# Design of Proportional Navigation Guidance for Air to Air Missiles 

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#### Abstract

The primary weapon used in modern air combat is a guided air to air missile. Many air to air missiles use proportional navigation (PN) guidance during the terminal phase. The main advantage of PN guidance is it requires minimal information regarding target motion. It is reliable and robust guidance scheme. The main objectives of this project are three fold 1) working of a guided missile, 2) PN guidance scheme and its suitability for air to air missiles. 3) Advantages and limitations of PN guidance. The above goals are met by implementing PN guidance law with the help of simplified interceptor and target missile models in MATLAB (SIMULINK). The effectiveness of PN guidance is also studied for various target engagement scenarios. Further this implementation has also been carried out in Xenomai - 2.6 (a Linux based open source Real Time Operating system) in C language.


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## I. Introduction

A guided missile is an unmanned explosive-carrying vehicle that moves above the earth's surface in a flight path controlled by an external or internal source. There are many kinds of guided missiles, but all have the same ultimate function: destroy enemy "targets", i.e., personnel, tanks, vehicles, airplanes, ships, and weapons, including attacking missiles.

The incorporation of energy source in a missile to provide the required force for its movement (propulsion), intelligence to go in the correct direction (guidance) and effective maneuvering (control) are mainly the technologies of guided missiles. They help in making a missile specific to a target, that is, they determine the size, range and state of motion of a missile.

## II Procedure for Paper Submission

## Simulation of Proportional Navigation Guidance in 2-D

Figure 2.1 represents the engagement geometry. The missile, with velocity magnitude $\mathrm{V}_{\mathrm{m}}$, is heading at an angle L + HE with respect to the line of sight. The angle L is the missile lead angle. The lead angle is the theoretical correct angle for the missile to be on a collision course triangle with the target. In other words, if the missile is on a collision triangle, no further acceleration commands are required for the missile to hit the target. The angle HE is the heading error, which represents the initial deviation of the missile from the collision triangle.

The line connecting the missile and the target is the line-of-sight. The line-of-sight makes an angle $\lambda$ with respect to the reference, and the length of the line-of-sight which is the instantaneous separation between the missile and the target satellite is denoted by $\mathrm{R}_{\text {тм }}$. The point of closest approach of the missile and the satellite is known as the miss distance. The closing velocity Vc is defined as the negative rate of change of the distance from the missile to the target.
$\mathrm{Vc}=-\mathrm{d}\left(\mathrm{R}_{\mathrm{TM}}\right) / \mathrm{dt}$
Therefore, at the end of engagement, when the missile and target are in closest proximity, the sign of Vc will change.


Figure 2.1. Two-dimensional missile-target engagement geometry
The closing velocity will be zero when $\mathrm{R}_{\mathrm{TM}}$ is a minimum. The desired acceleration command $\mathrm{n}_{\mathrm{c}}$, which is derived from the proportional navigation guidance law, is perpendicular to the instantaneous line of sight. In the simulated engagement model, the target can maneuver evasively with acceleration $\mathrm{n}_{\mathrm{T}}$. The target acceleration is perpendicular to the target velocity vector; thus, the angular velocity of the target is expressed as $\beta$ dot $=\mathrm{n}_{\mathrm{T}} / \mathrm{V}_{\mathrm{T}}$
----- (2.2)
where $\mathrm{V}_{\mathrm{T}}$ is the target velocity. The components of the target velocity in the inertial coordinate system are found by integrating the differential equation for the flight path angle of the target, $\beta$ and substituting in
$\mathrm{V}_{\mathrm{T} 1}=-\mathrm{V}_{\mathrm{T}} \cos (\beta)$
$\mathrm{V}_{\mathrm{T} 2}=\mathrm{V}_{\mathrm{T}} \sin (\beta)$
Target position components can be found by integrating the target velocity components. Therefore, the equations for the components of the target position are given by
$\mathrm{R}_{\mathrm{T} 1}=\int \mathrm{V}_{\mathrm{T} 1}$

Similarly the missile velocity and position equations are given by
$\mathrm{V}_{\mathrm{M} 1}=\int \mathrm{a}_{\mathrm{M} 1}$
$\mathrm{V}_{\mathrm{M} 2}=\int \mathrm{a}_{\mathrm{M} 2}$
$\mathrm{R}_{\mathrm{M} 1}=\int \mathrm{V}_{\mathrm{M} 1}$
$\mathrm{R}_{\mathrm{M} 2}=\int \mathrm{V}_{\mathrm{M} 2}$
Where $\mathrm{a}_{\mathrm{M} 1}$ and $\mathrm{a}_{\mathrm{M} 2}$ are the missile accelerations.
The relative missile-target separations are
$\mathrm{R}_{\mathrm{TM} 1}=\mathrm{R}_{\mathrm{T} 1}-\mathrm{R}_{\mathrm{M} 1}$
$\mathrm{R}_{\mathrm{TM} 2}=\mathrm{R}_{\mathrm{T} 2}-\mathrm{R}_{\mathrm{M} 2}$
From figure 3.1 the line-of-sight is
$\lambda=\tan ^{-1}\left(\mathrm{R}_{\mathrm{TM} 1} / \mathrm{R}_{\mathrm{TM} 2}\right)$
The relative velocity components in earth coordinates are defined as

$$
\begin{align*}
& \mathrm{V}_{\mathrm{TM} 1}=\mathrm{V}_{\mathrm{T} 1}-\mathrm{V}_{\mathrm{M} 1}  \tag{2.14}\\
& \mathrm{~V}_{\mathrm{TM} 2}=\mathrm{V}_{\mathrm{T} 2}-\mathrm{V}_{\mathrm{M} 2} \tag{2.15}
\end{align*}
$$

The line-of-sight rate is calculated by direct differentiation of the expression for the line-of-sight angle
$\lambda_{\_}$dot $=\left(\mathrm{R}_{\mathrm{TM} 1} \mathrm{~V}_{\mathrm{TM} 2}-\mathrm{R}_{\mathrm{TM} 2} \mathrm{~V}_{\mathrm{TM} 1}\right) / \mathrm{R}_{\mathrm{TM}}^{2}$
The relative separation between the missile and the target,
$\mathrm{R}_{\mathrm{TM}}$, can be expressed in terms of its inertial components by application of the distance formula, as
$\mathrm{R}_{\mathrm{TM}}=\operatorname{sqrt}\left(\mathrm{R}_{\mathrm{TM} 1}^{2}+\mathrm{R}_{\mathrm{TM} 2}^{2}\right)$
Since the closing velocity is defined as the negative rate of change of the missile-target separation, it can obtained from differentiating equation
$\mathrm{V}_{\mathrm{c}}=-\mathrm{d}\left(\mathrm{R}_{\mathrm{TM}}\right) / \mathrm{dt}=-\left(\mathrm{R}_{\mathrm{TM} 1} \mathrm{~V}_{\mathrm{TM} 1}+\mathrm{R}_{\mathrm{TM} 2} \mathrm{~V}_{\mathrm{TM} 2}\right) / \mathrm{R}_{\mathrm{TM}}$
The magnitude of the missile guidance command $n_{c}$ can then be found by
$\mathrm{n}_{\mathrm{c}}=\mathrm{N}^{\prime} \mathrm{Vc} \lambda \_\operatorname{dot}$
Since the acceleration command is perpendicular to the instantaneous line-of sight, the missile acceleration components can be found in earth coordinates
$\mathrm{a}_{\mathrm{M} 1}=-\mathrm{n}_{\mathrm{c}} \sin \lambda$
$\mathrm{a}_{\mathrm{M} 2}=\mathrm{n}_{\mathrm{c}} \cos \lambda$
----- (2.20)
A missile employing PNG is not fired at the target but is fired in a direction to lead the target. The initial angle of the missile velocity vector with respect to the line-of-sight is known as the missile lead angle L. The missile is fired at the expected intercept point. For the missile to be on a collision triangle, the theoretical missile lead angle is
$\left.\mathrm{L}=\sin ^{-1}\left(\mathrm{~V}_{\mathrm{T}} \sin (\beta+\lambda)\right) / \mathrm{V}_{\mathrm{M}}\right)$
In actuality, the location of the intercept point can only be approximated since there is no prior knowledge of the target maneuver. Any angular deviation of the missile from the collision triangle is known as heading error HE. The initial missile velocity components can therefore be calculated as
$\mathrm{V}_{\mathrm{M} 1}(0)=\mathrm{V}_{\mathrm{M}} \cos (\mathrm{L}+\mathrm{HE}+\lambda)$
$\mathrm{V}_{\mathrm{M} 2}(0)=\mathrm{V}_{\mathrm{M}} \sin (\mathrm{L}+\mathrm{HE}+\lambda)$
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## Simulation of Proportional navigation Guidance in 3-D

In the previous section, two-dimensional proportional navigation $(P N)$ guidance law for intercepting airborne targets is given. The proportional navigation law in three dimensions shows that it is necessary to measure the $L O S$ angular rate $d \lambda / d t$ in two axes that are orthogonal to the Line of Sight (LOS) to the target. Proportional navigation system geometry is shown Figure 2.2

The relative missile-target separations are
$\mathrm{R}_{\mathrm{TM} 1}=\mathrm{X}_{\mathrm{t}}-\mathrm{X}_{\mathrm{m}}$
$\mathrm{R}_{\mathrm{TM} 2}=\mathrm{Y}_{\mathrm{t}}-\mathrm{Y}_{\mathrm{m}}$.
$\mathrm{R}_{\mathrm{TM} 3}=\mathrm{Z}_{\mathrm{t}}-\mathrm{Z}_{\mathrm{m}}$


Figure 2.2. Three-dimensional missile-target engagement geometry
The relative velocity components in earth coordinates are defined as
$\mathrm{V}_{\mathrm{TM1}}=\mathrm{V}_{\mathrm{xt}}-\mathrm{V}_{\mathrm{xm}}$
$\mathrm{V}_{\mathrm{TM} 2}=\mathrm{V}_{\mathrm{yt}}-\mathrm{V}_{\mathrm{mm}}$
$\mathrm{V}_{\mathrm{TM} 3}=\mathrm{V}_{\mathrm{zt}}-\mathrm{V}_{\mathrm{zm}}$
The relative separation between the missile and the target, $\mathrm{R}_{\mathrm{TM}}$, can be expressed as
$R_{\text {TM }}=\operatorname{sqrt}\left(R^{2}{ }_{\text {TM }}+R^{2}{ }_{\text {TM } 2}+R^{2}{ }_{\text {TM }}\right)$
Since the closing velocity is defined as the negative rate of change of the missile-target separation, it can obtained from differentiating equation
$\mathrm{V}_{\mathrm{c}}=-\mathrm{d}\left(\mathrm{R}_{\mathrm{TM}}\right) / \mathrm{dt}=-\left(\mathrm{R}_{\text {TM1 }} \mathrm{V}_{\text {TM1 }}+\mathrm{R}_{\text {TM } 2} \mathrm{~V}_{\text {TM2 }}++\mathrm{R}_{\text {TМ3 }} \mathrm{V}_{\text {TM }}\right) / \mathrm{R}_{\text {TM }}$ ----- (2.31)
LOS angles in azimuth and elevation plane $\left(\varphi_{1}, \gamma_{1}\right)$ are calculated as
$\varphi_{1}=\tan ^{-1}\left(\mathrm{R}_{\text {TM } 2} / \mathrm{R}_{\text {TM1 }}\right)$
$\begin{array}{ll}\varphi_{1}=\tan & \left(\mathrm{R}_{\text {TМ }} / \mathrm{R}_{\text {TM1 }}\right) \\ \gamma_{1}=\tan ^{-1}\left(\mathrm{R}_{\text {TМ }} / \operatorname{sqrt}\left(\mathrm{R}^{2}{ }_{\text {TM }}+\mathrm{R}^{2}{ }_{\text {TM }}\right)\right) & ------(2.33)\end{array}$
The line-of-sight rates are calculated by direct differentiation of the expression for the line-of-sight angles
$\varphi_{1}$ dot $=\left(\mathrm{R}_{\text {TM1 }} * \mathrm{~V}_{\text {TM } 2}-\mathrm{R}_{\text {TM } 2} * \mathrm{~V}_{\text {TM1 }}\right) /\left(\mathrm{R}^{2}{ }_{\text {TM1 }}+\mathrm{R}^{2}{ }_{\text {TM2 }}\right) \quad$----- (2.34) $\gamma_{\perp}$ dot $=\left(\mathrm{V}_{\mathrm{TM} 3} *\left(\mathrm{R}^{2}{ }_{\mathrm{TM} 1}+\mathrm{R}^{2}{ }_{\mathrm{TM} 2}\right)-\right.$
$\left.\left(\mathrm{R}_{\mathrm{TM} 3} *\left(\mathrm{R}_{\mathrm{TM} 1} * \mathrm{~V}_{\mathrm{TM} 1}+\mathrm{R}_{\mathrm{TM} 2} * \mathrm{~V}_{\mathrm{TM} 2}\right)\right)\right) /\left(\mathrm{R}^{2}{ }_{\mathrm{TM}} *\left(\operatorname{sqrt}\left(\mathrm{R}^{2}{ }_{\mathrm{TM} 1}+\mathrm{R}^{2}{ }_{\mathrm{TM} 2}\right)\right)\right)$
The missile lateral accelerations can be given by
latax_losy $=\mathrm{N} * \mathrm{Vc} * \varphi \_\mathrm{dot}^{*} \cos \left(\gamma_{1}\right)$
latax_losz $=\mathrm{N}^{*} \mathrm{Vc}^{*} \gamma_{1}{ }^{-}$dot

## Simulink Model for 3d Simulation

Simulink includes a comprehensive block library of sinks, sources, linear and nonlinear components, and connectors. We can also customize and create our own blocks. Simulink blocks can be created from MATLAB code using Embedded MATLAB Function. The Embedded MATLAB Function block allows adding MATLAB functions to Simulink models for deployment to embedded processors.

This capability is useful for coding algorithms that are better stated in the textual language of the MATLAB software than in the graphical language of the Simulink product. This block works with a subset of the MATLAB language called the Embedded MATLAB subset, which provides optimizations for generating efficient, production-quality C code for embedded applications. We can specify input and output data to the Embedded MATLAB Function block in the function header as arguments and return value

## III FLOW CHART



Fig 3.1. Flow chart for Design of Proportional Navigation Guidance for Air To Air Missiles
IV Simulation Results
2D Engagements against Various Target Maneuvers Scenario 1: A Constant Velocity Target with missile heading error $\mathrm{HE}=-20 \mathrm{deg}, \mathrm{N}^{\prime}=3$

The simulation is conducted with the missile (interceptor) engaging a constant velocity target with heading error (HE) of -20 deg. The initial conditions are
Initial missile position in x - direction $\mathrm{X}_{\mathrm{m} 0}=0.0$ meter Initial missile position in $y$-direction $\mathrm{Y}_{\mathrm{m} 0}=10000.0$ meter Initial missile velocity $\mathrm{V}_{\mathrm{m} 0}=3000.0$ meter $/ \mathrm{sec}$ Initial target position in x -direction $\mathrm{X}_{\mathrm{t} 0}=40000.0$ meter Initial target position in $y$-direction $\mathrm{Y}_{\mathrm{t} 0}=10000.0$ meter


Figure 4.1 Scenario 1. Missile and Target x-position Initial target velocity $\mathrm{Vm} 0=1000.0$ meter/sec Heading Error HE=-20 deg Initial target flight path angle $\beta=0.0 \mathrm{deg}$


Figure 4.2. scenario 1. Missile and Target y-position


Figure 4.3. Scenario 1: 2D Missile and Target Engagement


Figure 4.4. Scenario 1: Missile and Target x-velocity


Figure 4.5. Scenario 1: Missile and Target y-velocity


Figure 4.6. Scenario 1: Closing Range (Miss Distance)


Figure 4.7. Scenario 2: 2D Missile and Target Engagement


Figure 4.8. Scenario 3: Missile and Target x-position


Figure 4.9. Scenario 3: Missile and Target y-position


Figure 4.10. Scenario 3: Missile and Target z-position


Figure 4.11. Scenario 3: 3D Missile and Target Engagement

## V Conclusion

The simulations developed in this work provide a very good generic model which can be easily modified for more complex missile systems. The simulation can be taken to several different levels. The target flight can be modified for different engagement scenarios. In this work we made an assumption that flight control system is unity. In the future, the simulation can be developed with the actual auto pilot, actuator and missile models. It will give further insight into the system.

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