ be arbitrary. Then $V_{A_R}(1) = V_R(1, 1) = V_R(x \to 1, 1 \to 1) \ge V_R(x, 1) = V_{A_R}(x)$. $V_{A_R}(y) = V_R(y,1) = V_R(y \vee (x * (x \to y)), y\vee 1) \ge V_R(x * (x \to y), 1 * 1) \ge V_R(x,1) \wedge V_R(x \to y, 1) = V_{A_R}(x) \wedge V_{A_R}(x \to y)$.

Definition 4.3

Let $A = [t_A, 1- f_A]$ be a vague filter (in short VF) of a residuated lattice L. The vague relation $R_A = [t_{R_A}, 1- f_{R_A}]$ on L which is defined by $V_{R_A}(x, y) = V_A(x \to y) \wedge V_A(y \to x)$ is called the vague relation induced by A.

Lemma 4.4

Let $A = [t_A, 1- f_A]$ be a VF of a residuated lattice L. Then

1. $V_A(x \to y) \le V_A[(x * z) \to (y * z)]$
2. $V_A(x \to y) \le V_A[(y \to z) \to (x \to z)]$
3. $V_A(x \to y) \le V_A[(x \wedge z) \to (y \wedge z)]$
4. $V_A(x \to y) \le V_A[(x \vee z) \to (y \vee z)]$ for all $x, y, z \in L$.

Proof:

(1) and (2) follows from Definitions.

3.Since $(x \wedge z) * (x \to y) \le (x * (x \to y)) \wedge (z * (y \to x)) \le y \wedge z$, then $(x \to y) \le (x \wedge z) \to (y \wedge z)$. Hence (3) holds.

4.$(x \vee z) * (x \to y) = (x * (x \to y)) \vee (z * (x \to y)) \le y \vee z$. Then $x \to z \le (x \vee z) \to (y \vee z)$. Hence (4) holds.

Theorem 4.5

Let $A = [t_A, 1- f_A]$ be a vague filter of a residuated lattice L. Then $R_A = [t_{R_A}, 1- f_{R_A}]$ is a VC on L.

Proof: Follows from Lemma 4.4.

Theorem 4.6

Let $A = [t_A, 1- f_A]$ be a vague filter on a residuated lattice L. Then $A_{R_A} = A$.

Proof:

Let $x \in L$. Since A is a vague filter of L, we have

$V_{A_{R_A}}(x) = V_{A_{R_A}}(x, 1) = V_A(x \to 1) \wedge V_A(1 \to x) = V_A(x)$. Hence $A_{R_A} = A$.

Theorem 4.7

Let $R = [t_R, 1- f_R]$ be a VC on a residuated lattice L. Then $R_{A_R} = R$.

Proof:

Let $x, y \in L$. Then $V_{R_{A_R}}(x, y) = V_{A_R}(x \to y) \wedge V_{A_R}(y \to x) = V_R(x \to y, 1) \wedge V_R(y \to x, 1) = V_R(x \to y, y \to y) \wedge V_R(y \to x, x \to x) \ge V_R(x, y)$. Therefore $R_{A_R} \supseteq R$. Conversely, we have $V_R(x, y) \ge V_R(x, x \vee y) \wedge V_R(x \vee y, y) \ge V_R(y * (y \to x), y) \wedge V_R(x, x * (x \to y)) \ge V_R(y * (y \to x), y * 1) \wedge V_R(x * 1, x * (x \to y)) \ge V_R(y \to x, 1) \wedge V_R(1, x \to y) = V_{A_R}(y \to x) \wedge V_{A_R}(x \to y) = V_{R_{A_R}}(x, y)$. Therefore $R_{A_R} \subseteq R$. Hence $R_{A_R} = R$.

Theorem 4.8: (correspondence theorem)

There is a bijection between the set of all vague congruence relations and the set of all vague filters $A = [t_A, 1- f_A]$ be a vague filter of a residuated lattice L such that $V_A(1) = 1$.

Proof:

Denote the set of all vague congruence relations on L by VC(L) and the set of all vague filters such that $V_A(1) = 1$ by VF(L). Define ψ : VC(L) $\to$ VF(L) by ψ (R) = A_R and χ : VF(L) $\to$ VC(L) by χ(A) = R_A. By Theorems 4.3 and 4.4. ψ and χ are well defined. By Theorem 4.6 and 4.7, ψ and χ are inverse of each other.

Definition 4.9

Let R be a vague congruence relation on a residuated lattice L and a $\in$ L. Define the complex mapping R_a: L $\to$ I x I as follows: $R_a(x) = R(a, x)$, for all x $\in$ L. Then R_a is a vague set and it is called a vague equivalence class of R containing a.

Proposition 4.10:

Let A = [t_A, 1- f_A] be a vague filter of a residuated lattice L and R_A be the VC induced by A. Then the following hold:

1. $(R_A)_a = (R_A)_b$ if and only if $t_A(a \to b) = t_A(b \to a) = t_A(1)$ and $1 - f_A(a \to b) = 1 - f_A(b \to a) = 1 - f_A(0)$,
2. $(R_A)_a = (R_A)_1$ if and only if $t_A(a) = t_A(1)$ and $1 - f_A(a) = 1 - f_A(0)$.

Proof:

1. Let $(R_A)_a = (R_A)_b$. We have $(R_A)_a(a) = (R_A)_b(a)$ and obtain that $V_a(a \to a) \wedge V_A(a \to a) = V_{R_A}(a, a) = V_{R_A}(a, b) = V_A(b \to a) \wedge V_A(a \to b)$. It follows that $t_A(b \to a) = t_A(a \to b) = t_A(1)$, $1 - f_A(b \to a) = 1 - f_A(a \to b) = 1 - f_A(0)$. Conversely, suppose that $t_A(b \to a) = t_A(a \to b) = t_A(1)$ and $1 - f_A(a \to b) = 1 - f_A(b \to a) = 1 - f_A(0)$. we know that $V_A(x \to a) \wedge V_A(a \to b) \le V_A((x \to a) * (a \to b)) \le V_A(x \to b)$ and $V_A(x \to b) \wedge V_A(b \to a) \le V_A((x \to b) * (b \to a)) \le V_A(x \to a)$. By using assumption, we have $V_A(x \to a) \le V_A(x \to b)$ and $V_A(x \to b) \le V_A(x \to a)$. Therefore $V_A(x \to b) = V_A(x \to a)$. Similarly, we can show that $V_A(b \to x) = V_A(a \to x)$. Thus $(R_A)_a(x) = (R_A)_b(x)$ for all x $\in$ L.
2. It follows from part (1)

Theorem 4.11

Let A = [t_A, 1- f_A] be a vague filter of a residuated lattice L. Define a $\cong_A$ b if and only if $(R_A)_a = (R_A)_b$. Then $\cong_A$ is a congruence relation on L.

Proof: The proof follows from Proposition 4.10.

Definition 4.12

Let A = [t_A, 1- f_A] be a vague filter of a residuated lattice L and R_A be the VC induced by A.

The set { $(R_A)_a$: a $\in$ L} is called the vague quotient set of L by R_A and denoted by L / R_A. On this set, we have $(R_A)_a * (R_A)_b = (R_A)_{a*b}$, $(R_A)_a \to (R_A)_b = (R_A)_{a \to b}$ and $(R_A)_a \wedge (R_A)_b = (R_A)_{a \wedge b}$, $(R_A)_a \vee (R_A)_b = (R_A)_{a \vee b}$.

Theorem 4.13

Let A= [t_A,1- f_A] be a vague filter of a residuated lattice L. Then L/R_A = (L/R_A,$\wedge$,$\vee$,$\to$, *, $0_\sim$, $1_\sim$) is a residuated lattice.

Proof:

We have $(R_A)_a = (R_A)_b$ and $(R_A)_c = (R_A)_d$ if and only if a $\cong_A$ b and c $\cong_A$ d. Since $\cong_A$ is the congruence relation on L by Theorem 4.11, then all above operations are well defined. It is easy to show that (L/R_A,$\wedge$,$\vee$,$\to$, *, $0_\sim$, $1_\sim$) is a bounded lattice , * is commutative, associative and has $1_\sim$ as an identity. The operation $\vee$ defines a relation $\le$ on L / R_A by $(R_A)_a \le (R_A)_b$ if and only if $(R_A)_{a \vee b} = (R_A)_b$ for all a, b $\in$ L. This relation is a partial order on L / R_A. Using Proposition 4.10, we have $(R_A)_a \le (R_A)_b$ if and only if a $\to$ b $\in$ U(t_A, $t_A(1)$) and a $\to$ b $\in$ L($1 - f_A$, 1- $f_A(0)$) for all a, b $\in$ L. Now, we will show that $(R_A)_a \le (R_A)_b \to (R_A)_c$ if and only if $(R_A)_a * (R_A)_b \le (R_A)_c$ for all a, b, c $\in$ L. we have $(R_A)_a \le (R_A)_b \to (R_A)_c \Leftrightarrow (R_A)_a \le (R_A)_{b \to c} \Leftrightarrow$ (a $\to$ (b $\to$ c)) $\in$ U(t_A, $t_A(1)$) and (a $\to$ (b $\to$ c)) $\in$ L($1 - f_A$, 1- $f_A(0)$) $\Leftrightarrow$ ((a * b) $\to$ c) $\in$ U(t_A, $t_A(1)$) and ((a * b) $\to$ c) $\in$ L($1 - f_A$, 1- $f_A(0)$) $\Leftrightarrow (R_A)_{a*b} \le (R_A)_c \Leftrightarrow (R_A)_a * (R_A)_b \le (R_A)_c$. This completes the proof.

Theorem 4.14

Let A= [t_A,1- f_A] be a vague filter of a residuated lattice L and L/R_A be the corresponding quotient algebra. Then the map Ω : L $\to$ L/R_A defined by $\Omega(a) = (R_A)_a$ for all a $\in$ L is a surjective homomorphism and ker(Ω) = U(t_A, $t_A(1)$) $\cap$ L($1 - f_A$, 1- $f_A(0)$) , where ker(Ω) = {x $\in$ L : h(x) = $(R_A)_1$}. Moreover, L/R_A is isomorphic to the commutative residuated lattice L / $\cong_A$.

Proof:

It follows from Definition 4.12 and Theorem 4.13, that Ω is surjective homomorphism. By Proposition 4.10, we have x $\in$ ker(Ω) if and only if $(R_A)_x = \Omega(x) = (R_A)_1$ if and only if $t_A(x) = t_A(1)$ and 1- f_A (x) = 1- $f_A(0)$ if and only if x $\in$ U(t_A, $t_A(1)$) $\cap$ L($1 - f_A$, 1- $f_A(0)$). Hence by Proposition 4.10, L/R_A is isomorphic to the commutative residuated lattice L/ $\cong_A$. Hence Proved.

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