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The Effect of the Rotation on the Onset of Convection in a Hele-Shaw Cell Saturated by a Newtonian Nanofluid: A Revised Model

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ABSTRACT

The aim of this paper, is to use a more realistic model which incorporates the effects of Brownian motion and the thermophoresis of nanoparticles for studying the effect of some control parameters on the onset of convective instability in a rotating Hele-Shaw cell filled of a Newtonian nanofluid layer and heated from below, this layer is assumed to have a low concentration of nanoparticles. The linear study which was achieved in this investigation shows that the thermal stability of Newtonian nanofluids depends of the Coriolis force generated by the rotation of the system, the Hele-Shaw cell parameter, the Brownian motion, the thermophoresis of nanoparticles, the buoyancy forces and other thermo-physical properties of nanoparticles. The studied problem will be solved analytically by converting our boundary value problem to an initial value problem, after this step we will approach numerically the searched solutions by polynomials of high degree to obtain a fifth-order-accurate solution.

Introduction

The nanofluid is considered as a homogeneous fluid containing colloidal suspensions of nano-sized particles named nanoparticles in the base fluid (water, ethylene glycol, oil). The nanoparticles used in nanofluids are generally prepared of metals, oxides, carbides, or carbon nanotubes. The purpose of using nanofluids is to obtain a higher value of heat transfer coefficient compared with that of the base fluid , this remarkable properties make them potentially useful in many practical applications , for example in modern science and engineering including rotating machineries like nuclear reactors, petroleum industry, biochemical and geophysical problems.

In the recent years, the problem of natural convection in a confined medium filled of a Newtonian nanofluid layer has been studied in different situations by several authors **[1-7]**. When the volumetric fraction of nanoparticles is constant at the horizontal walls limiting the layer, they found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on the relative distribution of nanoparticles between the top and bottom walls.

Today, the problem of natural convection for the nanofluids is studied by some authors **[8-13]** using a new type of boundary conditions for the nanoparticles which combines the contribution of the Brownian motion and the thermophoresis of the nanoparticles instead to impose a nanoparticle volume fraction at the boundaries of the layer.

The new model of boundary conditions assumes that the nanoparticle flux must be zero on the impermeable boundaries. D.A. Nield and A.V. Kuznetsov **[8]** are considered as the first ones who were used this type of boundary conditions for the nanoparticles. Until now, the precedent boundary conditions are used to study only the problem of natural convection in a porous (Darcy or Brinkman model) or non-porous medium Saturated by a nanofluid using the Galerkin weighted residuals method based only on some test functions.

Our work consists of studying the Rayleigh-Bénard problem in a rotating Hele-Shaw cell filled of a Newtonian nanofluid layer in the rigid-rigid case where the nanoparticle flux is assumed to be zero on the boundaries, our problem will be solved with a more accurate numerical method based on analytic approximations (power series method). In this investigation we assume that the effect of the rotation in the momentum equation is restricted to the Coriolis force and also the centrifugal acceleration is negligible compared to the buoyancy force. In this investigation we assume that the nanofluid is Newtonian and the parameters which appear in the governing equations are considered constant in the vicinity of the temperature of the cold wall $\mathbf{T_c^*}$ which we took it as a reference temperature. Finally we will impose that the flow is laminar and the radiation heat transfer mode between the horizontal walls will be negligible compared to other modes of heat transfer.

To show the accuracy of our method in this study, we will check some results treated by Chandrasekhar **[14]** , Guo and Kaloni **[15]** and D.Yadav and J. Lee **[16]** concerning the study of the convective instability of the regular fluids .Our numerical method is used in this investigation to give results with an absolute error of the order of 10^{-6} to the exact critical values characterizing the onset of the convection.

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Mathematical Formulation:

We consider a dilute layer saturated by an incompressible Newtonian nanofluid, vertically confined between two parallel rigid impermeable boundaries ($z^* = 0$ and $z^* = d$) and heated uniformly from below where the temperature is constant and the nanoparticle flux is zero on the boundaries (Fig 1), such that:

$$w^* = \frac{\partial w^*}{\partial z^*} = \frac{\partial v^*}{\partial x^*} - \frac{\partial u^*}{\partial y^*} = 0 \;\; ; \;\; T^* = T_0^* \;\; ; \;\; D_B \frac{\partial \chi^*}{\partial z^*} + \frac{D_T}{T_0^*}\frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = 0; 1$$

The nanofluid layer shall be infinitely extended in the x^*direction but confined in the y^* direction by vertical impermeable boundaries ($y^* = 0$ and $y^* = b$) such that $b \ll d$, this layer will be subjected to a uniform rotation characterized by an angular velocity $\vec{\Omega} = \Omega \vec{e}_z$ and also acted upon by the gravity force $\vec{g} = -g\vec{e}_z$ .The thermo-physical properties of nanofluid (viscosity, thermal conductivity, specific heat) are assumed constant in the vicinity of the temperature of the cold wall T_c^* except for the density variation in the momentum equation which is based on the Boussinesq approximations .The asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks).

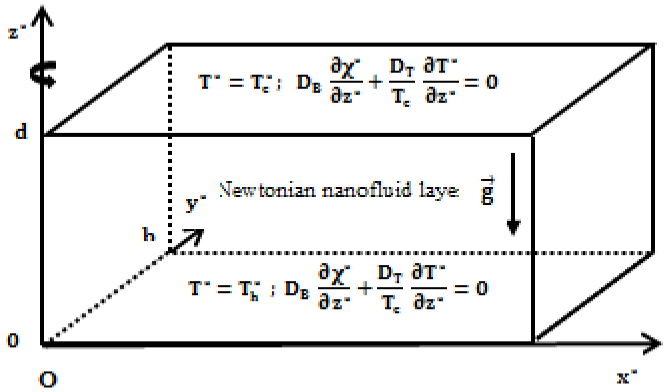

Figure 1. Physical configuration .

When the Hele-Shaw cell gap width is not sufficiently small with regard to the appearing wavelength of the instability, the correction to Darcy's law is needed **[15]** .Within the framework of the assumptions which were made by Buongiorno **[3]** and Tzou **[4,5]** in their publications for the Newtonian nanofluids we can write the basic equations of conservation which govern our problem under the Boussinesq and Hele-Shaw approximation in dimensionless form as follows:

$$\vec{\nabla}^*.\vec{V}^* = 0 \qquad (1)$$

$$\rho_0\left[\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\vec{V}^*\right] = -\vec{\nabla}^* P^* + \mu \vec{\nabla}^{*2}\vec{V}^* - \frac{\mu}{K}\vec{V}^* - 2\rho_0\vec{\Omega}\times\vec{V}^* + \{\rho_0[1-\beta(T^* - T_c^*)](1-\chi^*) + \rho_p\chi^*\}\vec{g} \qquad (2)$$

$$(\rho c)\left[\frac{\partial T^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)T^*\right] = \kappa \vec{\nabla}^{*2} T^* + (\rho c)_p\left[D_B \vec{\nabla}^*\chi^*.\vec{\nabla}^* T^* + \left(\frac{D_T}{T_c^*}\right)\vec{\nabla}^* T^*.\vec{\nabla}^* T^*\right] \qquad (3)$$

$$\frac{\partial \chi^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\chi^* = D_B \vec{\nabla}^{*2}\chi^* + \left(\frac{D_T}{T_c^*}\right)\vec{\nabla}^{*2} T^* \qquad (4)$$

Where $\vec{\nabla}^* = \frac{\partial}{\partial x^*}\vec{e}_{x^*} + \frac{\partial}{\partial y^*}\vec{e}_{y^*} + \frac{\partial}{\partial z^*}\vec{e}_{z^*}$ is the vector differential operator.

If we consider the following dimensionless variables:

$$(x^*; y^*; z^*) = d(x; y; z) \;\; ; \;\; t^* = \frac{d^2}{\alpha}t \;\; ; \;\; \vec{V}^* = \frac{\alpha}{d}\vec{V} \;\; ; \;\; P^* = \frac{\mu\alpha}{d^2}P \;\; ; \;\; T^* - T_c^* = (T_h^* - T_c^*)T \;\; ; \;\; \chi^* - \chi_0^* = \chi_0^*\chi$$

Then, we can get from the equations (1)-(4) the following adimensional forms:

$$\vec{\nabla}.\vec{V} = 0 \qquad (5)$$

$$P_r^{-1}\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -\vec{\nabla}(P + R_M z) + \vec{\nabla}^2\vec{V} - P_{HS}\vec{V} + \sqrt{T_A}(v\vec{e}_x - u\vec{e}_y) + [(1-\chi_0^*)P_{HS}R_{HS}T - R_N\chi - \chi_0^* P_{HS}R_{HS}T\chi]\vec{e}_z \qquad (6)$$

$$\frac{\partial T}{\partial t} + (\vec{V}.\vec{\nabla})T = \vec{\nabla}^2 T + N_B L_e^{-1}\vec{\nabla}\chi.\vec{\nabla}T + N_A N_B L_e^{-1}\vec{\nabla}T.\vec{\nabla}T \qquad (7)$$

$$\frac{\partial \chi}{\partial t} + (\vec{V}.\vec{\nabla})\chi = L_e^{-1}\vec{\nabla}^2\chi + N_A L_e^{-1}\vec{\nabla}^2 T \qquad (8)$$

Such that:

$$K=\frac{b^2}{12}\;;\;P_{HS}=\frac{d^2}{K}\;;\;P_r=\frac{\mu}{\rho_0\alpha}\;;\;L_e=\frac{\alpha}{D_B}\;;\;N_B=\frac{(\rho c)_p}{(\rho c)}\chi_0^*\;;\;T_A=\left(\frac{2\rho_0\Omega d^2}{\mu}\right)^2\;;\;\alpha=\frac{\kappa}{(\rho c)}$$

$$R_{HS}=\frac{\rho_0 g\beta(T_h^*-T_c^*)Kd}{\mu\alpha}\;;\;R_M=\frac{[\rho_0(1-\chi_0^*)+\rho_p\chi_0^*]gd^3}{\mu\alpha}\;;\;R_N=\frac{(\rho_p-\rho_0)\chi_0^* gd^3}{\mu\alpha}\;;\;N_A=\frac{D_T}{D_B T_c^*}\left(\frac{T_h^*-T_c^*}{\chi_0^*}\right)$$

Stability Analysis

Basic Solutions

The basic solution of our problem is a quiescent thermal equilibrium state, it's assumed to be independent of time where the equilibrium variables are varying only in the z-direction, therefore:

$$\vec{V}_b=\vec{0} \tag{9}$$

$$T_b=1\;;\;\frac{d\chi_b}{dz}+N_A\frac{dT_b}{dz}=0\quad\text{at}\quad z=0 \tag{10}$$

$$T_b=0\;;\;\frac{d\chi_b}{dz}+N_A\frac{dT_b}{dz}=0\quad\text{at}\quad z=1 \tag{11}$$

If we introduce the precedent results into equations (6)-(8), we obtain:

$$\vec{\nabla}(P_b+R_M z)=[(1-\chi_0^*)P_{HS}R_{HS}T-R_N\chi-\chi_0^*P_{HS}R_{HS}T\chi]\vec{e}_z \tag{12}$$

$$\frac{d^2T_b}{dz^2}+N_B L_e^{-1}\left(\frac{d\chi_b}{dz}\frac{dT_b}{dz}\right)+N_A N_B L_e^{-1}\left(\frac{dT_b}{dz}\right)^2=0 \tag{13}$$

$$\frac{d^2\chi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0 \tag{14}$$

After using the boundary conditions (10) and (11), we can integrate the equation (14) between 0 and z for obtaining:

$$\chi_b=N_A(1-T_b)+\chi_0 \tag{15}$$

Where χ_0 is the relative nanoparticle volume fraction at $z=0$, such that: $\chi_0=(\chi^*(0)-\chi_0^*)/\chi_0^*$.

If we take into account the expression (15), we can get after simplification of the equation (13):

$$\frac{d^2T_b}{dz^2}=0 \tag{16}$$

Finally, we obtain after an integrating of the equation (16) between 0 and 1:

$$T_b=1-z \tag{17}$$

$$\chi_b=N_A z+\chi_0 \tag{18}$$

Perturbation of the basic state

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows:

$$T=T_b+T'\;;\;\vec{V}=\vec{V}_b+\vec{V}'\;;\;P=P_b+P'\;;\;\chi=\chi_b+\chi' \tag{19}$$

In the framework of the Oberbeck-Boussinesq approximations, we can neglect the terms coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (6), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (19) into equations (5)-(8) the following linearized equations:

$$\vec{\nabla}.\vec{V}'=0 \tag{20}$$

$$P_r^{-1}\frac{\partial\vec{V}'}{\partial t}=-\vec{\nabla}P'+\vec{\nabla}^2\vec{V}'-P_{HS}\vec{V}'+\sqrt{T_A}(v'\vec{e}_x-u'\vec{e}_y)+(P_{HS}R_{HS}T'-R_N\chi')\vec{e}_z \tag{21}$$

$$\frac{\partial T'}{\partial t}-w'=\vec{\nabla}^2T'-N_A N_B L_e^{-1}\frac{\partial T'}{\partial z}-N_B L_e^{-1}\frac{\partial\chi'}{\partial z} \tag{22}$$

$$\frac{\partial\chi'}{\partial t}+N_A w'=N_A L_e^{-1}\vec{\nabla}^2T'+L_e^{-1}\vec{\nabla}^2\chi' \tag{23}$$

After application of the curl operator twice to the equation (21) and using the equation (20), we obtain the following equations:

$$P_r^{-1}\frac{\partial F'}{\partial t}=\vec{\nabla}^2F'-P_{HS}F'+\sqrt{T_A}\frac{\partial w'}{\partial z} \tag{24}$$

$$P_r^{-1}\frac{\partial}{\partial t}\vec{\nabla}^2w'=\vec{\nabla}^4w'-P_{HS}\vec{\nabla}^2w'+P_{HS}R_{HS}\vec{\nabla}_2^2T'-R_N\vec{\nabla}_2^2\chi'-\sqrt{T_A}\frac{\partial F'}{\partial z} \tag{25}$$

Where $\vec{\nabla}_2^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$ and $\mathbf{F}' = \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)$.

Analyzing the disturbances into normal modes, we can simplify the equations (22) - (25) by assuming that the perturbation quantities are of the form:

$$(\mathbf{w}', \mathbf{T}', \chi', \mathbf{F}') = \left(\mathcal{W}(z), \mathcal{T}(z), \mathcal{X}(z), \mathcal{F}(z)\right)\exp\left[i\left(a_x x + a_y y\right) + nt\right] \quad (26)$$

After introducing the expressions (26) into equations (22) - (25), we obtain:

$$P_r^{-1} n\mathcal{F} = (D^2 - a^2)\mathcal{F} - P_{HS}\mathcal{F} + \sqrt{T_A} D\mathcal{W} \quad (27)$$

$$P_r^{-1} n(D^2 - k^2)\mathcal{W} = (D^2 - a^2)^2\mathcal{W} - P_{HS}(D^2 - a^2)\mathcal{W} - a^2 P_{HS} R_{HS}\mathcal{T} + a^2 R_N \mathcal{X} - \sqrt{T_A}\, D\mathcal{F} \quad (28)$$

$$n\mathcal{T} - \mathcal{W} = (D^2 - a^2)\mathcal{T} - N_A N_B L_e^{-1} D\mathcal{T} - N_B L_e^{-1} D\mathcal{X} \quad (29)$$

$$n\mathcal{X} + N_A\mathcal{W} = N_A L_e^{-1}(D^2 - a^2)\mathcal{T} + L_e^{-1}(D^2 - a^2)\mathcal{X} \quad (30)$$

Where $a = \sqrt{a_x^2 + a_y^2}$ and $D = d/dz$.

The equations (27) - (30) will be solved subject to the following rigid-rigid boundary conditions:

$$\mathcal{W} = D\mathcal{W} = \mathcal{F} = \mathcal{T} = D(\mathcal{X} + N_A\mathcal{T}) = 0 \qquad \text{at} \quad z = 0\,;1 \quad (31)$$

Method of Solution

Numerical Method

Very recently, Nield and Kuznetsov **[8]** and Agarwal **[9]** observed that the oscillatory convection is ruled out for nanofluids with this new type of boundary conditions due to very large nanofluid Lewis number, so the stationary convection $(n = 0)$ is the predominant mode. Hence, the equations (27)-(30) become:

$$\sqrt{T_A} D\mathcal{W} + (D^2 - a^2)\mathcal{F} - P_{HS}\mathcal{F} = 0 \quad (32)$$

$$(D^2 - a^2)^2\mathcal{W} - P_{HS}(D^2 - a^2)\mathcal{W} - a^2 P_{HS} R_{HS}\mathcal{T} + a^2 R_N\mathcal{X} - \sqrt{T_A}\, D\mathcal{F} = 0 \quad (33)$$

$$\mathcal{W} + (D^2 - a^2)\mathcal{T} - N_A N_B L_e^{-1} D\mathcal{T} - N_B L_e^{-1} D\mathcal{X} = 0 \quad (34)$$

$$N_A\mathcal{W} - N_A L_e^{-1}(D^2 - a^2)\mathcal{T} - L_e^{-1}(D^2 - a^2)\mathcal{X} = 0 \quad (35)$$

We can solve the equations (32)-(35) which are subjected to the conditions (31), by making a suitable change of variables that makes the number of variables equal to the number of boundary conditions to obtain a set of ten first order ordinary differential equations which we can write it in the following form:

$$\frac{d}{dz} u_i(z) = a_{ij} u_j(z)\,; \quad 1 \le i, j \le 10 \quad (36)$$

The solution of the system (36) in matrix notation can be written as follows:

$$U = \exp(A)C \quad (37)$$

Where $B = \left(\left(b_{ij}(z)\right)_{\substack{1\le i\le 10\\ 1\le j\le 10}}\right)$, $U = \left(\left(u_i(z)\right)_{1\le i\le 10}\right)^T$ and $C = \left(\left(c_j\right)_{1\le j\le 10}\right)^T$.

If we assume that the matrix $\mathbf{B}$ is written in the following form:

$$B = \left(\left(u_i^j(z)\right)_{\substack{1\le i\le 10\\ 1\le j\le 10}}\right) \quad (38)$$

Therefore, the use of five boundary conditions at $z = 0$, allows us to write each variable $u_i(z)$ as a linear combination for five functions $u_i^j(z)$, such that:

$$b_{ij}(0) = u_i^j(0) = \delta_{ij} \quad (39)$$

Where δ_{ij} is the Kronecker delta symbol .

After introducing the new expressions of the variables $u_i(z)$ in the system (36), we will obtain the following equations:

$$\frac{d}{dz} u_i^j(z) = a_{il} u_l^j(z)\,; \quad 1 \le i, l, j \le 10 \quad (40)$$

For each value of j , we must solve a set of ten first order ordinary differential equations which are subjected to the initial conditions (39) , by approaching these variables with power series defined in the interval [0,1] and truncated at the order N , such that:

$$u_1^j(z) = \sum_{p=0}^{p=N} d_p^{1,j} z^p \qquad (41)$$

A linear combination of the solutions $u_1^j(z)$ satisfying the boundary conditions (31) at $z = 1$ leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_{HS}, a, T_A, P_{HS}, N_B, L_e, R_N, N_A) = 0 \qquad (42)$$

If we give to each control parameter $(T_A, P_{HS}, N_B, L_e, R_N, N_A)$ its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number R_{HS} which corresponds to a fixed wave number a and verifies the dispersion relation (42). After that, we will find a set of points (a, R_{HS}) which help us to plot our curve and find the critical value (a_c, R_{HSC}) which characterizes the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

Validation of the Method

The main aim of our study consists to study the influence of a uniform rotation on the convective instability in a Hele-Shaw cell filled of a Newtonian nanofluid layer in the rigid-rigid case. Our study shows that the thermal stability of Newtonian nanofluids depends on six parameters: $T_A, P_{HS}, N_B, L_e, R_N$ and N_A.

The truncation order N which corresponds at the convergence of our method is determined, when the five digits after the comma of the critical Hele-Shaw Rayleigh number R_{HSC} for the nanofluids and the regular fluids remain unchanged (Tables 1 and 2) .To validate our method, we compared our results respectively with those obtained by Chandrasekhar [14] , Guo and Kaloni [15] and D.Yadav and J. Lee [16] concerning the classical Rayleigh-Bénard problem in a rotating medium and the Rayleigh-Bénard problem in a Hele-Shaw cell in a non-rotating medium for the regular fluids (Tables 3 and 4).

Table 1. The stationary instability threshold of the regular fluids for different values of the Taylor number T_A and the truncation order N in the case where P_{HS} =10

N	$T_A = 6000$			$T_A = 8000$			$T_A = 10000$	
	a_c	R_{HSC}		a_c	R_{HSC}		a_c	R_{HSC}
36	4.15175	395.50422		4.34366	442.72134		4.50613	486.57282
37	4.15175	395.50419		4.34367	442.72113		4.50620	486.57215
38	4.15175	395.50419		4.34367	442.72114		4.50619	486.57197
39	4.15175	395.50419		4.34367	442.72115		4.50619	486.57210
40	4.15175	395.50419		4.34367	442.72114		4.50619	486.57206
41	4.15175	395.50419		4.34367	442.72115		4.50619	486.57207
42	4.15175	395.50419		4.34367	442.72115		4.50619	486.57207
43	4.15175	395.50419		4.34367	442.72115		4.50619	486.57207
44	4.15175	395.50419		4.34367	442.72115		4.50619	486.57207
45	4.15175	395.50419		4.34367	442.72115		4.50619	486.57207
46	4.15175	395.50419		4.34367	442.72115		4.50619	486.57207

Table 2. The stationary instability threshold of a nanofluid (H_2O + Al_2O_3) characterized by N_B=0.0075, Le=5000, R_N=0.1 and N_A=5 (D.Yadav et al. [17]) for different values of the Taylor number T_A and the truncation order N in the case where P_{HS} =10

N	$T_A = 6000$			$T_A = 8000$			$T_A = 10000$	
	a_c	R_{HSC}		a_c	R_{HSC}		a_c	R_{HSC}
36	2.33515	29.14817		3.28921	103.03361		3.70296	159.09402
37	2.33515	29.14818		3.28922	103.03379		3.70297	159.09534
38	2.33515	29.14818		3.28922	103.03374		3.70298	159.09493
39	2.33515	29.14818		3.28922	103.03375		3.70297	159.09501
40	2.33515	29.14818		3.28922	103.03375		3.70297	159.09501
41	2.33515	29.14818		3.28922	103.03375		3.70297	159.09500
42	2.33515	29.14818		3.28922	103.03375		3.70297	159.09500
43	2.33515	29.14818		3.28922	103.03375		3.70297	159.09500
44	2.33515	29.14818		3.28922	103.03375		3.70297	159.09500
45	2.33515	29.14818		3.28922	103.03375		3.70297	159.09500
46	2.33515	29.14818		3.28922	103.03375		3.70297	159.09500

Table 3. The comparison of critical values of Rayleigh number and the corresponding wave number with Chandrasekhar [13] for the regular fluids for different values of the Taylor number T_A in the case where P_{HS} = 0

T_A	Chandrasekhar			Present study		
	a_c	R_{HSC}		a_c	R_{HSC}	N
0	3.117	1707.762		3.11632	1707.76177	28
10	3.10	1713		3.12087	1712.67407	28
100	3.15	1756.6		3.16081	1756.34730	27
500	3.30	1940.5		3.31925	1940.19924	29
1000	3.50	2151.7		3.48471	2151.34119	30
10000	4.80	4713.10		4.78484	4712.04201	42

Table 4. The comparison of critical values of Rayleigh number and the corresponding wave number with Guo and Kaloni [14] and D.Yadav and J. Lee [15] for the regular fluids for different values of the Hele-Shaw cell parameter P_{HS} in the case where $T_A = 0$

P_{HS}	Guo and Kaloni		D.Yadav and J. Lee		Present study		
	a_c^2	R_{HSC}	a_c^2	R_{HSC}	a_c^2	R_{HSC}	N
1	9.70	1752.20	9.734	1751.871	9.73587	1752.21039	28
10	9.92	215.06	9.923	215.02	9.91699	215.06029	28
100	10.47	60.36	10.439	60.395	10.43952	60.39782	38

According to the above results, we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. In this study we find that the convergence of the results depends greatly on the truncation order N of the power series, the Hele-Shaw cell parameter P_{HS} and also of the Taylor number T_A .

5. Results and Discussion:

To study the effect of a parameter $(P_{HS}, N_B, L_e, R_N, N_A)$ on the onset of the convective instability in a rotating Hele-Shaw cell filled of a Newtonian nanofluid layer (for example : Water + Alumina) we must fix the others and determine the variation of the critical Hele-Shaw Rayleigh number R_{HSC} as a function of the Taylor number T_A in the interval $[6 \times 10^3, 10^4]$ for different values of this parameter (Figs 2-6) and then compare the obtained results with those of the regular fluids. To ensure the accuracy of our study, we will take as truncation order:

- $N = 43$ for the regular fluids.
- $N = 44$ for the nanofluids.

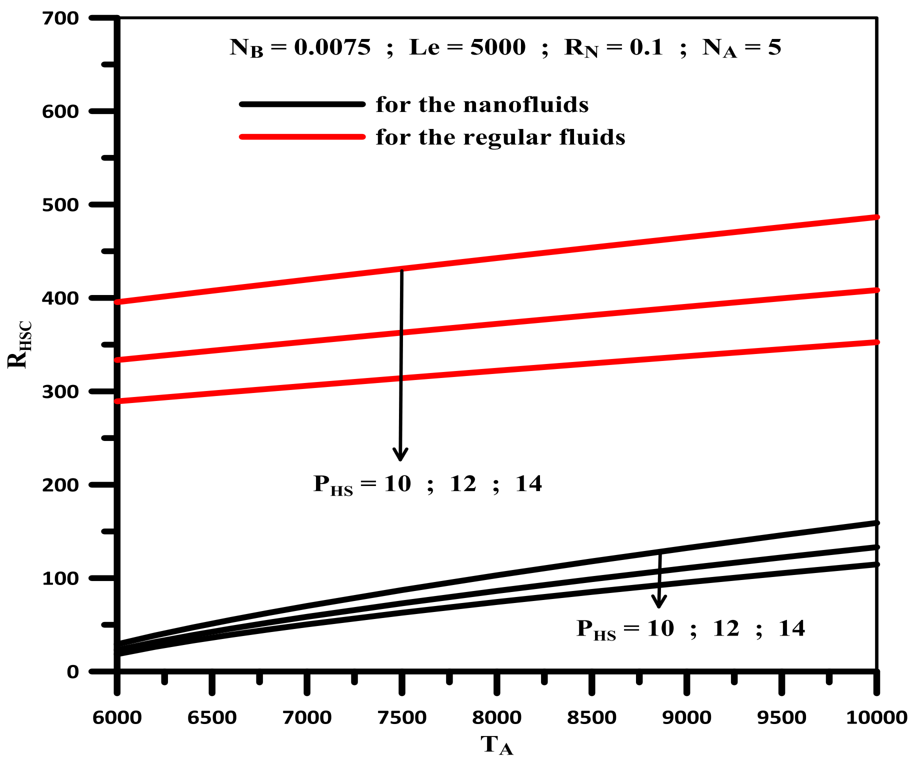

Figure 2. Plot of R_{HSC} as a function of T_A for different values of P_{HS}

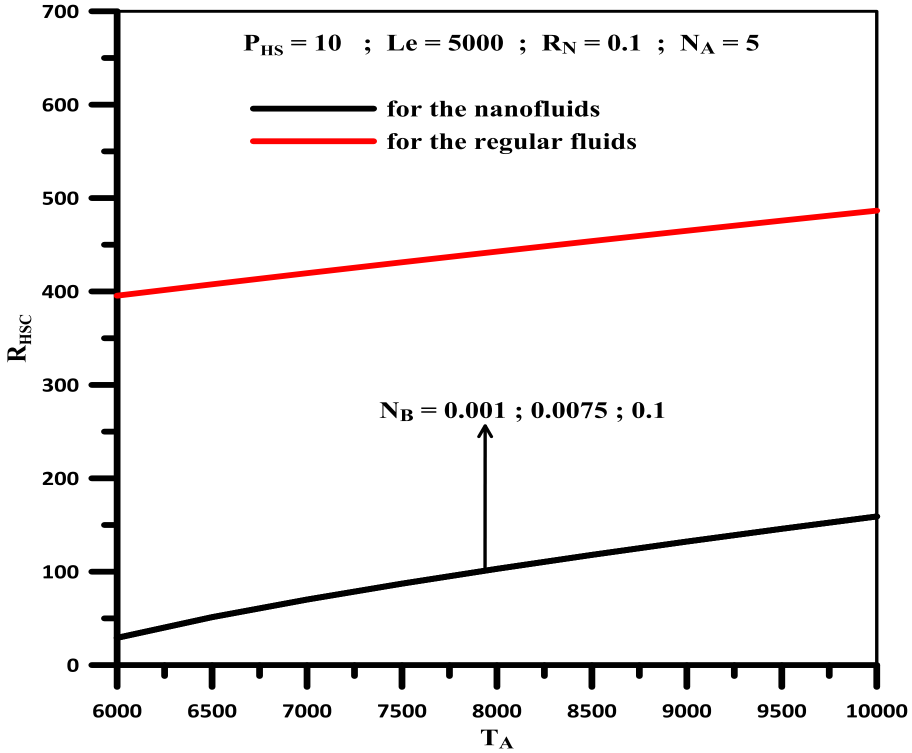

Figure 3. Plot of R_{HSC} as a function of T_A for different values of N_B

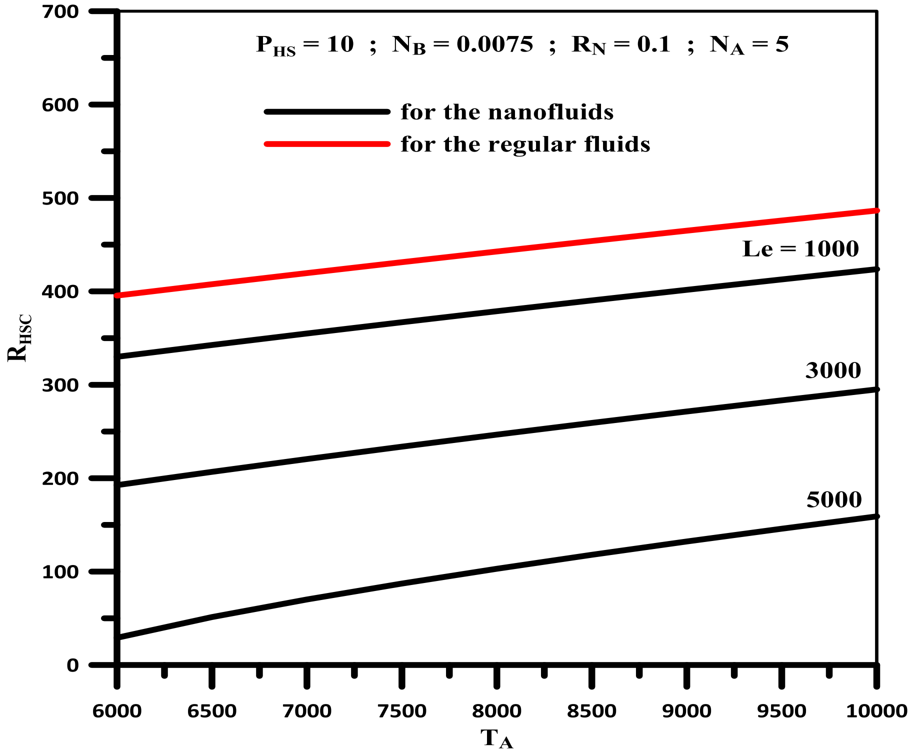

Figure 4. Plot of R_{HSC} as a function of T_A for different values of Le

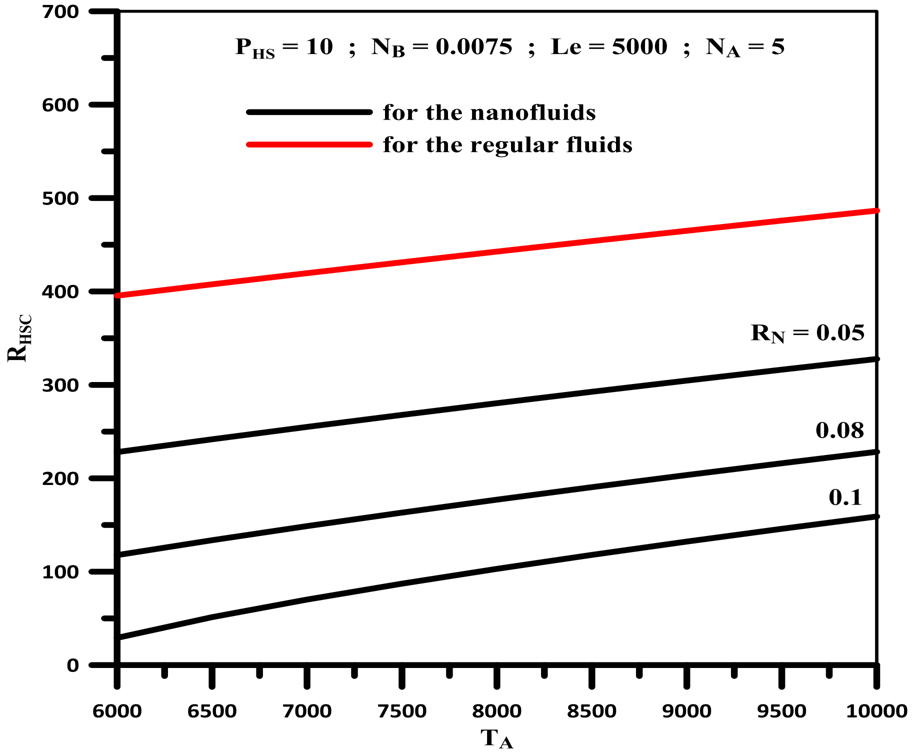

Figure 5. Plot of R_{HSC} as a function of T_A for different values of R_N

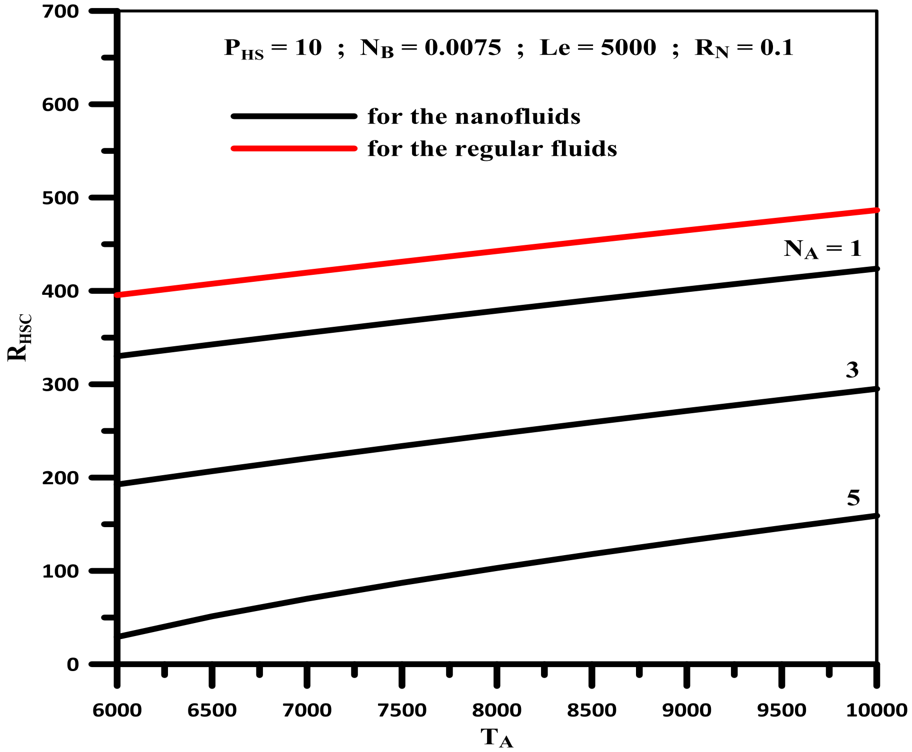

Figure 6. Plot of R_{HSC} as a function of T_A for different values of N_A

Generally the variation in the critical Hele-Shaw Rayleigh number R_{HSC} with the Taylor number T_A is an increasing function whatever the value taken for the parameters P_{HS} , N_B , L_e , R_N and N_A , so the presence of the Coriolis forces allows us to reduce the effect of the buoyancy forces , hence the Taylor number T_A has a stabilizing effect.

From the Fig 2 we conclude that an increase in the Hele-Shaw cell parameter P_{HS} allows us to accelerate the onset of the convection, so the permeability of the Hele-Shaw cell K has a stabilizing effect. Hence, an infinite horizontal layer is more stable compared with that in the Hele-Shaw cell.

The Fig 3 shows that the modified particle-density increment N_B has almost no effect on the convective instability of the nanofluids, this result may be explained by its low value $(N_B \sim 10^{-3} - 10^{-1})$ which appears only in the perturbed energy equation (22) as a product with the inverse of the Lewis number $(L_e \sim 10^2 - 10^3)$ near the temperature gradient and the volume fraction gradient of nanoparticles, so the effect of this parameter on the onset of convection in nanofluids will be very small which we can neglect it .

From The Figs 4 and 5 we conclude that an increase either in the Lewis number L_e or in the concentration Rayleigh number R_N allows us to accelerate the onset of the convection, hence they have a destabilizing effect .Therefore, to ensure the stability of the nanofluids, we can use the nanofluids which are having a less thermal diffusivity or containing less dense nanoparticles.

In this investigation, we find also that an increase in the volume fraction of nanoparticles destabilizes the nanofluids, because an increase in this parameter, increases also the Brownian motion and the thermophoresis of nanoparticles, which cause the destabilizing effect, this result confirm that the regular fluids are more stable than the nanofluids.

When the modified diffusivity ratio N_A increases, the temperature difference between the horizontal plates also increases. The Fig 6 shows that an increase in the modified diffusivity ratio N_A allows us to decrease the critical Hele-Shaw Rayleigh number R_{HSC} ,this result can be explained by the increase in the Buoyancy forces which destabilizes the system.

Conclusions

In this paper, we have examined the effect of a uniform rotation on the onset of convection in a Hele-Shaw cell filled of a Newtonian nanofluid layer, heated uniformly from below and cooled from above for rigid-rigid boundaries in the case where the nanoparticle flux is zero on the boundaries. The contribution of the Brownian motion and the thermophoresis in the equation expressing the buoyancy effect coupled with the conservation of nanoparticles have a major effect on the onset of convection compared with their contributions in the thermal energy equation .

The resulting eigenvalue problem is solved analytically and numerically using the power series method (PSM). The behavior of various parameters like the Taylor number T_A , the Hele-Shaw cell parameter P_{HS} , the modified particle-density increment N_B , the Lewis number L_e , the concentration Rayleigh number R_N and the modified diffusivity ratio N_A on the onset of convection has been analysed. The principal results of this investigation can be summarized as follows:

- The presence of the Coriolis forces allows us to stabilize the nanofluids, such that an increase in the Taylor number T_A induces also an increase in the critical Hele-Shaw Rayleigh number R_{HSC}.
- An increase in the permeability of the Hele-Shaw cell K allows us to increase also the critical Hele-Shaw Rayleigh number R_{HSC} . Hence, this parameter has a stabilizing effect.
- An infinite horizontal layer of a nanofluid (also of a regular fluid) is more stable compared with that in the Hele-Shaw cell.
- To ensure the stability of the nanofluids, we can use the nanofluids which are having a less thermal diffusivity, a low concentration of nanoparticles or consisting of less dense nanoparticles.
- An increase in the volume fraction of nanoparticles, in the Brownian motion, in the thermophoresis of nanoparticles or in the Buoyancy forces allows us to destabilize the nanofluids.
- The regular fluids are more stable than the nanofluids.

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