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Weibel Instability in Plasma Low Temperature

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ABSTRACT

A Numerical scheme for the Weibel instability of low temperature has been developed which is a modification of the Darwin model. The Darwin model neglects the ion contribution and the theoretical model used considers homogeneous plasma in the presence of a wave low amplitude laser in the dipole approximation using the formalism of kinetic theory. In this study the coupling of the magnetic field generated by self-instability with the laser-wave field is taken into account, described by the Fokker-Planck equation. This equation is solved analytically zero order and disrupted order and distribution functions electrons were explicitly calculated. The dispersion relation of modes Weibel, who holds account of the term of coupling the quasi-static magnetic field with the high frequency field the laser wave, has been established More specifically, typical physical parameters in the experiments melting by laser; it has been revealed a reduction in the spectral range of the unstable modes and a reduction of two orders of magnitude of the rate instabilities.

I-Introduction

The Weibel instability [1] is a micro-convective instability. It corresponds to the excitation of electromagnetic modes in plasma characterized by anisotropy in temperature whichcorresponds toplasma described by an anisotropic velocity distribution function. This anisotropy can be generated by several mechanisms, namely: the heat transport [2], the expansion of the plasma [3] and the inverse bremsstrahlung absorption [4].

The presence of strong magnetic fields in a mega-gauss Range in laser irradiated targets could be detrimental to the process of ablative implosion, necessary for achieving thermonuclear fusion reactions. Indeed, several effects could be induced by these fields, such as the anomalous reduction of the electron heat flux from the laser energy deposition layer to the ablation surface, reduction of the mass ablation rate filamentation of the plasma flow, etc. Various mechanisms responsible for producing such B fields have been reported in the literature: thermoelectric effects [5], nonlinear effects [6], Rayleigh-Taylor [7] and electromagnetic instabilities [1], etc.

In this paper, we deal with the Weibel instability due to the inverse bremsstrahlung absorption in homogeneous plasmas and laser-produced plasmas. It has been shown by Weibel.That anisotropic distribution functions in the velocity space may drive unstable electromagnetic modes. For a symmetrical angular distribution function about the x axis, $f(v,v,x)$ a positive second anisotropic distribution function $T_x > T_\perp$ drives unstable $k_\perp$ modes, whereas a negative second anisotropic distribution function $T_x < T_\perp$ drives unstable k_x modes [8].

The present work aims to the investigation of the Weibel instability induced by inverse bremsstrahlung absorption in the laser fusion plasma. This requires the investigation of the dispersion relation for low-frequency electromagnetic modes in plasma heated by a laser pulse.

The spatio-temporal dependence of the high-frequency laser pulse is supposed as a normal mode. It results high light new terms in the dispersion relation which can contribute to the Weibel modes instability. We will keep all terms and study their role in the study of the growth rate of the Weibel instability.

II-Basic Equation

Throughout this work we use the Fokker-Planck (FP) equation which describes, in particular, the thermal transport and the light energy absorption in fully ionized plasmas. Let us now compute the electron distribution function of an unmagnetized plasma in the presence of an oscillating electric field. As we aim at obtaining in this section the IBA contribution to the Weibel source, we consider for simplicity, homogeneous plasmas in order to avoid the Weibel sources due to the thermal transport and the plasma expansion [3]. Following theBraginski[9] notations, the FP equation for the electrons reads:

$$\frac{\partial f}{\partial t}+\vec{v}.\frac{\partial f}{\partial \vec{r}}-\frac{e}{m_e}\left(\vec{E}+\vec{v}\times\vec{B}\right)\frac{\partial f}{\partial \vec{v}}=C_{ei}(f,f)+C_{ee}(f) \quad (1)$$

where f is the distribution functions of electrons,$m_e$is the electron mass, eis the elementary charge, $\vec{E}$ and $\vec{B}$ are respectively the electric and the magnetic fields present in the plasma, $C_{ee}(f)$ and $C_{ei}(f)$ are respectively of electron-electron collision and electron-ion operators [10].

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In order to take into account the high-frequency (HF) response of plasma electrons to the laser field excitation, we split up the distribution function into a low-frequency (SF).

The rating on the indices «s» and «h» refers to secular time scales low frequency and high frequency respectively and will be used throughout this work.

The separation of time scales in equation (1) leads to the following high-frequency and low-frequency kinetic equations:

$$\frac{\partial f_h}{\partial t} - \frac{e}{m_e}\vec{E}_h.\frac{\partial f_s}{\partial \vec{v}} = \frac{e}{m_e}\left(\vec{E}_s + \vec{v}\times\vec{B}_s\right).\frac{\partial f_h}{\partial \vec{v}} + C_{ei}(f_h) \tag{2}$$

$$\frac{\partial f_s}{\partial t} + \vec{v}.\frac{\partial f_s}{\partial \vec{r}} - \frac{e}{m_e}\left(\vec{E}_s + \vec{v}\times\vec{B}_s\right).\frac{\partial f_s}{\partial \vec{v}} - C_{ei}(f_s) = \langle\frac{e}{m_e}\vec{E}_h.\frac{\partial f_h}{\partial \vec{v}}\rangle \tag{3}$$

The symbol <> denotes the average over the laser wave cycle time, $T = 2\pi/\omega$.These equations make a system of two coupled equations. Note here that the terms in the electric field $\vec{E}_s$ and magnetic field $\vec{B}_s$ reflect the inclusion of the, low frequency electromagnetic field effect in our study. In particular, the first term on the right hand side of the equation (2) reflects the coupling of quasi-static fields with the laser field. Let us recall here that this field present in the plasma is generated by the mechanism of the Weibel instability. These terms do not appear in the work reported in reference [3].

II-1-Resolution of the high Frequency Fokker- Planck equation

We consider the high frequency approximation, where the laser wave frequency ω_l is greater than the collisions frequency ν_{ei}

This approximation is largely justified in the laser fusion plasma experiments. For example of typical Parameters :(electron temperature; overage free scaling and laser wave length respectively)

$T_e = 1KeV, \lambda_{ei} = 1\mu m\ and\ \lambda_L = 1.06\mu m$; We have $\omega_l/\nu_{ei} \gg 1.$

In this case, the high frequency electronic distribution function is calculated as follow:

$$f_h(\vec{v}) = -\frac{e}{m_e}\frac{i}{\omega_l}E_{hJ}.\frac{\partial f_s}{\partial \vec{v}} - \frac{e^2}{m_e^2}\left(\vec{E}_s + \vec{v}\times\vec{B}_s\right)\frac{\partial}{\partial v_i}\left[E_{hJ}.\frac{\partial f_s}{\partial v_j}\right] + \frac{e}{m_e}\frac{\nu_{ei}}{\omega_l^2}E_{hJ}.\frac{\partial f_s}{\partial v_j} \tag{4}$$

where we used the Einstein notation means that the repeated indices represent the summation over the indices The expression (4) represents the high-frequency component of the electronic distribution function which depends on its low-frequency component f_s .

II-2-Resolution of the low frequency Fokker-Planck equation

We will now solve the low frequency (Fokker-Planck 3) using the expression of the distribution function of high frequency f_h established in the previous paragraph. Substituting (4) in (3), we obtain then after some algebra:

$$\frac{\partial f_s}{\partial t} + \vec{v}.\frac{\partial f_s}{\partial \vec{r}} - \frac{e}{m_e}\left(\vec{E}_s + \vec{v}\times\vec{B}_s\right).\frac{\partial f_s}{\partial \vec{v}} + \nu_{ei}(v)[f_S] = S_{BI} \tag{5}$$

where

$$S_{BI} = -\frac{1}{2}\frac{e^2}{m^2\omega_0}E_{0l}E_{0j}\frac{\partial}{\partial v_l}\left[\nu_{ei}(v)\frac{\partial f_s}{\partial v_j}\right] - \frac{1}{2}\frac{e}{m}\frac{e^2}{m^2\omega_0^2}E_{0l}E_{0j}E_{si}\frac{\partial}{\partial v_l}\left[\frac{\partial^2 f_s}{\partial v_i \partial v_j}\right] - \frac{1}{2}\frac{e}{m}\frac{e^2}{m^2\omega_0^2}E_{0l}E_{0j}\frac{\partial}{\partial v_l}\left[\left(\vec{v}\times\vec{B}_s\right)_i\frac{\partial^2 f_s}{\partial v_i \partial v_j}\right] \tag{6}$$

The next step of our calculation is to linearize the equation (5) by setting

$$f_s = f_s^{(0)} + f_s^{(1)} \tag{7}$$

The evolution of the second anisotropy equation $f_{s2}^{(0)} > 0$ is obtained by projecting the equation (5) on the Legendre polynomial $P_l(\mu)$ [11].

$$f_{s2}^{(0)} = \frac{1}{3\sqrt{5}}\left(\frac{v_0}{v_T}\right)^2\left(3 + \frac{v^2}{v_T^2}\right)f_M \tag{8}$$

This result shows that the second anisotropy is positive $f_{s2}^{(0)} > 0$, which corresponds to a temperature anisotropy where $T_x > T_\perp$ and $T_\perp$denote the temperature in the direction ox and the direction perpendicular to ox. Indeed, the plasma is heated preferentially along the laser wave field direction: ox.

III-Analysis of the Weibel instability

This paragraph is devoted to the analysis of the Weibel instability. We determine the dispersion relation of the Weibel modes and deduce the growth rateof Weibel instability. For this it is necessary to calculate the perturbed distribution function due to an electromagnetic disturbance.

III-1-Calculation of the perturbed distribution function.

The evolution equation of the perturbed function is obtained from low frequency equationFokker_Planck (3) by considering the first term order, so:

$$\frac{\partial f_s^{(1)}}{\partial t} + \vec{v}.\frac{\partial f_s^{(1)}}{\partial \vec{r}} - \frac{e}{m_e}\left(\vec{E}_s + \vec{v}\times\vec{B}_s\right).\frac{\partial f_s^{(0)}}{\partial v} + \nu_{ei}(v)\left[f_s^{(1)}\right] = I_{BI}\left(f_s^{(1)}\right) + I_{BI}\left(f_s^{(0)}\right) \tag{9}$$

where

$$I_{BI}\left(f_s^{(1)}\right) = \frac{1}{2}v_0^2\frac{\partial}{\partial v_x}\left[\nu_{ei}(v)\frac{\partial f_s^{(1)}}{\partial v}\right]$$

After some manipulation and mathematical equation (11) becomes

$$\nu_{ei}(v)f_{sl.m}^{(1)} + ikv\left[\frac{l^2-m^2}{4l^2-1}\right]f_{sl-1.m}^{(1)} + ikv\left[\frac{(l+1)^2-m}{4(l+1)^2-1}\right]f_{sl+1.m}^{(1)} = S_{El,m} + S_{Bl,m} \tag{10}$$

Note also, for the right-hand side of equation (10) is zero. The result is a recurrence relation between the components:

$$\nu_{ei}(v)f_{sl.m}^{(1)} = -ikv^4\left[\frac{l^2-m^2}{4l^2-1}\right]f_{sl-1.m}^{(1)} - ikv^4\left[\frac{(l+1)^2-m^2}{4(l+1)^2-1}\right]f_{sl+1.m}^{(1)}\ pour\ l \geq 3 \tag{11}$$

We will now solve the system of equations (9) using mathematical techniques based on continued fractions. Solving kinetic equations with continued fractions [11] was used for the first time in [8], where the collisional propagator in the Fokker-Planck equation has been explicitly reversed on the basis of spherical harmonics. By applying these results to equation (13), and after some mathematical manipulations, we obtain:

$$f_{s3.m}^{(1)} = -ikv^4\left[\frac{9-m^2}{35}\right]^{\frac{1}{2}} F_{3,m} f_{s2.m}^{(1)} \quad (12)$$

Where $F_{3,m}$ is the continued fraction defined the following recurrence relation:

$$F_{l,m}(k,v) = \left[v_{ei}(v) + k^2 v^8 \frac{(l+1)^2 - m^2}{4(l+1)^2 - 1} F_{l+1,m}\right]^{-1} \quad (13)$$

We must note here that equation (12) is the exact solution of the infinite hierarchy of equations (11). It gives a relationship between components $f_{s3.m}^{(1)}$ and $f_{s2.m}^{(1)}$ that includes the contribution of all components $f_{sl.m}^{(1)}$ with $l > 3$. The explicit expressions of the components $f_{sl\pm1}^{(1)}$ are then obtained as:

$$f_{sl\pm1}^{(1)} = \pm 4\sqrt{\frac{2\pi}{3}}\frac{e}{m_e} E_s v_t^2 y^2 F_{1,1} f_M \mp i\frac{e}{m_e}\frac{\sqrt{5}}{Kv_t}\sqrt{\frac{\pi}{15}} B_s \frac{v_0^2}{v_t^2}\left(1 - vF_{1,1}\right)(3+y)y^{-\frac{1}{2}} f_M$$

(14)

III-2-Determination of the dispersion relation.

The calculation of the dispersion relation of electromagnetic modes Weibel in the semi-collisional approximation [12.13] can be calculated using the perturbed Fokker-Planck equation coupled with Maxwell's equations presented as follows:

$$\vec{\nabla} \times \overrightarrow{E_s} = -\frac{\partial \vec{B}_s}{\partial t} \quad (15)$$

and

$$\vec{\nabla} \times \overrightarrow{B_s} = -\mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}_s}{\partial t} \quad (16)$$

where $\vec{j}$ is the current density defined by

$$\vec{J} = -e\int \vec{v} f_s^{(1)}\, d\vec{v} \quad (17)$$

By considering that thespatio-temporal dependence of the field $\vec{E}_s$ and $\overrightarrow{B_s}$ as a Fourier mode:$\sim\mathbf{exp}(i\omega t + kz)$, equations (16) and (17) can be represented as:

$$kE_s = \omega B_s \quad (18)$$

$$kB_s = -ie\mu_0 \int v_x f_s^{(1)}\, d\vec{v} \quad (19)$$

By developing the function $f_s^{(1)}$, on the spherical harmonicsbasis Y_l^m, the equation (19) reads as:

$$kB_s = -ie\mu_0\sqrt{\frac{2\pi}{3}}\int_0^\infty v^3\left(f_{s1,-1}^{(1)} - f_{s1,1}^{(1)}\right)dv \quad (20)$$

We deduce then the dispersion relation of the Weibel modes as:

$$\frac{k^2c^2}{\omega_p^2} = i\frac{8\sqrt{2}}{3\sqrt{\pi}}\omega v_t^3\int_0^\infty y^3 F_{1,1}\exp(-y)\,dy + \frac{2}{3\sqrt{\pi}}\frac{v_0^2}{v_t^2}\int_0^\infty y^{1/2}(3+y)\left(1 - vF_{1,1}\right)ep(-y)\,d\,y$$

(21)

Here, $\omega = \omega_r + i\gamma$, is the frequency of Weibel mode and γ being the growth rate which is derived in the linear approximation from equation (21) as:

$$\gamma(k) = -\frac{k^2c^2}{\omega_p^2}\frac{1}{v}\frac{1}{\int_0^\infty y^3 F_{1,1}(k\lambda_{ei},y)\exp(-y)dy} + \frac{v_0^2}{v_t^2}\frac{1}{v_t^3}\frac{\int_0^\infty y^{\frac{1}{2}}(3+y)\left(1-vF_{1,1}\right)ep(-y)dy}{\int_0^\infty y^3 F_{1,1}(k\lambda_{ei},y)\exp(-y)dy} \quad (22)$$

IV-Discussion

The first term of equation (22) is a loss termwhich. It corresponds to a loss term due to collisions between particles in the collisional limit ($k\lambda_{ei} \ll 1$) while in the non-collisional limit($k\lambda_{ei} \gg 1$) , it describes the Landau damping of electromagnetic modes. The second term $\sim v_0$, corresponds to the Weibel instability source. This Equation gives explicitly the growth rate of the Weibel instabilityexcited by inverse bremsstrahlung absorption in laser fusion plasmalow temperature. This expression contains continuous fractions which are numerically calculated [5]. We present on Figures 1, the spectra of thegrowth rate of Weibel unstable mode for typical parameters of plasma and laser.

We point out that the profile of the spectrum $\gamma(k)$ present a maximum. This can be interpreted by the competition between the loss effects (collisions and Landau damping) and the inverse bremsstrahlung Weibel instability growing.

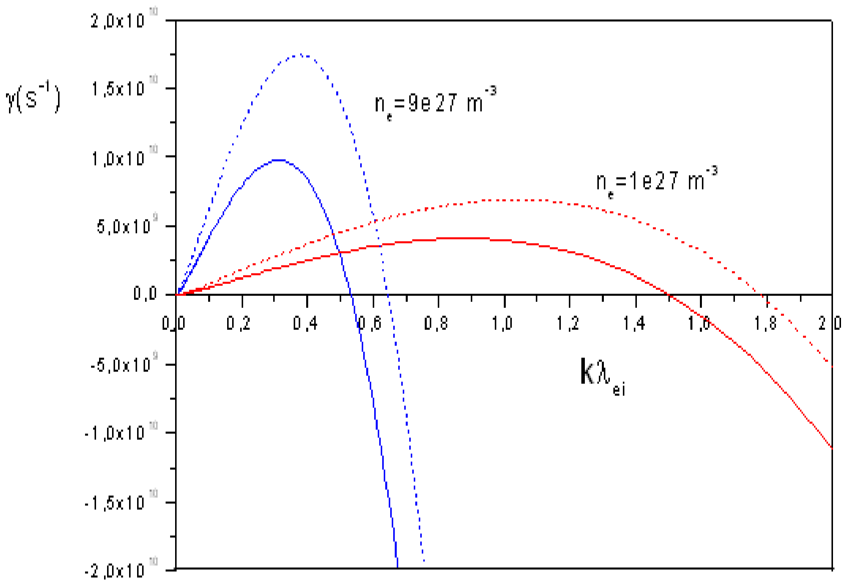

***Figure1**: Growth rate of Weibel instability γ as function of the collision parameter $k\lambda_{ei}$ for typical physical parameters. Where the dotted line corresponds to the case without $I_{BI}(f_{s0}^{(0)})$ term.*

$$\boldsymbol{straight\ solid\ red}: \boldsymbol{n_e} = \mathbf{10^{27}}\boldsymbol{m^{-3}}, \boldsymbol{T_e} = \mathbf{1}\boldsymbol{KeV}, \frac{\boldsymbol{v_0}}{\boldsymbol{v_t}} = \mathbf{0.1}\ and\ \boldsymbol{Z} = \mathbf{4}$$
$$\boldsymbol{straight\ dash\ red}: \boldsymbol{n_e} = \mathbf{10^{27}}\boldsymbol{m^{-3}}, \boldsymbol{T_e} = \mathbf{2}\boldsymbol{KeV}, \frac{\boldsymbol{v_0}}{\boldsymbol{v_t}} = \mathbf{0.2}\ and\ \boldsymbol{Z} = \mathbf{4}$$
$$\boldsymbol{straight\ solid\ blue}: \boldsymbol{n_e} = \mathbf{9.10^{27}}\boldsymbol{m^{-3}}, \boldsymbol{T_e} = \mathbf{2}\boldsymbol{KeV}, \frac{\boldsymbol{v_0}}{\boldsymbol{v_t}} = \mathbf{0.1}\ and\ \boldsymbol{Z} = \mathbf{4}.$$
$$\boldsymbol{straight\ dash\ bleu}: \boldsymbol{n_e} = \mathbf{9.10^{27}}\boldsymbol{m^{-3}}, \boldsymbol{T_e} = \mathbf{2}\boldsymbol{KeV}, \frac{\boldsymbol{v_0}}{\boldsymbol{v_t}} = \mathbf{0.2}\ and\ \boldsymbol{Z} = \mathbf{4}.$$

V-Conclusion

In this work we have presented atheoretical investigation and a numerical analysis (Figures 1) of the Weibel instability in the laser fusion plasma low temperature using the Fokker-Plank model. The effect due to generated static magnetic field is computed. It has been shows that by taking into account the coupling of the self-generated magnetic field by theWeibel instability with the laser wave field leads to a significant decreasing in the spectral range of unstable Weibel modes. Also this coupling undergoes a reduction in the growth rate values of Weibel instable modes. However, the values of the growth rate remain in the order of $\mathbf{10^{10}}\boldsymbol{s^{-1}}$. It is found that this reduction is independent of the values of the electron density andtemperature . We believe that this is due to the choice of thecollisions operator where the collision frequency is considered constant. This approximation is not realistic since it is known that collisions role is to reduce the anisotropy of the plasma by letting it to a steady state described by an isotropic distribution function. Therefore, collisions are more important over the anisotropies of the plasma are low.

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