38199

R.K.Dhal et al./ Elixir Appl. Math. 91 (2016) 38199-38212

Available online at www.elixirpublishers.com (Elixir International Journal)



Applied Mathematics



Elixir Appl. Math. 91 (2016) 38199-38212

Analytical solution of Unsteady MHD free Convection and mass transfer past a Vertical Porous Plate with Slip Flow Region

R.K.Dhal^{1,*}, Banamali Jena² and P. M. Sreekumar³ ¹J.N.V. Paralakhemundi, Gajapati, Orissa-761201, India. ²JNV Betaguda, Odisha-761201, India. ³JNV Joura, Morena, M.P., India.

ARTICLE INFO

Article history: Received: 2 January 2016; Received in revised form: 30 January 2016; Accepted: 5 February 2016;

Keywords

MHD, Mass Transfer, Heat Transfer, Chemical Reaction, Radiation, Heat sources, Soret Number and Slip flow Region.

Introduction

MHD plays an important role in power generation, space propulsions, cure of diseases, control of thermonuclear reactor, boundary layer control in field of aerodynamics. In past few years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of hydrodynamics. Convection in porous medium has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomenon of mass transfer is also very common in the theory of stellar structure and observable effects are detectable on solar surface. The effect of magnetic field on free convection flow is important in liquid-metals and ionized gases. To study such applications which are closely associated with magneto-chemistry requires a complete understanding of the equation of state and transfer properties such as diffusion, the shear stress, thermal conduction, electrical conduction, etc. Some of these properties will undoubtedly be influenced by the presence of external magnetic field. The phenomena of MHD flows with unsteady oscillatory free convective flows play an important role in aerospace technology and in chemical engineering turbo-machinery. Moreover considerable interest has been shown in radiation interaction with convection for heat and mass transfer fluids. This is due to the significant role of thermal radiation in the surface heat transfer, which is small. That velocity of solid surface is proportional to shear stress at the surface [1]. i.e

$$\mathbf{V}_{\mathbf{x}} = \mathbf{h}' \frac{\partial \mathbf{V}_{\mathbf{x}}}{\partial \mathbf{y}}$$

where \mathbf{h}' is the slip co-efficient. If $\mathbf{h}' = \mathbf{0}$, then no slip condition will obtain. If $\mathbf{h}' \neq \mathbf{0}$ (finite) fluid slip occurs at the wall. The above relation is linear but one can establish a nonlinear relationship of the slip flow. The fluid slippage phenomena of the solid boundaries appear in many applications such as micro channels or nanochannels and in application when surface is coated with special coatings such as thick monolayer of hydrophobic octa decyltrichlorosilance have been discuss by Derek et.al [2]. Singh et.al [3] discussed the effects of MHD Free convective flow of a viscous fluid through a porous medium bounded by an oscillated porous plate in slip flow regime with mass transfer. Magnetohydrodynamic unsteady free convection flow with mass transfer through a porous medium has been analyzed by Mahapatra et.al [1]. The effects of fluid slippage at the wall for Coutte flow are considered by Marques et.al [6] under steady state condition and only for gases. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi [4]. Das et.al [5] derived Numerical solution of Mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction. Senapati and Dhal [7] have studied magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in the presence of chemical reaction.

ABSTRACT

The unsteady free convection flow of a laminar viscous, incompressible, electrically conducting with chemical reactive species and heat generation fluid past a semi-infinite vertical plate embedded in uniform porous medium with slip flow region has been studied. The governing equations are solved analytically using perturbation techniques. The influences of the various parameters on the flow field, Temperature field, Mass concentration field, Skin friction, Rate of heat transfer and Rate of mass transfer are extensively discussed through Graphs and Tables.

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When the mass flux contains a term that depends on the temperature gradient then the Soret effect focuses our study on the effect of free convection in the presence of a second order fluid. Convection in binary fluids is considerably more complicated than that in pure fluids. Even when a concentration gradient is not externally imposed, it can be created by the applied thermal gradient via Soret effect. Mbeledogu et.al [6] have discussed the unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radioactive heat transfer. Ahmmed et al. [8] have discussed Numerical Study on MHD free convection and mass transfer flow on flat plate. Ahmmed et. al [9] have studied the unsteady MHD free convection and mass transfer flow past a vertical porous plate which has been investigated analytically by using perturbation technique.

In this problem, attempt has been made to investigate the Effect of chemical reaction on Unsteady MHD free Convection and mass transfer past a Semi infinite Vertical Plate embedded in a Porous medium with Slip Flow Region.

Formulation of the problem

Consider a two dimensional unsteady free convection flow of a laminar viscous, incompressible, electrically conducting with chemical reactive species and heat generation fluid past a semi-infinite vertical plate embedded in uniform porous medium with slip flow region. A magnetic field of uniform strength B_0 is applied in the normal to the plate in the presence of pressure gradient and thermal diffusion and thermal radiation effects are taken into account. The fluid is assumed to be gray, absorbing – emitting but not scattering medium. The radiative heat flux is considered negligible along χ' direction in comparison with γ' direction. Reynolds number and induced magnetic field are negligible. Viscous and Darcy resistance terms are taken into account in the constant permeability porous medium. Let T'_w and C'_w be respectively the temperature and the molar species concentration of the fluid at the plate and T'_{∞} and C'_{∞} be respectively the equilibrium temperature and equilibrium molar species concentration is considered only in the body force term and the flow variable are the functions of y' and t' only. By usual Boussinesq's approximation, the unsteady flow is governed by the following equations on the balances of mass, linear momentum, energy and concentration species:

$$\frac{\partial v}{\partial y'} = \mathbf{0} \tag{1}$$

$$\rho \left| \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial v'} \right| = \frac{\partial P'}{\partial x'} + \mu \frac{\partial^2 u}{\partial v'^2} - \rho \beta - \frac{\mu}{\kappa'} u' - \sigma B_0^2 u'$$
⁽²⁾

$$\frac{\partial T'}{\partial t'} + \nu' \frac{\partial T}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p} \left(\frac{\partial q'}{\partial y'} \right) - \frac{Q_0}{\rho c_p} \left(T' - T'_{\infty} \right)$$
(3)

$$\frac{\partial c'}{\partial t'} + \nu' \frac{\partial c}{\partial y'} = D_M \frac{\partial^2 c'}{\partial {y'}^2} + D_T \frac{\partial^2 T'}{\partial {y'}^2} - \mathbf{R}' (\mathbf{C}' - \mathbf{C}'_{\infty})$$
⁽⁴⁾

with the following boundary conditions

$$\begin{aligned} \mathbf{u}' &= \mathbf{h}' \frac{d\mathbf{u}'}{d\mathbf{y}'} , \mathbf{T}' = \mathbf{T}'_{\mathbf{w}} + \epsilon (\mathbf{T}'_{\mathbf{w}} - \mathbf{T}'_{\infty}) \mathbf{e}^{\mathbf{n}'\mathbf{t}'} , \mathbf{C}' &= \mathbf{C}'_{\mathbf{w}} + \epsilon (\mathbf{C}'_{\mathbf{w}} - \mathbf{C}'_{\infty}) \mathbf{e}^{\mathbf{n}'\mathbf{t}'} \quad \text{at } \mathbf{y}' = \mathbf{0} \\ \mathbf{u}' &= \mathbf{U}_{\mathbf{0}} \big(\mathbf{1} + \mathbf{e}^{\mathbf{n}'\mathbf{t}'} \big), \qquad \mathbf{T}' = \mathbf{T}'_{\infty} , \qquad \mathbf{C}' = \mathbf{C}'_{\infty} \quad \text{at} \quad \mathbf{y}' \to \infty \end{aligned}$$

where u' and v' are velocity components along x' and y' directions respectively, ρ is the fluid density, μ is viscosity, C_P is specific heat at constant pressure, K' is the permeability of the porous medium, D_M is the coefficient of chemical molecular diffusivity, D_T is the coefficient of thermal diffusivity, g is the acceleration due to gravity, Q_0 is the heat sink/source, R' is the rate of chemical reaction, h' is refraction parameter and q'_r is radioactive heat flux.

From the equation (1), we consider the velocity in the exponential form

$$v' = -v_0(1 + \varepsilon A e^{n't'})$$

where A is a real positive constant, ε and ε A are less than unity and v_0 is mean variable suction velocity which has non-zero positive constant.

(6)

In the free stream, we have

$$\rho \frac{\partial U'_{\infty}}{\partial t'} = \frac{\partial P'}{\partial x'} - \rho_{\infty} g - \frac{\mu}{K'} U'_{\infty} - \sigma B_0^2 U'_{\infty}$$
(7)
Also, by using Equation of state, we have

$$(\boldsymbol{\rho}_{\infty} - \boldsymbol{\rho}) = \mathbf{g}\boldsymbol{\beta}(\mathbf{T}' - \mathbf{T}'_{\infty}) + \mathbf{g}\boldsymbol{\beta}_{\mathbf{c}}(\mathbf{C}' - \mathbf{C}'_{\infty})$$
(8)

Eliminate $\frac{\partial P'}{\partial x'}$ using equation (2), (7) and (8), we get

$$\rho\left[\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right] = \rho\frac{\partial U'_{\infty}}{\partial t'} + \mu\frac{\partial^2 u'}{\partial {y'}^2} + g\beta(\mathbf{T}' - \mathbf{T}'_{\infty}) + g\beta_c(\mathbf{C}' - \mathbf{C}'_{\infty}) + \frac{\mu}{K'}(U'_{\infty} - u') + \sigma B_0^2(U'_{\infty} - u')$$
⁽⁹⁾

The heat flux by using Roseland approximation is given by

$$q'_r = \frac{4\sigma'}{3k'_1} \frac{\partial T'}{\partial y'} \tag{10}$$

where σ' and k'_1 are respectively Boltzman constant and mean absorption coefficient.

By assuming the temperature difference within the flow as very small such that T'^4 may be express as the linear function of temperature by neglecting the higher power of T'^4_{co} using Taylor series, thus

$$T'^4 = 4T'^3_{\infty} - 3T'^4_{\infty} \tag{11}$$

By using equations (3), (10) and (11), we get

$$\frac{\partial \mathbf{T}'}{\partial \mathbf{t}'} + \mathbf{v}' \frac{\partial \mathbf{T}'}{\partial \mathbf{y}'} = \frac{\mathbf{k}}{\rho C_p} \frac{\partial^2 \mathbf{T}'}{\partial {\mathbf{y}'}^2} - \frac{16\sigma' \mathbf{T}_{\infty}'^3}{3\mathbf{k}_1' \rho C_p} \left(\frac{\partial^2 \mathbf{q}'}{\partial {\mathbf{y}'}^2} \right) - \frac{\mathbf{Q}_0}{\rho C_p} \left(\mathbf{T}' - \mathbf{T}_{\infty}' \right)$$
(12)

Let us introduce the non-dimensional variables

$$\mathbf{u} = \frac{\mathbf{u}'}{\mathbf{u}_{0}}, \mathbf{y} = \frac{\mathbf{y}'\mathbf{v}_{0}}{\mathbf{v}}, \mathbf{\theta} = \frac{\mathbf{T}' - \mathbf{T}'_{\infty}}{\mathbf{T}'_{\mathbf{w}} - \mathbf{T}'_{\infty}}, \mathbf{\varphi} = \frac{\mathbf{C}' - \mathbf{C}'_{\infty}}{\mathbf{C}'_{\mathbf{w}} - \mathbf{C}'_{\infty}}, \mathbf{K} = \frac{\mathbf{k}'\mathbf{v}_{0}^{2}}{\mathbf{v}^{2}}, \mathbf{Gm} = \frac{\mathbf{vg}\beta_{c}(\mathbf{C}'_{\mathbf{w}} - \mathbf{C}'_{\infty})}{\mathbf{v}_{0}^{2}\mathbf{U}_{0}}, \mathbf{h} = \mathbf{h}'\frac{\mathbf{v}_{0}}{\mathbf{v}},$$

$$\mathbf{Gr} = \frac{\mathbf{vg}\beta(\mathbf{T}'_{\mathbf{w}} - \mathbf{T}'_{\infty})}{\mathbf{v}_{0}^{2}\mathbf{U}_{0}}, \mathbf{Sc} = \frac{\mathbf{v}}{\mathbf{D}_{\mathbf{M}}}, \mathbf{Pr} = \frac{\mathbf{v}\rho\mathcal{C}_{p}}{\mathbf{k}}, \mathbf{M} = \frac{\sigma B_{0}^{2}\mathbf{v}}{\rho\mathbf{v}_{0}^{2}}, \mathbf{R} = \frac{\mathbf{v}\mathbf{R}'}{\mathbf{v}_{0}^{2}}, \mathbf{t} = \frac{\mathbf{t}'\mathbf{v}_{0}^{2}}{\mathbf{v}}, \mathbf{n}' = \frac{\mathbf{n}\mathbf{v}_{0}^{2}}{\mathbf{v}},$$

$$\left. \right\}$$

$$\left. \left\{ \mathbf{M} = \frac{\mathbf{v}\mathbf{u}^{2}\mathbf{v}}{\mathbf{v}^{2}}, \mathbf{R} = \frac{\mathbf{v}\mathbf{R}'}{\mathbf{v}_{0}^{2}}, \mathbf{T} = \frac{\mathbf{v}\mathbf{v}^{2}}{\mathbf{v}}, \mathbf{T}' = \frac{\mathbf{v}\mathbf{v}\mathbf{v}^{2}}{\mathbf{v}}, \mathbf{T}' = \frac{\mathbf{v}\mathbf{v}\mathbf{v}\mathbf{v}^{2}}{\mathbf{v}}, \mathbf{T}' = \frac{\mathbf$$

$$Q = \frac{Q_0 v}{\rho v_0^2 C_p}, N = \frac{4\sigma' T'_{\infty} (T'_{w} - T'_{\infty})}{k'_1 k}, U'_{\infty} = U_{\infty} U_0$$

Then the governing equations (12), (9), (4) with boundary condition (5) reduce to

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{dU_{\infty}}{dt} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\varphi + \left(M + \frac{1}{\kappa}\right) (U_{\infty} - u)$$
(14)

$$\frac{\partial\theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial\theta}{\partial y} = \frac{1}{Pr} \left(1 + \frac{4N}{3} \right) \frac{\partial^2 \theta}{\partial y^2} - Q\theta$$
⁽¹⁵⁾

$$\frac{\partial\varphi}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial\varphi}{\partial y} = \frac{1}{s_c} \frac{\partial^2\varphi}{\partial y^2} + S_0 \frac{\partial^2\theta}{\partial y^2} - R\varphi$$
(16)

with corresponding boundary conditions

$$u = h \frac{\partial u}{\partial y}, \theta = 1 + \varepsilon e^{nt}, \varphi = 1 + \varepsilon e^{nt} \text{ at } y = 0$$

$$u \to 1 + \varepsilon e^{nt}, \theta \to 0, \varphi \to 0 \text{ as } y \to \infty$$

$$(17)$$

where Pr is the Prandtl number, Gr is the Grashof number, Gm is modified Grashof number, Sc is the Schmidt number, M is the Magnetic parameter, K is the non dimensional permeability parameter of porous medium, R is the non dimensional chemical reaction parameter, N is radiation parameter, S_0 is Soret number, Q is Source parameter, n is frequency of oscillation and h is the non dimensional refraction parameter.

Method of Solution

In order to solve the equation from (14)-(16) with boundary condition (17) of the problem using perturbation, we assume the solutions of the following forms

$$\begin{aligned} u &= u_0(y) + u_1(y)\varepsilon e^{nt} + 0(\varepsilon^2) \\ \theta &= \theta_0(y) + \theta_1(y)\varepsilon e^{nt} + 0(\varepsilon^2) \\ \varphi &= \varphi_0(y) + \varphi_1(y)\varepsilon e^{nt} + 0(\varepsilon^2) \end{aligned}$$
(18)

Where u_0 , θ_0 and φ_0 are respectively mean velocity, mean temperature and mean concentration.

By substituting the above equations of (18) into the equations (14)-(17) and then equating harmonic and non harmonic terms by neglecting higher order terms of $\mathbf{0}(\varepsilon^2)$, we obtain the following equations for u_0 , θ_0 , φ_0 , u_1 , θ_1 and φ_1 as given below:

$$-\frac{\partial u_{0}}{\partial y} = \frac{\partial^{2} u_{0}}{\partial y^{2}} + G_{r}\theta_{0} + G_{m}\varphi_{0} + \left(M + \frac{1}{\kappa}\right)(u_{0} - 1)$$

$$u_{1}(n-1) - A\frac{\partial u_{0}}{\partial y} - \frac{\partial u_{1}}{\partial y} = \frac{\partial^{2} u_{1}}{\partial y^{2}} + G_{r}\theta_{1} + G_{m}\varphi_{1} + \left(M + \frac{1}{\kappa}\right)(u_{1} - 1)$$

$$-\frac{\partial \theta_{0}}{\partial y} = \frac{1}{P_{r}}\left(1 + \frac{4N}{3}\right)\frac{\partial^{2} \theta_{0}}{\partial y^{2}} - Q\theta_{0}$$

$$n\theta_{1} - A\frac{\partial \theta_{0}}{\partial y} = \frac{1}{P_{r}}\left(1 + \frac{4N}{3}\right)\frac{\partial^{2} \theta_{1}}{\partial y^{2}} - Q\theta_{1}$$

$$-\frac{\partial \varphi_{0}}{\partial y} = \frac{1}{S_{c}}\frac{\partial^{2} \varphi_{0}}{\partial y^{2}} + S_{0}\frac{\partial^{2} \theta_{1}}{\partial y^{2}} - R\varphi_{0}$$

$$n\varphi_{1} - A\frac{\partial \varphi_{0}}{\partial y} = S_{0}\frac{\partial^{2} \theta_{1}}{\partial y^{2}} + \frac{1}{S_{c}}\frac{\partial^{2} \varphi_{1}}{\partial y^{2}} - R\varphi_{1}$$
with corresponding boundary conditions
$$(19)$$

$$u_{0} = h \frac{\partial u_{0}}{\partial y}, u_{1} = h \frac{\partial u_{1}}{\partial y}, \theta_{0} = 1, \theta_{1} = 1, \varphi_{0} = 1, \varphi_{1} = 1 \text{ at } y = 0$$

$$u_{0} = 1, u_{1} = 1, \theta_{0} = 0, \theta_{1} = 0, \varphi_{0} = 0, \varphi_{1} = 0 \text{ as } y \to \infty$$

$$(20)$$

Finally by solving equations in (19) using boundary conditions (20), we get

$$\theta = e^{-a_1 y} + \varepsilon e^{nt} ((1 - b_1) e^{-a_2 y} + b_1 e^{-a_1 y})$$
⁽²¹⁾

$$\varphi = (b_3 e^{-a_3 y} + b_2 e^{-a_1 y}) + \varepsilon e^{nt} (b_7 e^{-a_4 y} + b_4 e^{-a_3 y} + b_5 e^{-a_2 y} + b_6 e^{-a_1 y})$$
(22)

$$u = (b_8 e^{-a_5 y} + b_9 e^{-a_1 y} + b_{10} e^{-a_3 y} + b_{11} e^{-a_1 y} + 1) + \varepsilon e^{nt} \left(\frac{b_{18} e^{-a_6 y} + b_{13} e^{-a_2 y} + b_{14} e^{-a_1 y} + b_{15} e^{-a_4 y}}{+b_{16} e^{-a_3 y} + b_{17} e^{-a_5 y} + b_{12}} \right)$$
(23)

The dimensional rate of heat transfer,

$$Nu = \left(\frac{\partial\theta}{\partial y}\right)_{y=0} a_1 + \varepsilon e^{nt} \left((1 - b_1)a_2 + a_1b_1 \right)$$
(24)
The dimensionless rate of mass transfer,

$$Sh = \left(\frac{\partial\varphi}{\partial y}\right)_{y=0} = (a_3b_3 + a_1b_2) + \varepsilon e^{nt} (a_4b_7 + a_3b_4 + a_2b_5 + a_1b_6)$$
(25)

The non-dimensional Skin friction at the wall from the equations (23) is given by

$$\begin{aligned} \tau_{0} &= \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\left(b_{8}a_{5} + b_{9}a_{1} + b_{10}a_{3} + a_{1}b_{11}\right) - \varepsilon e^{nt} \left(\frac{b_{18}a_{6} + b_{13}a_{2} + a_{1}b_{14} + a_{4}b_{15}}{+a_{3}b_{16} + a_{5}b_{17}}\right) \\ \text{where} \\ a_{1} &= \frac{1 + \sqrt{1 + \frac{4Q}{P_{\Gamma}}\left(1 + \frac{4N}{3}\right)}}{\frac{2}{P_{\Gamma}\left(1 + \frac{4N}{3}\right)}}, a_{2} &= \sqrt{\frac{(Q+n)Pr}{(1 + \frac{4N}{3})}}, a_{3} &= \frac{1 + \sqrt{1 + \frac{4R}{5c}}}{\frac{2}{5c}}, \\ a_{4} &= \sqrt{(R+n)Sc}, a_{5} &= \frac{1 + \sqrt{1 - 4\left(M + \frac{1}{K}\right)}}{2}, a_{6} &= \frac{1 + \sqrt{1 - 4\left(M + \frac{1}{K} - n\right)}}{2}, \\ b_{1} &= \frac{a_{1A}}{\frac{1}{P_{\Gamma}\left(1 + \frac{4N}{3}\right)a_{1}^{2} + a_{1}(Q+n)}}, b_{2} &= \frac{-s_{0}a_{1}^{2}}{\frac{a_{1}^{2}}{2} - a_{1} - R}, b_{3} &= 1 - b_{2}, b_{4} &= \frac{Ab_{3}a_{3}}{\frac{a_{3}^{2}}{5c} - (R+n)}, \\ b_{5} &= \frac{S_{0}(1-b)a_{1}^{2}}{\frac{a_{5}^{2}}{2} - (R+n)}, b_{6} &= \frac{Ab_{2}a_{1} - S_{0}b_{1}a_{1}^{2}}{\frac{a_{7}^{2}}{5c} - a_{1} - R}, b_{7} &= 1 - b_{4} - b_{5} - b_{6}, b_{9} &= \frac{-Gr}{a_{1}^{2} - a_{1} + \left(M + \frac{1}{K}\right)}, \\ b_{10} &= \frac{-Gmb_{3}}{a_{3}^{2} - a_{3} + \left(M + \frac{1}{K}\right)}, b_{11} &= \frac{-Gmb_{2}}{a_{1}^{2} - a_{1} + \left(M + \frac{1}{K}\right)}, \\ b_{10} &= \frac{-Gmb_{3}}{a_{3}^{2} - a_{3} + \left(M + \frac{1}{K}\right)}, b_{11} &= \frac{-Gmb_{2}}{a_{1}^{2} - a_{1} + \left(M + \frac{1}{K}\right)}, \\ b_{13} &= \frac{-(Gr(1-b_{1}) + Gmb_{5})}{a_{2}^{2} - a_{2} + M + \frac{1}{K} - n}, b_{14} &= \frac{-(Grb_{1} + Gmb_{6} - Aa_{1}b_{9} - Ab_{11}a_{1})}{a_{1}^{2} - a_{1} + M + \frac{1}{K} - n}, \\ b_{15} &= \frac{-Gmb_{7}}{a_{4}^{2} - a_{4} + M + \frac{1}{K} - n}, b_{16} &= \frac{-Gmb_{4} + Ab_{10}a_{3}}{a_{3}^{2} - a_{3} + M + \frac{1}{K} - n}, \\ b_{17} &= \frac{Ab_{8}a_{5}}{a_{5}^{2} - a_{5} + M + \frac{1}{K} - n}{a_{5}^{2} - a_{5} + M + \frac{1}{K} - n}, b_{12} &= \frac{-\left(N + \frac{1}{K}\right)}{ha_{6} + 1}}, b_{13}b_{1} + b_{14} + b_{15} + b_{16} + b_{17} + h(b_{13}a_{2} + b_{14}a_{1} + b_{15}a_{4} + b_{16}a_{3} + b_{17}a_{5})} \\ b_{18} &= -\left(\frac{(b_{12} + b_{13} + b_{14} + b_{15} + b_{16} + b_{17} + h(b_{13}a_{2} + b_{14}a_{1} + b_{15}a_{4} + b_{16}a_{3} + b_{17}a_{5})}{ha_{6} + 1}\right) \right)$$

Graphical Results and Discussion

In this paper, we have studied the Effect of unsteady free convection flow of a laminar viscous, incompressible, electrically conducting with chemical reactive species and heat generation fluid past a semi-infinite vertical plate embedded in uniform porous medium with slip flow region. The effect of the parameters Gr, Gm, M, K, R, Sc, h, n, t, Q, So, N, Pr, A and Sc on flow characteristics have been studied and shown by means of graphs. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocities, heat and mass concentration are taken w.r.t. \mathbf{y} and the graphs of Skin friction are taken w.r.t time (t), where as the values of Nusselt number and Sherwood Number are displayed in Tables. To be realistic, the values of Schmidt number (Sc) are chosen 0.22, 0.3, 0.65 and 0.78 and the values of Prandtl Number are choosen for 0.71, 1, 2 and 7 for different fluids.

Velocity profiles: The velocity profiles are depicted in Figs 7-12. Figure-(7) shows the effect of **Sc and Pr** on Velocity profile at any point of the fluid, when Gr=2, Gm=2, N=2, R=2, Q=2, So=3, M=0.3, K=4, n=2, h=2, A=0.02 and $\mathbf{t} = \mathbf{0.5}$. It is noticed that the velocity increases with the increase of Schmidt number (Sc) and Prandtl number (Pr).

Figure-(8) shows the effect of the parameters **Gr and Gm** on Velocity profiles at any point of the fluid, when Sc=0.22, Pr=0.71, N=2, R=2, Q=2, So=3, M=0.3, K=4, n=2, h=2, A=0.02 and $\mathbf{t} = \mathbf{0.5}$. It is noticed that the velocity increases with the increase of Modified Grashof Number (Gm) where as decreases with the increase of Grashof Number (Gr).

Figure-(9) shows the effect of the parameters **Q** and **N** on Velocity profile at any point of the fluid, when Gr=2, Gm=2, Sc=0.22, R=2, Pr=0.71, So=3, M=0.3, K=4, n=2, h=2, A=0.02 and $\mathbf{t} = \mathbf{0.5}$. It is noticed that the velocity increases with the increase of Source parameter (Q) where as decreases with radiation parameter (N).

Figure-(10) shows the effect of the parameters **So and R** on Velocity profile at any point of the fluid, when Gr=2, Gm=2, N=2, Pr=0.71, Q=2, Sc=0.22, M=0.3, K=4, n=2, h=2, A=0.02 and $\mathbf{t} = \mathbf{0.5}$. It is noticed that the velocity increases with the increase of Soret Number (So) and Chemical reaction parameter (R).

Figure-(11) shows the effect of the parameters **M** and **K** on Velocity profile at any point of the fluid, when Gr=2, Gm=2, N=2, Pr=0.71, Q=2, Sc=0.22, So=3, n=2, h=2, A=0.02 and $\mathbf{t} = \mathbf{0}.\mathbf{5}$. It is noticed that the velocity increases with the increase of Magnetic parameter (**M**) where as decreases with permeability of porous medium (K).

Figure-(12) shows the effect of the parameters **n**, **t**, **h** and **A** on Velocity profile at any point of the fluid, when Gr=2, Gm=2, N=2, Pr=0.71, Q=2, Sc=0.22, M=0.3, K=4, So=3, and R=2. It is noticed that the velocity increases with the increase of oscillating parameter (n), Time (t) and Suction constant (A) where as decreases with refraction parameter (h).

Temperature Profile: The Temperature profiles are depicted in Figures 1-3.

Figure-(1) shows the effect of the parameters Pr on Temperature profile at any point of the fluid, when N=2, Q=2, n=2, and $\mathbf{t} = \mathbf{0}$. **5**. It is noticed that the temperature falls with the increase of Prandtl Number (Pr).

Figure-(2) shows the effect of the parameters Q and N on Temperature profiles at any point of the fluid, when Pr=0.71, n=2, and $\mathbf{t} = \mathbf{0}$. 5. It is noticed that the temperature falls with the increase of Radiation parameter (N), whereas rises with Source parameter (Q).

Figure-(3) shows the effect of the parameters n and t on Temperature profiles at any point of the fluid, when Pr=0.71, N=2, and Q = 2. It is noticed that the temperature rises in the increase of Oscillation parameter (n) and time (t).

Mass concentration profile: The Mass concentration profiles are depicted in Figures4-6.

Figure-(4) shows the effect of the parameters **Sc** on mass concentration profile at any point of the fluid when N=2, R=2, Q=2, So=3, Pr=0.71, n=2, and $\mathbf{t} = \mathbf{0}.\mathbf{5}$. It is noticed that the mass concentration decreases with the increase of Schmidt number (Sc).

Figure-(5) shows the effect of the parameters So and R on mass concentration profile at any point of the fluid when N=2, Q=2, Sc=0.22, Pr=0.71, n=2 and $\mathbf{t} = \mathbf{0}$. **5**. It is noticed that the mass concentration decreases with the increase of Soret Number (So) whereas increases with the increase of Chemical reaction parameter (R).

Figure-(6) shows the effect of the parameters **n** and **t** on mass concentration profile at any point of the fluid, when N=2, R=2, Q=2, So=3, Pr=0.71, Sc=0.22_{*} It is noticed that the mass concentration increases with the increase of Oscillating parameter (n) and time (t).

Skin friction: The Skin frictions are depicted in Figs 13-15.

Figure-(13) illustrates the effect of the parameters Gr, Gm, Pr and Sc on Skin friction at plate of the fluid w.r.t. time (t), when N=2, R=2, Q=2, So=3, h=2, A=0.02, M=0.3, K=4 and n=2. It is noticed that Skin friction at plate decreases with the increase of Modified Grashof number (Gm), whereas increases with the increase of Grashof number (Gr), Prandtl number (Pr) and Schmidt number (Sc).

Figure-(14) illustrates the effect of the parameters M, K, Q and N on Skin friction at plate of the fluid w.r.t. time (t), when Gr=2, Pr=0.71, Gm=2, Sc=0.22, h=2, A=0.02, So=3, R=2 and n=2. It is noticed that Skin friction at plate decreases with the increase of permeability of porous medium (K) and radiation parameter (N), whereas increases with the increase of Magnetic parameter (Q).

Figure-(15) illustrates the effect of the parameters So, h, R and n on Skin friction at plate of the fluid w.r.t. time (t), when Gr=2, Pr=0.71, Gm=2, Sc=0.22, A=0.02, M=0.3, K=4,Q=2 and N=2. It is noticed that Skin friction at plate decreases with the increase of Soret number (So), whereas increases with the increase of Chemical reaction parameter (R), refraction parameter (h) and oscillating parameter (n).

Nusselt Number: Table-(1) illustrates the effect of the parameters Pr, Q, N and n on Nusselt number at plate. It is observed that Nusselt number increases at the plate with the increase of Oscillating parameter (n), Source parameter (Q) and Prandtl number (Pr), whereas decreases with the increase of Radiation parameter(N).

Table-(2) illustrates the effect of the parameters Sc, Pr, Q, N, So, n and R on Sherwood Number at the plate of the fluid. It is noticed that Sherwood Number at the plate increases with the increase of Schmidt number (Sc), Radiation parameter (N), Soret number (So) and Reaction parameter (R) whereas decreases with the increase of Prandtl number (Pr) and Source parameter (Q).

Table 1. Numerical values of the Rate of Heat Transfer (Nu)

Sl.No	Pr	Q	Ν	n	Nu
01	0.71	2	2	2.4	1.0606
02	1.00	2	2	2.4	1.2933
03	2.00	2	2	2.4	1.9632
04	0.71	4	2	2.4	1.4249
05	0.71	6	2	2.4	1.7064
06	0.71	2	4	2.4	0.6792
07	0.71	2	6	2.4	0.5323
08	0.71	2	2	2.6	1.0685
09	0.71	2	2	2.8	1.0779

Sl.No	Sc	Pr	Q	Ν	R	So	n	Sh
01	0.22	0.71	2	2	2	2	2.4	0.5125
02	0.3	0.71	2	2	2	2	2.4	0.6798
03	0.6	0.71	2	2	2	2	2.4	0.9315
04	0.22	1.00	2	2	2	2	2.4	0.4913
05	0.22	2.00	2	2	2	2	2.4	0.2427
06	0.22	0.71	4	2	2	2	2.4	0.4384
07	0.22	0.71	6	2	2	2	2.4	0.3337
08	0.22	0.71	2	4	2	2	2.4	0.7089
09	0.22	0.71	2	6	2	2	2.4	0.7525
10	0.22	0.71	2	2	4	2	2.4	0.9194
11	0.22	0.71	2	2	6	2	2.4	1.1564
12	0.22	0.71	2	2	2	4	2.4	0.1747
13	0.22	0.71	2	2	2	6	2.4	0.1631
14	0.22	0.71	2	2	2	2	2.6	0.7168
15	0.22	0.71	2	2	2	2	2.8	0.6248

Table 2. Numerical values of the Rate of Mass Transfer (Sh)



Figure 1. Effect of Pr on Temperature profile (θ), when N=2, Q=2, n=2, and t = 0.5.



Figure 2. Effect of N and Q on Temperature profile (θ), when Pr=0.71, n=2, and t = 0.5.



Figure 3. Effect of n and t on Temperature profile (θ), when Pr=0.71, N=2, and Q = 2.



Figure 4. Effect of Sc on Mass concentration profile (φ), when N=2, R=2, Q=2, So=3, Pr=0.71, n=2, and t = 0.5.



Figure 5. Effect of So and R on Mass concentration profile (φ), when N=2, Q=2, Sc=0. 22, Pr=0.71, n=2, and t = 0.5.



Figure 6. Effect of *n* and *t* on Mass concentration profile (ϕ), when N=2, R=2, Q=2, So =3, Pr=0.71, Sc=0.22.



Figure 7. Effect of Sc and Pr on Velocity profile (u), when Gr=2, Gm=2, N=2, R=2, Q=2, So =3, M=0.3, K=4, n=2, h=2, A=0.02 and t = 0.5.



Figure 8. Effect of Gr and Gm on Velocity profile (u), when Sc=0. 22, Pr=0.71, N=2, R=2, Q=2, So =3, M=0.3, K=4, n=2, h=2, A=0.02 and t = 0.5.



Figure 9. Effect of Q and N on Velocity profile (u), when Gr=2, Gm=2, Sc=0. 22, R=2, Pr=0.71, So =3, M=0.3, K=4, n=2, h=2, A=0.02 and t = 0.5.



Figure 10. Effect of So and R on Velocity profile (u), when Gr=2, Gm=2, N=2, Pr=0.71, Q=2, Sc=0. 22, M=0.3, K=4, n=2, h=2, A=0. 02 and t = 0.5.



Figure 11. Effect of *M* and *K* on Velocity profile (u), when Gr=2, Gm=2, N=2, Pr=0.71, Q=2, Sc=0. 22, So=3, R=2, n=2, h=2, A=0. 02 and t = 0.5.



Figure 12. Effect of *n*, *t*, *h* and *A* on Velocity profile (*u*), when Gr=2, Gm=2, N=2, Pr=0.71, Q=2, Sc=0.22, M=0.3, K=4, So=3 and R=2.



Figure 13. Effect of Sc, Gm, Gr and Pr on Skin Friction (τ), when N=2, R=2, Q=2, So=3, A=0.02, M=0.3, K=4, h=2 and n=2.



Figure 14. Effect of M, K, Q and N on Skin Friction (T), when Gr=2, Pr=0.71, Gm=2, Sc=0. 22, h=2, A=0. 02, So=3, R=2 and n=2.



Figure 15. Effect of *So*, *R*, *h* and *n* on Skin Friction (τ), when Gr=2, Pr=0. 71, Gm=2, Sc=0. 22, A=0. 02, M=0. 3, K=4, Q=2 and N=2.

Conclusions

The above study shows up the following results of physical interest on velocity and concentration distribution of flow field.

- The magnetic field and Chemical reaction slows down the velocity of fluid due to the magnetic pull of Lorentz force and heavier diffusion of chemical reactive species.
- The velocity of fluid accelerated by modified Grashoff number and enhance by Porosity parameter.
- Mass concentration decreased by Reactive diffusion species.
- Skin friction at the plate decreases by Lorentz force and Reactive diffusion species

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