



Certain class of graph with odd and even ratio edge antimagic Labeling

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ABSTRACT

In this paper the existence of odd and even ratio edge antimagic labeling for double triangular snakes ($2\Delta_k$ -snake), $2m\Delta_1$ -snake, $2m\Delta_k$ -snake and kC_4 -snake are proved.

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Keywords

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 $2m\Delta_k$ -snake,
 kC_4 -snake.

Introduction

The graphs considered here are finite, undirected and simple. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G respectively. Also p and q denote the number of vertices and edges of G respectively. Rosa [7] defined a triangular snake (or Δ -snake) as a connected graph in which all blocks are triangles and the block-cut-point graph is a path. Let Δ_k -snake be a Δ -snake with k blocks while $k n\Delta$ -snake is a Δ -snake with k blocks and every block has n number of triangles with one common edge.

Max-min edge antimagic labeling was first introduced by J.Jayapriya and D.Muruganandam in the year 2012, seeking application in Welding Technology [4]. In the year 2013[5] Jayapriya showed the existences of Max-min edge antimagic labeling for the graphs path, cycle, star, sunflower etc. Max-min edge antimagic labeling was renamed as ratio edge antimagic labeling [6]. In this paper the existence of odd and even ratio edge antimagic labeling, for double triangular snakes, $2m\Delta_k$ -snake and kC_4 -snake graphs are shown.

Definition 1.1 [4]: Let $G(V, E)$ be a simple graph with p vertices and q edges. A bijective function $f: V(G) \rightarrow \{1, 3, 5, \dots, 2p-1\}$ is said to be odd ratio edge antimagic labeling if for every edge uv in E , the edge weights

$$\lambda(uv) = \frac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}} \text{ are distinct.}$$

Definition 1.2 [4]: Let $G(V, E)$ be a simple graph with p vertices and q edges.

A bijective function $f: V(G) \rightarrow \{2, 4, 6, \dots, 2p\}$ is said to be even ratio edge antimagic labeling if for every edge uv in E ,

$$\lambda(uv) = \frac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}} \text{ are distinct.}$$

2. Some Class of Triangular Snake Graph with Odd and Even Ratio Edge Antimagic Labeling

Theorem 2.1: The double triangular snake ($2\Delta_k$ -snake) graph admits odd and even ratio edge antimagic labeling.

Proof: Let $G(V, E)$ be a $2\Delta_k$ -snake graph. The graph G consists of the vertices $V = \{u_1, u_2, \dots, u_k\} \cup \{v_1, v_2, \dots, v_{k+1}\} \cup \{w_1, w_2, \dots, w_k\}$ and the edges $E = \{u_i v_i; 1 \leq i \leq k\} \cup \{u_i v_{i+1}; 1 \leq i \leq k\} \cup \{v_i w_i; 1 \leq i \leq k\} \cup \{w_i v_{i+1}; 1 \leq i \leq k\}$.

Let us consider the function $f: V(G) \rightarrow \{1, 3, 5, \dots, 2p-1\}$, such that $f(u_i) = 2i-1; 1 \leq i \leq k$.

$f(v_i) = 4k + (2i-1); 1 \leq i \leq k+1. f(w_i) = 2k + (2i-1); 1 \leq i \leq k$. Now the edge weights are calculated as follows.

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For $1 \leq i \leq k$, $\lambda(u_i v_i) = \frac{4k+2i-1}{2i-1}$. Clearly for $1 \leq i, j \leq k$, $i \neq j$, $\lambda(u_i v_i) \neq \lambda(u_j v_j)$, if $\lambda(u_i w_i) = \lambda(u_j w_j)$ then

$$\frac{4k+2i-1}{2i-1} = \frac{4k+2j-1}{2j-1} \text{ which implies } i = j, \text{ which is a contradiction. Therefore all edge labels are distinct.}$$

For $1 \leq i \leq k$,

$$\lambda(u_i v_{i+1}) = \frac{4k+(2i+1)}{(2i-1)}. \text{ Clearly for } 1 \leq i, j \leq k, \lambda(u_i v_{i+1}) \neq \lambda(u_j v_{j+1}), \text{ if } \lambda(u_i v_{i+1}) = \lambda(u_j v_{j+1}) \text{ then}$$

$$\frac{4k+(2i+1)}{(2i-1)} = \frac{4k+(2j+1)}{(2j-1)} \text{ which implies } i = j, \text{ which is a contradiction. Therefore all edge labels are distinct. For } 1 \leq i \leq k,$$

$$\lambda(u_i w_i) = \frac{4k+2i-1}{2k+2i-1}. \text{ Clearly for } 1 \leq i, j \leq k, i \neq j, \lambda(u_i w_i) \neq \lambda(u_j w_j), \text{ if } \lambda(u_i w_i) = \lambda(u_j w_j) \text{ then}$$

$$\frac{4k+2i-1}{2k+2i-1} = \frac{4k+2j-1}{2k+2j-1} \text{ which implies } i = j, \text{ which is a contradiction. Therefore all edge labels are distinct. For } 1 \leq i \leq k,$$

$$\lambda(w_i v_{i+1}) = \frac{4k+(2i+1)}{2k+(2i-1)}. \text{ Clearly for } 1 \leq i, j \leq k, i \neq j, \lambda(w_i v_{i+1}) \neq \lambda(w_j v_{j+1}), \text{ if } \lambda(w_i v_{i+1}) = \lambda(w_j v_{j+1}) \text{ then } \frac{4k+(2i+1)}{2k+(2i-1)} = \frac{4k+(2j+1)}{2k+(2j-1)}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(v_i v_{i+1}) = \frac{4k+(2i+1)}{4k+(2i-1)}.$$

Suppose $\lambda(v_i v_{i+1}) = \lambda(v_j v_{j+1})$ then $\frac{4k+(2i+1)}{4k+(2i-1)} = \frac{4k+(2j+1)}{4k+(2j-1)}$ which implies $i = j$, which is a contradiction. Therefore all edge labels

are distinct. Thus the double triangular snake ($2\Delta_k$ -snake) graph admits odd ratio edge antimagic labeling.

To prove the existences of even ratio edge antimagic labeling, let us define

$$f: V(G) \rightarrow \{2, 4, 6, \dots, 2p\}, \text{ such that } f(u_i) = 2i; 1 \leq i \leq k.$$

$$f(v_i) = 4k+2i; 1 \leq i \leq k+1.$$

$$f(w_i) = 2k+2i; 1 \leq i \leq k.$$

Now the edge weights are calculated as follows.

$$\text{For } 1 \leq i \leq k, \lambda(u_i v_i) = \frac{4k+2i}{2i}. \text{ Clearly for } 1 \leq i, j \leq k, i \neq j, \lambda(u_i v_i) \neq \lambda(u_j v_j), \text{ if } \lambda(u_i w_i) = \lambda(u_j w_j) \text{ then } \frac{4k+2i}{2i} = \frac{4k+2j}{2j} \text{ which}$$

implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(u_i v_{i+1}) = \frac{4k+2i+2}{2i}. \text{ Clearly for } 1 \leq i, j \leq k, \lambda(u_i v_{i+1}) \neq \lambda(u_j v_{j+1}), \text{ if } \lambda(u_i v_{i+1}) = \lambda(u_j v_{j+1}) \text{ then } \frac{4k+2i+2}{2i} = \frac{4k+2j+2}{2j}$$

which implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(u_i w_i) = \frac{4k+2i}{2k+2i}. \text{ Clearly for } 1 \leq i, j \leq k, i \neq j, \lambda(v_i w_i) \neq \lambda(v_j w_j), \text{ if } \lambda(v_i w_i) = \lambda(v_j w_j) \text{ then } \frac{4k+2i}{2k+2i} = \frac{4k+2j}{2k+2j} \text{ which}$$

implies $i = j$, which is a contradiction. Therefore all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(w_i v_{i+1}) = \frac{4k+2i+2}{2k+2i}. \text{ Clearly for } 1 \leq i, j \leq k, i \neq j, \lambda(w_i v_{i+1}) \neq \lambda(w_j v_{j+1}), \text{ if } \lambda(w_i v_{i+1}) = \lambda(w_j v_{j+1}) \text{ then}$$

$$\frac{4k+2i+2}{2k+2i} = \frac{4k+2j+2}{2k+2j} \text{ which implies } i = j, \text{ which is a contradiction. Therefore all edge labels are distinct.}$$

$$\text{For } 1 \leq i \leq k, \lambda(v_i v_{i+1}) = \frac{4k+(2i+2)}{2k+(2i)}.$$

For $1 \leq i \leq k$, if $i \neq j$, $\lambda(v_i v_{i+1}) \neq \lambda(v_j v_{j+1})$. Suppose $\lambda(v_i v_{i+1}) = \lambda(v_j v_{j+1})$ then $\frac{4k + (2i + 2)}{2k + (2i)} = \frac{4k + (2j + 2)}{2k + (2j)}$ which implies $i = j$, which is

a contradiction. Therefore all edge labels are distinct. Thus the double triangular snakes ($2\Delta_k$ -snake) graph admits even ratio edge antimagic labeling.

Theorem 2.2: $2m\Delta_1$ -snake graph admits odd and even ratio edge antimagic labeling for $m \geq 1$.

Proof: Let $G(V, E)$ be a $2m\Delta_1$ -snake graph.

The graph G consists of the vertex $V = \{u_1, u_2\} \cup \{v_1^1, v_1^2, \dots, v_1^m\} \cup \{w_1^1, w_1^2, \dots, w_1^m\}$ and the edges $E = \{u_1 u_2\} \cup \{u_1 v_1^i; 1 \leq i \leq m\} \cup \{u_2 v_1^i; 1 \leq i \leq m\} \cup \{u_1 w_1^i; 1 \leq i \leq m\} \cup \{u_2 w_1^i; 1 \leq i \leq m\}$.

Let us consider the function $f: V(G) \rightarrow \{1, 3, 5, \dots, 2p-1\}$, such that $f(w_1^i) = 2i-1; 1 \leq i \leq m$.

$$f(v_1^i) = 2m + (2i-1); 1 \leq i \leq m.$$

$$f(u_1) = 4m + 1 \text{ and } f(u_2) = 4m + 3.$$

Now the edge weights are calculated as follows.

$$\text{For } 1 \leq i \leq m, \lambda(u_1 w_1^i) = \frac{4m+1}{2i-1}.$$

$$\lambda(u_2 w_1^i) = \frac{4m+3}{2i-1}.$$

$$\lambda(u_1 v_1^i) = \frac{4m+1}{2m+(2i-1)}.$$

$$\lambda(u_2 v_1^i) = \frac{4m+3}{2m+(2i-1)}.$$

Thus all edge labels are distinct. Hence $2m\Delta_1$ -snake are odd ratio edge antimagic for $m \geq 1$.

To prove the existence of even ratio edge antimagic labeling, let us define

$f: V(G) \rightarrow \{2, 4, 6, \dots, 2p\}$, such that $f(w_1^i) = 2i; 1 \leq i \leq m$.

$$f(v_1^i) = 2m + 2i; 1 \leq i \leq m.$$

$$f(u_1) = 4m + 2 \text{ and } f(u_2) = 4m + 4.$$

Now the edge weights are calculated as follows.

$$\text{For } 1 \leq i \leq m, \lambda(u_1 w_1^i) = \frac{4m+2}{2i}.$$

$$\lambda(u_2 w_1^i) = \frac{4m+4}{2i}.$$

$$\lambda(u_1 v_1^i) = \frac{4m+2}{2m+2i}.$$

$$\lambda(u_2 v_1^i) = \frac{4m+4}{2m+2i}.$$

Thus all edge labels are distinct. Hence $2m\Delta_1$ -snake are even ratio edge antimagic for $m \geq 1$.

Theorem 2.3: The $2m\Delta_k$ -snake admits odd and even ratio edge antimagic labeling.

Proof. Let $G(V, E)$ be a $2m\Delta_k$ -snake graph. The graph $G(V, E)$ consists of $k(2m+1)+1$ vertices. Let $V(G) = \{u_1, u_2, \dots, u_{k+1}\} \cup \{v_1^1, v_1^2, \dots, v_1^m\} \cup \{v_2^1, v_2^2, \dots, v_2^m\} \cup \dots \cup \{v_k^1, v_k^2, \dots, v_k^m\} \cup \{w_1^1, w_1^2, \dots, w_1^m\} \cup \{w_2^1, w_2^2, \dots, w_2^m\} \cup \dots \cup \{w_k^1, w_k^2, \dots, w_k^m\}$ and edges as

$$E(G) = \{v_k u_i; 1 \leq i \leq m, 2 \leq k \leq m\} \cup \{v_k^i u_{i+1}; 1 \leq i \leq m, 2 \leq k \leq m\} \cup \{w_k^i u_i; 1 \leq i \leq m, 2 \leq k \leq m\} \cup \{w_k^i u_{i+1}; 1 \leq i \leq m, 2 \leq k \leq m\}$$

To prove that $2m\Delta_k$ -snake admits odd ratio edge antimagic labeling let us define, $f: V(G) \rightarrow \{1, 3, 5, \dots, 4km+2k+1\}$, such that

$$f(v_j^i) = 2m(j-1) + (2i-1); 1 \leq j \leq k, 1 \leq i \leq m.$$

$$f(w_j^i) = 2m(k+j-1) + (2i-1); 1 \leq j \leq k, 1 \leq i \leq m.$$

$$f(u_j) = 4km + (2j-1); 1 \leq j \leq k+1.$$

The edge weights are calculated as follows:

$$\text{For } 1 \leq j \leq k-1, 1 \leq i \leq m, \lambda(u_j v_j^i) = \frac{4km + 2j - 1}{2m(j-1) + (2i-1)}.$$

$$\text{Clearly } \lambda(u_j v_j^i) \neq \lambda(u_{j+1} v_j^i), \text{ if } \lambda(u_j v_j^i) = \lambda(u_{j+1} v_j^i) \text{ then, } \frac{4km + 2j - 1}{2m(j-1) + (2i-1)} = \frac{4km + 2j + 1}{2m(j-1) + (2i-1)}.$$

This implies $2 = 0$, which is a contradiction. Thus edge labels are distinct.

$$\text{For } 1 \leq j \leq k, 1 \leq i \leq m, \lambda(u_j w_j^i) = \frac{4km + 2j - 1}{2m(k+j-1) + (2i-1)}.$$

$$\text{Clearly } \lambda(u_j w_j^i) \neq \lambda(u_{j+1} w_j^i).$$

$$\text{If } \lambda(u_j v_j^i) = \lambda(u_{j+1} v_j^i), \text{ then } \frac{4km + 2j - 1}{2m(k+j-1) + (2i-1)} = \frac{4km + 2j + 1}{2m(k+j-1) + (2i-1)}.$$

This implies $2 = 0$, which is a contradiction. Thus edge labels are distinct.

$$\text{For } 1 \leq j, l \leq k, \text{ clearly for } l \neq j, \lambda(u_j u_{j+1}) \neq \lambda(u_l u_{l+1}).$$

$$\text{If } \lambda(u_j u_{j+1}) = \lambda(u_l u_{l+1}) \text{ then } \frac{4km + 2j + 1}{4km + 2j - 1} = \frac{4km + 2l + 1}{4km + 2l - 1}.$$

This implies $l = j$, which is a contradiction. Thus edge labels are distinct.

Therefore $2m\Delta_k$ -snake graph admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic labeling, let us define

$$g: V(G) \rightarrow \{2, 4, 6, \dots, 4km + 2k + 2\}, \text{ such that } g(v_1^i) = 2i; 1 \leq i \leq m$$

$$g(v_j^i) = 2m(j-1) + 2i; 1 \leq j \leq k, 1 \leq i \leq m.$$

$$g(w_j^i) = 2m(k+j-1) + 2i; 1 \leq j \leq k, 1 \leq i \leq m.$$

$$g(u_j) = 4km + 2j; 1 \leq j \leq k+1.$$

The edge weights are calculated as follows:

$$\text{For } 1 \leq j \leq k, 1 \leq i \leq m, \lambda(u_j v_j^i) = \frac{4km + 2j}{2m(j-1) + 2i}.$$

$$\text{Clearly } \lambda(u_j v_j^i) \neq \lambda(u_{j+1} v_j^i),$$

$$\text{if } \lambda(u_j v_j^i) = \lambda(u_{j+1} v_j^i) \text{ then } \frac{4km + 2j}{2m(j-1) + 2i} = \frac{4km + 2j + 2}{2m(j-1) + 2i}.$$

This implies $2 = 0$, which is a contradiction. Thus edge labels are distinct.

$$\text{For } 1 \leq j \leq k, 1 \leq i \leq m, \lambda(u_j w_j^i) = \frac{4km + 2j}{2m(k+j-1) + 2i}.$$

$$\text{Clearly } \lambda(u_j w_j^i) \neq \lambda(u_{j+1} w_j^i),$$

$$\text{if } \lambda(u_j v_j^i) = \lambda(u_{j+1} v_j^i) \text{ then } \frac{4km + 2j}{2m(k+j-1) + 2i} = \frac{4km + 2j + 2}{2m(k+j-1) + 2i}.$$

This implies $2 = 0$, which is a contradiction. Thus edge labels are distinct.

$$\text{For } 1 \leq j, l \leq k, \text{ clearly for } l \neq j, \lambda(u_j u_{j+1}) \neq \lambda(u_l u_{l+1}).$$

$$\text{If } \lambda(u_j u_{j+1}) = \lambda(u_l u_{l+1}) \text{ then } \frac{4km + 2j}{4km + 2j} = \frac{4km + 2l}{4km + 2l}. \text{ This implies } l = j, \text{ which is a contradiction. Thus edge labels are distinct. Therefore}$$

$2m\Delta_k$ -snake graph admits even ratio edge antimagic labeling.

Theorem 2.4: The kC_4 -snake graphs are odd and even ratio edge antimagic labeling.

Proof. Let $G(V, E)$ be a kC_4 -snake graph where $k \geq 1$. This graph has $3k+1$ vertices and $4k$ edges. Let

$$V(G) = \{w_i; 1 \leq i \leq k+1\} \cup \{u_i; 1 \leq i \leq k\} \cup \{v_i; 1 \leq i \leq k\},$$

$$E(G) = \{w_i v_i; 1 \leq i \leq k\} \cup \{w_i u_i; 1 \leq i \leq k\} \cup \{w_{i+1} v_i; 1 \leq i \leq k\} \cup$$

$$\{w_{i+1} u_i; 1 \leq i \leq k\}.$$

To prove that kC_4 -snake admits ratio edge antimagic labeling let us define,

$$f: V(G) \rightarrow \{1, 3, 5, \dots, 6k+1\} \text{ such that } f(v_i) = 2i-1; 1 \leq i \leq k,$$

$$f(u_i) = 2k+2i-1; 1 \leq i \leq k, \quad f(w_i) = 4k+2i-1; 1 \leq i \leq k+1.$$

The edge weights are calculated as follows:

$$\text{For } 1 \leq i \leq k, \lambda(w_i v_i) = \frac{4k+2i-1}{2i-1}.$$

For $1 \leq i, j \leq k$, clearly $\lambda(w_i v_i) \neq \lambda(w_j v_j)$ and $i \neq j$.

$$\text{If } \lambda(w_i v_i) = \lambda(w_j v_j), \text{ then } \frac{4k+2i-1}{2i-1} = \frac{4k+2j-1}{2j-1}.$$

This implies $i = j$, which is a contradiction.

$$\text{For } 1 \leq i, j \leq k, \lambda(w_i u_i) = \frac{4k+2i-1}{2k+2i-1}.$$

For $1 \leq i, j \leq k$, clearly $\lambda(w_i u_i) \neq \lambda(w_j u_j)$ and $i \neq j$.

$$\text{If } \lambda(w_i u_i) = \lambda(w_j u_j) \text{ then } \frac{4k+2i-1}{2k+2i-1} = \frac{4k+2j-1}{2k+2j-1}. \text{ This implies } i = j,$$

which is a contradiction.

$$\text{For } 1 \leq i, j \leq k, \lambda(w_{i+1} v_i) = \frac{4k+1+2i}{2i-1}.$$

For $1 \leq i, j \leq k$, clearly $\lambda(w_{i+1} v_i) \neq \lambda(w_{j+1} v_j)$ and $i \neq j$.

$$\text{If } \lambda(w_{i+1} v_i) = \lambda(w_{j+1} v_j) \text{ then } \frac{4k+1+2i}{2i-1} = \frac{4k+1+2j}{2j-1}.$$

This implies $i = j$, which is a contradiction.

$$\text{For } 1 \leq i, j \leq k, \lambda(w_{i+1} u_i) = \frac{4k+2i+1}{2k+2i-1}.$$

For $1 \leq i, j \leq k$, clearly $\lambda(w_{i+1} u_i) \neq \lambda(w_{j+1} u_j)$ and $i \neq j$.

$$\text{If } \lambda(w_{i+1} u_i) = \lambda(w_{j+1} u_j) \text{ then } \frac{4k+2i+1}{2k+2i-1} = \frac{4k+2j+1}{2k+2j-1}.$$

This implies $i = j$, which is a contradiction. Thus all edge labels are distinct

For $1 \leq i, j \leq k$, clearly $\lambda(w_i v_i) \neq \lambda(w_{i+1} v_i)$ and $i \neq j$.

$$\text{If } \lambda(w_i v_i) = \lambda(w_{i+1} v_i) \text{ then } \frac{4k+2i-1}{2i-1} = \frac{4k+2i+1}{2i-1} \text{ implies } 2 = 0, \text{ which is a contradiction.}$$

$$\text{Also } \lambda(w_i u_i) \neq \lambda(w_{i+1} u_i), \text{ if } \lambda(w_i u_i) = \lambda(w_{i+1} u_i) \text{ then } \frac{4k+2i-1}{2k+2i-1} = \frac{4k+2i+1}{2k+2i-1} \text{ implies } 2 = 0, \text{ which leads to contradiction.}$$

Thus all edge labels are distinct. Therefore kC_4 -snake graph admits odd ratio edge antimagic labeling.

To prove the existence of even ratio edge antimagic labeling, let us define

$$g: V(G) \rightarrow \{2, 4, 6, \dots, 6k+2\}, \text{ such that}$$

$$g(v_i) = 2i; 1 \leq i \leq k,$$

$$g(u_i) = 2k+2i; 1 \leq i \leq k,$$

$$g(w_i) = 4k + 2i; 1 \leq i \leq k+1.$$

The edge weights are calculated as follows:

$$\text{For } 1 \leq i \leq k, \lambda(w_i v_i) = \frac{4k + 2i}{2i}.$$

For $1 \leq i, j \leq k$, clearly $\lambda(w_i v_i) \neq \lambda(w_j v_j)$ and $i \neq j$.

$$\text{If } \lambda(w_i v_i) = \lambda(w_j v_j), \text{ then } \frac{4k + 2i}{2i} = \frac{4k + 2j}{2j}. \text{ This implies } i = j,$$

which is a contradiction. Thus all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(w_i v_{i+1}) = \frac{4k + 2i}{2i + 2}.$$

$$\text{For } 1 \leq i, j \leq k, \text{ clearly } \lambda(w_i v_{i+1}) \neq \lambda(w_j v_{j+1}). \text{ If } \lambda(w_i v_{i+1}) = \lambda(w_j v_{j+1}) \text{ then } \frac{4k + 2i}{2i + 2} = \frac{4k + 2j}{2j + 2}. \text{ This implies } i = j, \text{ which is a}$$

contradiction. Thus all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(w_i u_i) = \frac{4k + 2i}{2k + 2i}.$$

For $1 \leq i, j \leq k$, clearly $\lambda(w_i u_i) \neq \lambda(w_j u_j)$ and $i \neq j$.

$$\text{If } \lambda(w_i u_i) = \lambda(w_j u_j) \text{ then } \frac{4k + 2i}{2k + 2i} = \frac{4k + 2j}{2k + 2j}.$$

This implies $i = j$, which is a contradiction.

Thus all edge labels are distinct.

$$\text{For } 1 \leq i \leq k, \lambda(w_i u_{j+1}) = \frac{4k + 2i}{2k + 2i + 2}. \text{ Clearly } \lambda(w_i v_i) \neq \lambda(w_{i+1} v_i), \text{ if } \lambda(w_i v_i) = \lambda(w_{i+1} v_i) \text{ then } \frac{4k + 2i}{2i} = \frac{4k + 2i + 2}{2i} \text{ implies } 2 = 0,$$

which leads to a contradiction. Thus edge labels are distinct.

$$\text{Clearly } \lambda(w_i u_i) \neq \lambda(w_{i+1} u_i), \text{ if } \lambda(w_i u_i) = \lambda(w_{i+1} u_i) \text{ then } \frac{4k + 2i}{2k + 2i} = \frac{4k + 2i + 2}{2k + 2i} \text{ implies } 2 = 0, \text{ which leads to a contradiction. Thus edge}$$

labels are distinct. Therefore kC_4 -snake graph admits even ratio antimagic labeling.

Conclusion

Thus existence of odd and even ratio edge antimagic labeling, for some class of graph is proved.

References

- [1] M.E. Abdel-Aal, Odd Harmonic labeling of cyclic snakes, International Journal on Applications of Graph Theory in wireless Ad hoc Networks and Sensor Networks (GRAPH-HOC), Vol. 5, No. 3, September 2013, pp. 1–11.
- [2] E.M. Badr and M.E. Abdel-Aal, Odd and graceful labeling for the subdivision of double triangles graphs, International Journal of Soft Computing, Mathematics and Control (IJSCMC), Vol. 2, No. 1, February 2013, pp. 1–8.
- [3] J.A. Gallian, A dynamic survey of graph labeling, Electr. J. Combin., Vol. 19, 2012.
- [4] J. Jayapriya, D. Muruganandam, K. Thirusangu, Analyzing welding process using graph labeling techniques, European Journal of Scientific Research, Vol. 79, No. 2 (2012), pp. 253–257.
- [5] J. Jayapriya, K. Thirusangu, Max-min edge magic and antimagic labeling, European Scientific Journal, Vol. 3, No. 3 (2013), pp. 253–259.
- [6] J. Jayapriya and K. Thirusangu, Odd and even ratio edge antimagic labeling, Journal of Advances in Mathematics, Vol. 3, No. 2, pp. 178–183, 2013.
- [7] A. Rosa, Cyclic Steiner triple systems and labeling of triangular cacti, Scientia, Vol. 5, 1967, pp. 87–95.