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Design of coding schemes with low peak to average power ratio for OFDM systems

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ABSTRACT
A coding scheme for OFDM transmission is proposed, exploiting a connection between pairs of Golay complementary sequences and second-order Reed-Muller codes. Using a realization of 16-QAM or 64 QAM constellation as the vector sum or two or three QPSK constellation respectively we construct 16 QAM and 64 QAM sequences having low PAPR. In this paper, we further examine the squared Euclidean distance of these *M*-QAM sequences and their variations. Our aim here is to combine the block coded modulation (BCM) and Golay complementary sequences to trade off the PAPR, the code rate, and the squared Euclidean distance of *M*-QAM OFDM signals

Introduction

Today's needs of bandwidth and flexibility are imposing the use of efficient modulations that may be fit to the characteristics of wireless channels. This is one of the reasons why multicarrier modulation techniques are finding growing interest for Wireless Local Area Networks (WLAN). Recent WLAN standards, such as Hiperlan type 2 and IEEE 802.11a [1], have adopted them for transmission of high bit rates in these networks. The choice of OFDM (Orthogonal Frequency Division Multiplexing) is due to its good performance in multipath environments and the number of sub-carriers has been chosen so as to mitigate the indoor channel effects. Basically, OFDM divides bandwidth into several orthogonal sub-carriers and sends information into these sub-carriers. Nevertheless, one of the main disadvantages of this modulation is its high PAR (Peak-to-Average power Ratio) requiring the use of linear HPAs (High Power Amplifiers) that are very power-inefficient and have an enormous impact on equipment's autonomy. Since OFDM is a good technique to mitigate multipath effects, it is interesting to try to improve its disadvantage and reduce the high PAR of this kind of signals. There are a lot of techniques for this purpouse like Complementary Golay Sequences [2], Partial Transmit Sequences (PTS) [3], Selective Mapping (SLM)[3], Tone Reservation [4], Clustered Transmission [5] and Orthogonal Pilot Sequences (OPS) [6]. Golay Sequences have been chosen for two reasons.

First, with this technique, PAR is limited up to 3 dB, independently of the number of carriers and input data. This is very important, because we know in advance how is the dynamic range for HPA. Second is the error correction capability of these codes that allow us to improve the whole system.

Construction of 16 QAM and 64 QAM OFDM codes with low papr and large euclidean distance

In general, to construct a length n BCM modulation code over a QPSK constellation, we choose two length n binary codes B_1, as the first level component code, and B_2 as the second level component code, with proper minimum Hamming distance d_1 and d_2 respectively. Since the minimum Hamming distance of a coset of a code is equal to the minimum distance of the code itself, i.e., d(B_2) = d $(e + B_2)$ for any e, we can also choose an arbitrary coset of B_2 as the second level component code for each codeword in B_1. In other words, if B_1 = (x_1, x_2 .... x_M) consisting M code words is the first level component code, then it is possible to use the following M cosets of B_2 {t_1+B_2, t_2+B_2,..., t_M+B_2} as the second level component codes for each codeword $x_1 \in B_1$. The

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overall minimum squared Euclidean distance of this QPSK BCM code consisting of B_1 and $\{t_1+B_2, t_2+B_2, \ldots, t_M+B_2\}$ is equal to D_E^2 = min {1.d($B_1 1$) , 2.d($B_1 2$) }.E^2 .

We will denote the associated component-codes of this QPSK code C by B_1 and B_2 instead of B_1 and t_v+B_2 .

To construct a 16-QAM OFDM code using block coded modulation scheme with low PAPR, we take two low PAPR QPSK codes C_1 and C_2 by

$$C_1 = \bigcup_{i=1}^{b_1} \{2x_1(i) \oplus 4uG_1 : u_i \in Z_4\}$$

$$C_2 = \bigcup_{i=1}^{b_2} \{2y_1(i) \oplus 4uG_3 : u_i \in Z_4\}$$

The two associated binary codes of C_1 are

B_1 = <G1> and $B_2 = \bigcup_{i=1}^{b_1} ((x_i) + B_1)$

and the two associated binary codes of C_2 are

$B_3 = \langle G_3 \rangle$ and $B_4 = C_1 = \bigcup_{i=1}^{b_2} ((y_i) + B_3)$.

To keep the resulting 16-QAM BCM with high code rate and large Euclidean distance., we might choose these four binary component codes with the property of d($B_1 1$) = 2d($B_1 2$) = 4d($B_1 3$) =8d(B_4)

16-QAM and 64 QAM OFDM Sequences

Let C= $2Q_\pi + RM_4$(1,m), consisting of $\frac{m!}{2}4^{m+1}$ Golay Sequence of length 2^m over Z_4 where $Q_{\pi} = x_{\pi(1)}x_{\pi(2)} + \cdots + x_{\pi(m-1)}x_{\pi(m)}$. The set C can be decomposed into two binary codes $B_1 = RM_2$ (1,m) and $B_2 = Q_\pi + RM_2$ (1,m) with the property that PAPR(C)=2 and d(B1)=2d(B2)=2^{m-1}. We will use the set of these 4-ary Golay Complementary sequences and its modification as the building blocks for the construction of 16- QAM and 64 –QAM OFDM sequences with low PAPR and large squared Euclidean distance. Here Q_π denotes all $\frac{m!}{2}$ second order Boolean polynomials of the form $x_{\pi(1)}x_{\pi(2)}+\ldots+x_{\pi(m-1)}x_{\pi(m)}$ and Q=$x_1x_2 + \ldots + x_{m-1}x_m$ as one of them.

16 –QAM OFDM Codes From Four BPSK Sequences

Let two QPSK component codes C_1 and C_2 be $2Q_\pi + RM_4$(1,m), then the resulting 16-QAM code Z_1 can be constructed from four BPSK component codes B_1, B_2, B_3 and B_4 where C_1 and C_2 respectively is associated with its component codes B_1 and B_2, B_3 and B_4 respectively. Since $B_1 = B_3 = RM_2$(1,m) and $B_2 = B_4 = Q_\pi + RM_2$(1,m) $\in RM_2$(2,m), the size of 16 QAM code Z_1 of length n=2^m is $\left(\frac{m!}{2}\right)^2 ((4)^{m+1})^2$ and the squared Euclidean Distance is $D_E^2(Z_1)$ = min (1. 2^{m-1},2.2^{m-2},4.2^{m-1}, 8.2^{m-2})= 2^{m-1}. It has been shown that the average power of C is 2.5n and the PAPR of Z_1 is bounded above by ($\frac{1}{\sqrt{2}}\sqrt{2n} + \sqrt{2}\sqrt{2n}$)2/2.5 n= 3.6, shown in the first code of Table 1. This is the 16-QAM OFDM code constructed .[9] We can trade off the code rate, the PAPR, and the squared Euclidean distance of a 16-QAM code by proper choice of QPSK codes C_1 and C_2, where the code rate of a code C of length n is defined as R(C)=$\log_2 \frac{|C|}{n}$ bits/symbol . We then use the modifications of 4ary- Golay sequences satisfying these criteria as the component codes to keep the resulting 16-QAM OFDM codes with low PAPR. For Example, let $B_1 = RM_2$(0,m) and B_2 = ($x_1x_2 + x_2x_3 + \ldots + x_{m-1}x_m$)+ RM_2(1,m) be one specific coset consisting of 2^{m+1} binary golay sequences and C_2 =

$2Q_\pi + RM_4$ (1,m); then the resulting 16-QAM code Z_2 has PAPR($Z_\downarrow 2) = 3.6$ $D_E^2(Z_2)$ = min (1. 2^m ,2.2^{m-1} ,4.2^{m-1} , 8.2^{m-2})= 2^m , and $|Z_2|$ = 2.$2^{m+1} \frac{m!}{2} 4^{m+1}$, shown in the second code of Table1.

Table 1. 16 QAM OFDM codes with PAPR=3.6

	Z_1
B_1	RM(1,m)
B_2	Q_π +RM (1,m)
B_3	RM(1,m)
B_4	Q_π +RM (1,m)
size	$\left(\frac{m!}{2}\right)^2 4^{2(m+1)}$
PAPR	≤3.6
D_E^2	2^{m-1}

	Z_2
B_1	RM(0,m)
B_2	Q+RM(1,m)
B_3	RM(1,m)
B_4	Q_π +RM (1,m)
size	$\frac{m!}{2} 2^{m+2} 4^{m+1}$
PAPR	≤3.6
D_E^2	2^m

If we choose $B_1 = RM_2$ (0,m), $B_2 = Q + RM_2$ (1,m) and $C_2 = \square(2 \oplus \square_\downarrow \pi + RM_4$ (1,m-1)) $\square(2Q\square_\downarrow \pi + RM_4$ (1,m-1)), the resulting 16 –QAM OFDM code Z_3 is a length 2^m code of size$|Z_3|$=2.$2^{m+1}\left(\frac{(m-1)!}{2}\right)^2 [\![(4]\!]^m)^2$ and $D_\downarrow E^\uparrow 2\ (Z_\downarrow 3$)=min(1.2^m , 2.2^{m-1} , 4.2^{m-2} , 8.2^{m-3})=2^m .

The PAPR of Z_3 is bounded above by ($\frac{1}{\sqrt{2}}\sqrt{2n}$ + $\sqrt{2}\sqrt{4n}$)/5n=5.86 shown in the first code of Table 2.

Table 2. 16 QAM OFDM codes with PAPR=5.86

	Z_3
B_1	RM(0,m)
B_2	Q+RM(1,m)
B_3	RM (1,m-1) RM(1,m-1) $\oplus$
B_4	$\square(Q\square_\downarrow \pi$ +RM(1,m-1)) $\oplus$ $\square(Q\square_\downarrow \pi$ +RM(1,m-1))
size	$\left(\frac{(m-1)!}{2}\right)^2 (2)^{(m+2)} (4^m)^2$
PAPR	≤5.86
D_E^2	2^m

	Z_4
B_1	RM(1,m)
B_2	Q_π +RM(1,m)
B_3	RM (1,m-1) RM(1,m-1) $\oplus$
B_4	$\square(Q\square_\downarrow \pi$ +RM(1,m-1)) $\oplus$ $\square(Q\square_\downarrow \pi$ +RM(1,m-1))
Size	$(\frac{m}{2})!$ $\left(\frac{(m-1)!}{2}\right)^2$ $(4)^{m+1} [\![(4]\!]^m)^2$
PAPR	≤5.86
	2^{m-1}

We then take C_2 = ($2Q_\pi + RM_4$ (1, m-1)$\oplus$ ($2Q_\pi + RM_4$ (1, m-1)) with PAPR (C_2) ≤ 4 for the first 16-QAM OFDM code Z_4 and C_2 = ($2Q_\pi + RM_4$ (1, m-2)) $\oplus$ ($2Q_\pi + RM_4$ (1, m-2)) with

PAPR (C_2) ≤ 8 for the second 16-QAM OFDM code Z_5 . The PAPR of Z_5 is bounded by $\frac{1}{\sqrt{2}}\sqrt{2n}$ + $\sqrt{2}\sqrt{8n}$)/2.5n=10 shown in table 3.

Table 3. 16 QAM OFDM codes with PAPR=10

	Z_5
B_1	RM(1,m)
B_2	Q_π + RM (1,m)
B_3	RM (1,m-2) $\oplus$ RM(1,m-2) $\oplus$ RM (1,m-2) $\oplus$ RM(1,m-2) $\oplus$
B_4	$\square(Q\square_\downarrow\pi$ +RM(1,m-2)) $\oplus$ $\square(Q\square_\downarrow\pi$ +RM(1,m-2)) $\oplus$ $\square(Q\square_\downarrow\pi$ +RM(1,m-2)) $\square(Q\square_\downarrow\pi$ +RM(1,m-2))
size	$\frac{m}{2}!\left(\frac{(m-2)!}{2}\right)^4 (4)^{m+1}((4)^{m-1})^4$
PAPR	≤10
D_E^2	2^{m-1}

We have constructed 16 QAM codes Z2 – Z5 by a proper selection of BPSK component subcodes. Now we compare the PAPR, squared Euclidean distance, code rate of the constructed codes for different lengths n.

Table 4. Comparison of 16-QAM OFDM codes

Code		C_c	Z_1	Z_2	Z_3	Z_4	Z_5
PAPR		2.8	3.6	3.6	5.86	5.86	10
n=16	D_E^2	8	8	16	16	8	8
	R	1.218	1.698	1.224	1.573	2.047	2.350
n=32	D_E^2	16	16	32	32	16	16
	R	0.752	1.119	0.778	1.068	1.409	1.758
n=64	D_E^2	32	32	64	64	32	32
	R	0.451	0.703	0.476	0.685	0.911	1.201
n=128	D_E^2	64	64	128	128	64	64
	R	0.265	0.426	0.284	0.422	0.565	0.773
n=256	D_E^2	128	128	256	256	128	128
	R	0.153	0.252	0.166	0.252	0.340	0.478

64-QAM OFDM codes from QPSK sequences

The 64-QAM OFDM constellation symbols, denote by S_{64QAM} can be written as the sum of three QPSK symbols:

$$S_{64QAM} = \frac{1}{\sqrt{2}} S_{QPSK} + \sqrt{2} S_{QPSK} + \sqrt{2} S_{QPSK}$$

As with 16 QAM we can trade off the size ,the PAPR, the squared Euclidean distance of a 64 QAM OFDM code by an appropriate selection of C_1, C_2 ,C_3.

If we let $B_1 = RM_2$ (0 , m) and $B_2 = (x_1x_2 + x_2x_3 + \ldots + x_{m-1}x_m) + RM_2$ (1 , m) = Q+ RM_2 (1 , m). The resulting OFDM code is shown in Table 5. We can further increase the size of the 64 QAM OFDM code .The resulting code is shown in Table 6.

Table 5. 64- QAM OFDM codes with PAPR=4.67

B_1	RM(1,m)
B_2	Q_π +RM(1,m)
B_3	RM (1,m)
B_4	Q_π +RM(1,m)
B_5	RM(1,m)
B_6	Q_π +RM(1,m)
size	$\left(\frac{m!}{2}\right)^{3} [(4]^{m+1})^{3}$
PAPR	≤4.67
D_E^2	2^{m-1}

B_1	RM(0,m)
B_2	Q +RM(1,m)
B_3	RM (1,m)
B_4	Q_π +RM(1,m)
B_5	RM(1,m)
B_6	Q_π +RM(1,m)
Size	$\left(\frac{m!}{2}\right)^{2} [2^{m+2}(4]^{m+1})^{2}$
PAPR	≤4.67
	2^{m}

Table 6. 64- QAM OFDM codes with PAPR=7.14

B_1	RM(0,m)
B_2	$Q +$ RM (1,m)
B_3	RM(1,m)
B_4	Q_π +RM (1,m)
B_5	RM (1,m-1) RM (1,m-1) ⊕
B_6	(Q_π +RM(1,m-1)) (Q_π +RM(1,m-1)) ⊕
size	$(\frac{m}{2})! \left(\frac{(m-1)!}{2}\right)^{2} 2^{m+2} 4^{m+1}(\square) 4^{m^2}$
PAPR	≤7.14
D_E^2	2^{m}

Table 7. 64- QAM OFDM codes with PAPR=13.32

B_1	RM(1,m)
B_2	$Q_\pi +$ RM (1,m)
B_3	RM (1,m-1) RM (1,m-1) ⊕
B_4	(Q_π +RM(1,m-1)) (Q_π +RM(1,m-1)) ⊕
B_5	RM (1,m-2) RM (1,m-2) ⊕ RM (1,m-2) ⊕ RM (1,m-2) ⊕
B_6	(Q_π +RM(1,m-2)) ⊕ (Q_π +RM(1,m-2)) ⊕ (Q_π +RM(1,m-2)) ⊕ (Q_π +RM(1,m-2))
size	$(\frac{m}{2})! \left(\frac{(m-1)!}{2}\right)^{2} \left(\frac{(m-2)!}{2}\right)^{4} 2^{m+2} 4^{m+1} [(4]^{m})^{2} [(4]^{m-1})^{4}$

PAPR	≤13.32
D_E^2	2^{m-1}

Conclusion

We have considered in this paper the reduction of peak-to average power ratio in OFDM systems for 16-QAM and 64-QAM signals. We examined the squared Euclidean distance of a quaternary code over QPSK modulation and then analyzed the 16-QAM OFDM sequences by combining the block coded modulation techniques and some variations of Golay complementary sequences. Some 16-QAM and 64-QAM OFDM sequences with low PAPR and large squared Euclidean distance are presented.

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