

Rotation Effect on Unsteady MHD Flow Past an Impulsively Started Vertical Plate with Variable Mass diffusion in Porous Medium

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ABSTRACT

This paper deals with rotation effects on unsteady free convective flow past an impulsively started vertical plate with variable mass diffusion in the presence of uniform magnetic field. The problem is solved analytically using the Laplace transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction have been tabulated.

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Introduction

Natural convection flow occurs due to concentration differences, due to temperature differences or the combination of the two. Many transport processes frequently exist in chemical processing industries such as food processing and polymer production etc. Convection in porous media has great applications particularly in the field of agricultural engineering for irrigation processes; in the petroleum technology to study the petroleum transport; in the chemical engineering for filtration and purification processes, and the flow through filtering devices etc. Further, MHD flows are of significant interest and have been attracting the attention of many researchers due to their applications in the fast growing fields of science and technology. It is applied to study the stellar and solar structure, interstellar matter, radio propagation through ionosphere, in MHD pumps, MHD bearings etc.

In view of the important applications in industry and engineering, the study of uniform MHD flow on bodies of various geometries has been carried out by many researchers. For instance, Stewartson[1] worked on the impulsive motion of a flat plate in a viscous fluid. Deka *et al*[4] studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Raptis and Perdikis[6] have considered the radiation and free convection flow past a moving plate through a porous medium bounded by an infinite vertical plate. A boundary layer analysis for the natural convection past a horizontal plate in a porous medium saturated with a nanofluid is analyzed by Gorla and Chamkha[12]. Further, Ibrahim and Makinde[8] considered chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Again, they[10] studied the radiation effect on chemically reacting magnetohydrodynamics (MHD) boundary layer flow of heat and mass transfer through a porous vertical flat plate. Chamkha *et al*[9] worked on the effect of thermally stratified ambient fluid on MHD convective flow along a moving non-isothermal vertical plate. Radiation effect on

mixed convection along a vertical plate with uniform surface temperature was studied by Hossain and Takhar[5]. Chamkha[7] considered hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through a porous medium. Effects of mass transfer and free convection on the flow past an impulsively started vertical plate was studied by Soundalgekar[2]. Further the Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha *et al*[3]. Further Rajput and Kumar[11] considered rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. Recently Das *et al*[13] considered unsteady hydromagnetic flow of a heat absorbing dust fluid past a permeable vertical plate with ramped temperature.

This paper investigates the rotation effects on unsteady MHD flow past an impulsively started vertical plate with variable mass diffusion in porous medium. The results are shown with the help of graphs and table.

Mathematical Analysis

Consider an unsteady MHD flow of a viscous incompressible electrically conducting fluid past an impulsively started vertical plate with variable mass diffusion. The fluid and the plate rotate as a rigid body with a uniform angular velocity $\bar{\omega}$ about \bar{z} -axis in the presence of an applied uniform magnetic field B_0 normal to the plate. The fluid motion is induced due to the impulsive movement of the plate as well as the free convection due to heating of the plate. Initially, at time $\bar{t} \leq 0$, the fluid and the plate are at rest and at a uniform concentration \bar{C}_∞ . At time $\bar{t} > 0$, the plate starts moving with a velocity u_0 in its own plane and the concentration of the plate is raised to \bar{C}_w . As the plate occupying the plane $\bar{z}=0$ is of infinite extent, all the physical quantities depend only on \bar{z} and \bar{z} . As the fluid is electrically conducting whose magnetic Reynolds number is very small and hence the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one.

Under the above assumptions, the governing equations with Boussinesq's approximations are as follows:

$$\frac{\partial \bar{u}}{\partial t} - 2\bar{\Omega}\bar{v} = g\beta(\bar{C} - \bar{C}_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial z^2} - \frac{\sigma B_o^2}{\rho} \bar{u} - \frac{\mu}{K} \bar{u}, \quad (1)$$

$$\frac{\partial \bar{v}}{\partial t} - 2\bar{\Omega}\bar{u} = \nu \frac{\partial^2 \bar{v}}{\partial z^2} - \frac{\sigma B_o^2}{\rho} \bar{v} - \frac{\mu}{K} \bar{v}, \quad (2)$$

$$\frac{\partial \bar{C}}{\partial t} = D \frac{\partial^2 \bar{C}}{\partial z^2}. \quad (3)$$

The boundary conditions taken are as under

$$\left. \begin{aligned} \bar{t} \leq 0: \bar{u} = 0, \bar{v} = 0, \bar{C} = \bar{C}_\infty \nabla z, \\ \bar{t} > 0: \bar{u} = u_o, \bar{v} = 0, \bar{C} = \bar{C}_\infty + (\bar{C}_w - \bar{C}_\infty) \left(\frac{u_o^2}{\nu} \bar{t} \right) (at \bar{z} = 0), \\ \bar{u} \rightarrow 0, \bar{C} \rightarrow \bar{C}_\infty (as \bar{z} \rightarrow \infty), \end{aligned} \right\} \quad (4)$$

where the symbols used are \bar{C} - concentration of the fluid, \bar{C}_∞ - concentration of the fluid far away from the plate, \bar{C}_w - concentration at the wall, B_o - external magnetic field, \bar{u} - primary velocity of the fluid, \bar{v} - secondary velocity of the fluid, u_o - velocity of the Plate, K - permeability parameter, \bar{z} - spatial coordinate normal to the plate, \bar{t} - time, β - volumetric coefficient of thermal expansion, α - thermal diffusivity, g - acceleration due to gravity, ρ - density, ν - kinematic viscosity, σ - Stefan-Boltzmann constant and $\bar{\Omega}$ - rotation parameter.

To obtain the equations in dimensionless form, the following non-dimensional quantities are introduced:

$$\left. \begin{aligned} u = \frac{\bar{u}}{u_o}, v = \frac{\bar{v}}{u_o}, t = \frac{u_o^2}{\nu} \bar{t}, K = \frac{u_o^2}{\nu^2} \bar{K}, z = \frac{u_o}{\nu} \bar{z}, \\ G_m = \frac{g\beta\nu(\bar{C}_w - \bar{C}_\infty)}{u_o^3}, \Omega = \frac{\nu}{u_o^2} \bar{\Omega}, S_c = \frac{\nu}{D}, \\ c = \frac{(\bar{C} - \bar{C}_\infty)}{(\bar{C}_w - \bar{C}_\infty)}, M = \frac{\sigma B_o^2 \nu}{\rho u_o^2}, \end{aligned} \right\} \quad (5)$$

where u is dimensionless primary velocity of the fluid, v - dimensionless secondary velocity of the fluid, z - dimensionless spatial coordinate normal to the plate, c - dimensionless concentration, S_c - Schmidt number, G_m - mass Grashof number, t - dimensionless time, Ω - dimensionless rotation parameter and M - magnetic field parameter. The equations 1, 2 and 3 become

$$\frac{\partial u}{\partial t} - 2\Omega v = G_m c + \frac{\partial^2 u}{\partial z^2} - (M + \frac{1}{K})u, \quad (6)$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} - (M + \frac{1}{K})v, \quad (7)$$

$$\frac{\partial c}{\partial t} = \frac{1}{S_c} \frac{\partial^2 c}{\partial z^2}. \quad (8)$$

The corresponding boundary conditions given in eq. 4 become

$$\left. \begin{aligned} t \leq 0: u = 0, v = 0, c = 0(\nabla z), \\ t > 0: u = 1, v = 0, c = t(at z = 0), \\ u \rightarrow 0, c \rightarrow 0(as z \rightarrow \infty). \end{aligned} \right\} \quad (9)$$

To solve above system, take $q = u + iv$. Then using equations 6 and 7, we get,

$$\frac{\partial q}{\partial t} = G_m c + \frac{\partial^2 q}{\partial z^2} - mq, \quad (10)$$

where

$$m = M + 2\Omega i + \frac{1}{K}.$$

The boundary conditions given in eq. 9 are reduced to

$$\left. \begin{aligned} t \leq 0: q = 0, c = 0(\nabla z), \\ t > 0: q = 1, c = t(z = 0), \\ q \rightarrow 0, c \rightarrow 0(z \rightarrow \infty). \end{aligned} \right\} \quad (11)$$

The governing non-dimensional partial differential equations 8 and 10 subject to the above boundary conditions prescribed in equation 11 are solved using the Laplace transform technique. The solution obtained is as under

$$\begin{aligned} q(z, t) = & \frac{1}{2} a e^{bt-z\sqrt{d}} \left\{ 1 + \operatorname{Erf} \left(\frac{2t\sqrt{d}-z}{2\sqrt{t}} \right) + e^{2z\sqrt{d}} \operatorname{Erfc} \left(\frac{2t\sqrt{d}+z}{2\sqrt{t}} \right) \right\} + \\ & \frac{1}{2} (1-a) e^{-z\sqrt{m}} \left\{ 1 + \operatorname{Erf} \left(\frac{2t\sqrt{m}-z}{2\sqrt{t}} \right) + e^{2z\sqrt{m}} \operatorname{Erfc} \left(\frac{2t\sqrt{m}+z}{2\sqrt{t}} \right) \right\} - \\ & \frac{1}{2} a e^{bt-z\sqrt{bS_c}} \left\{ 1 + \operatorname{Erf} \left(\frac{2t\sqrt{b}-z\sqrt{S_c}}{2\sqrt{t}} \right) + e^{2z\sqrt{bS_c}} \operatorname{Erfc} \left(\frac{2t\sqrt{b}+z\sqrt{S_c}}{2\sqrt{t}} \right) \right\} \\ & + a \operatorname{Erfc} \left(\frac{z\sqrt{S_c}}{2\sqrt{t}} \right) \end{aligned}$$

$$c(z, t) = \frac{1}{2} e^{-z\sqrt{S_c}} \left\{ \operatorname{Erfc} \left(\frac{z-2t\sqrt{S_c}}{2\sqrt{t}} \right) + e^{2z\sqrt{S_c}} \operatorname{Erfc} \left(\frac{z+2t\sqrt{S_c}}{2\sqrt{t}} \right) \right\}$$

where

$$a = \frac{G_m}{m}, b = \frac{m}{S_c - 1}, d = m + b.$$

Skin Friction

The skin-friction components τ_x and τ_y are obtained as

$$\begin{aligned} \tau_x + i\tau_y = & - \left(\frac{\partial q}{\partial t} \right)_{z=0} = \frac{1}{2} a \sqrt{d} e^{bt} \left\{ 1 + \operatorname{Erf}(\sqrt{dt}) + \operatorname{Erfc}(\sqrt{dt}) \right\} \\ & - a e^{bt} \left\{ - \frac{e^{-dt}}{\sqrt{\pi t}} + \sqrt{d} \operatorname{Erfc}(\sqrt{dt}) \right\} + \frac{a \sqrt{S_c}}{\sqrt{\pi t}} \\ & + \frac{1}{2} (1-a) \sqrt{m} \left\{ 1 + \operatorname{Erf}(\sqrt{mt}) + \operatorname{Erfc}(\sqrt{mt}) \right\} \\ & - (1-a) \left\{ - \frac{e^{-mt}}{\sqrt{\pi t}} + \sqrt{m} \operatorname{Erfc}(\sqrt{mt}) \right\} \\ & + a e^{bt} \left\{ - \frac{\sqrt{S_c} e^{-bt}}{\sqrt{\pi t}} + \sqrt{bS_c} \operatorname{Erfc}(\sqrt{bt}) \right\} \\ & - \frac{1}{2} a \sqrt{bS_c} e^{bt} \left\{ 1 + \operatorname{Erf}(\sqrt{bt}) + \operatorname{Erfc}(\sqrt{bt}) \right\} \end{aligned}$$

Result and Discussion

In order to get a physical insight of the problem, a representative set of numerical results is shown graphically in figures 1 to 12.

Primary velocity profiles are shown in figures 1 to 5. From fig. 1, it is clear that the primary velocity increases when G_m is increased (keeping other parameters $K = 0.5$, $S_c = 2.01$, $M = 0.5$, $\Omega = 0.5$, $t = 0.2$ constant). Primary velocity profile for different values of K is shown in fig. 2, and it shows that primary velocity increases with increase in K . Also primary velocity decreases with increase in M (Fig. 3) and Ω (Fig. 4). But it increases with t (Fig. 5).

Secondary velocity profiles are shown in figures - 6 to 10. Figure 6 shows that secondary velocity increases when G_m is increased. Figure 7 shows that it also increases with K . From Fig. 8, it is observed that the secondary velocity

decreases when M increases. Figures 9 and 10 show that it increases with Ω and t .

Concentration profiles are illustrated in Fig 11 and 12 for different values of S_c and time. In fig. 11, it can be seen that the concentration of the fluid is inversely proportional to the value of Schmidt number S_c . Thus, the increase in S_c reduces the concentration in the system. This is because there would be a decrease of concentration boundary layer thickness with the increase of Schmidt number S_c . Also concentration in boundary layer increases with time (Fig. 12).

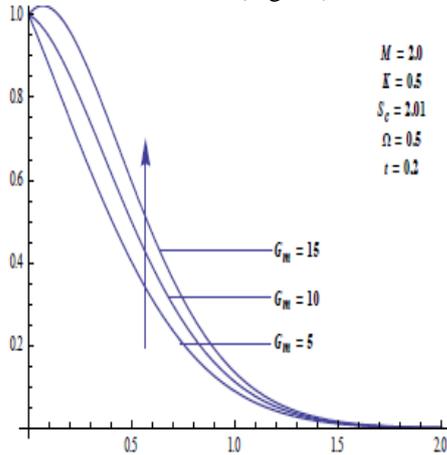


Figure 1. $u/v/s z$

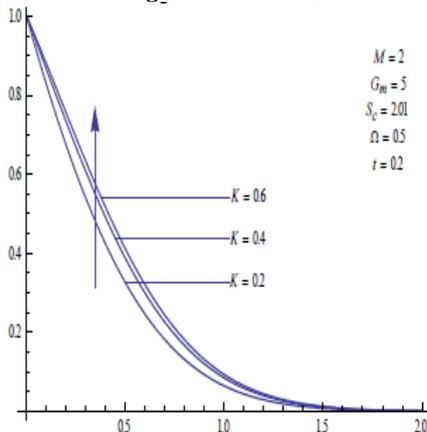


Figure 2. $u/v/s z$

The effects of various parameters on the skin-friction are shown in table 1. It is found from Table 1, that the value of τ_x increases when the values of Ω , M , G_m and S_c are increased (keeping other parameters fixed) but if values of t and K are increased, it gets decreased. Also, it is observed that τ_y decreases with M and S_c and it is increased when t , G_m , Ω and K are increased.

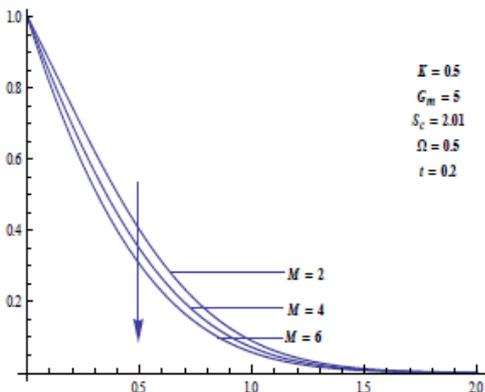


Figure 3. $u/v/s z$

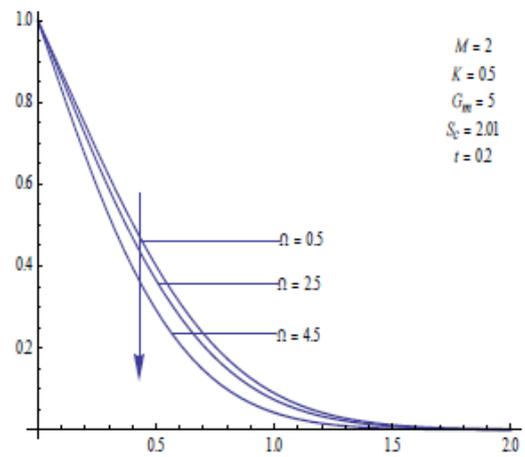


Figure 4. $u/v/s z$

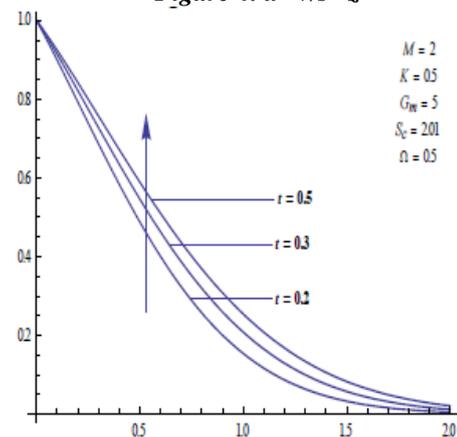


Figure 5. $u/v/s z$

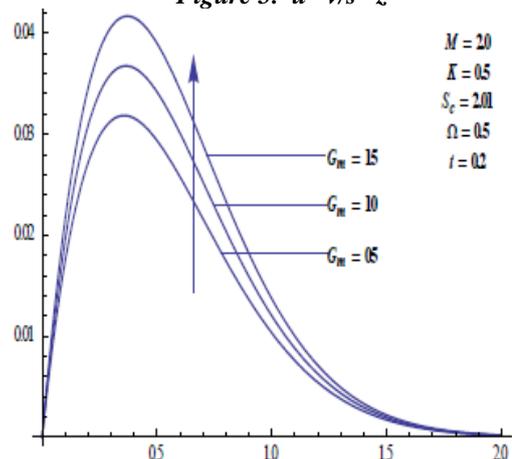


Figure 6. $v/v/s z$

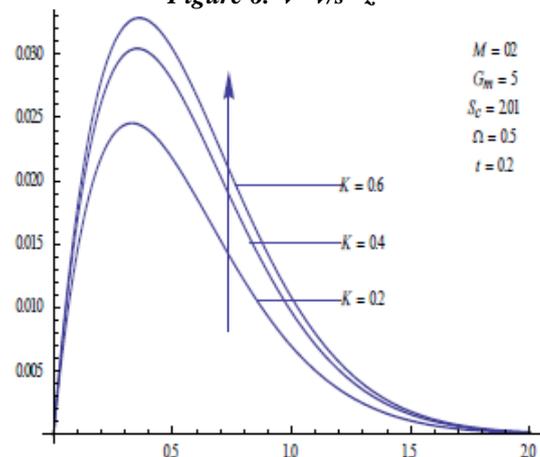


Figure 7. $v/v/s z$

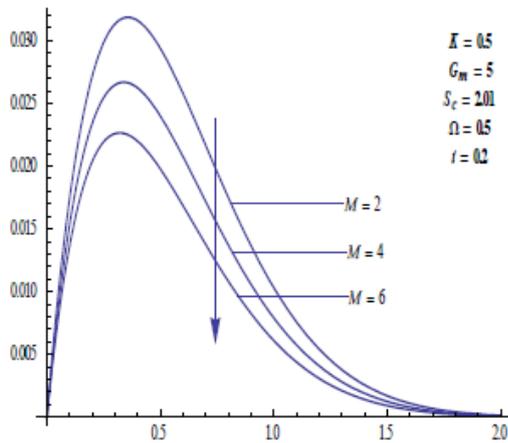


Figure 8. $v/vs z$

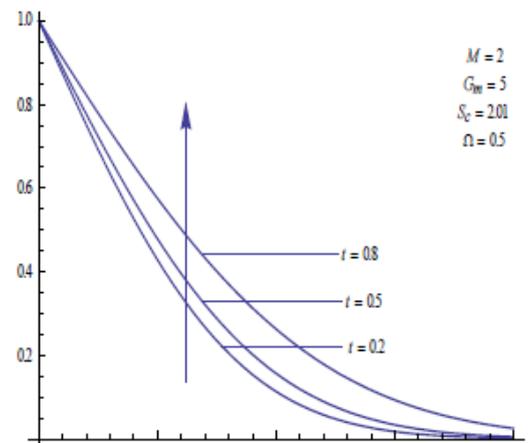


Figure 12. $c/vs z$

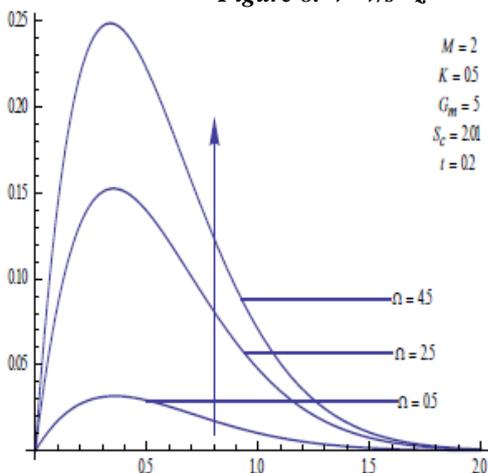


Figure 9. $v/vs z$

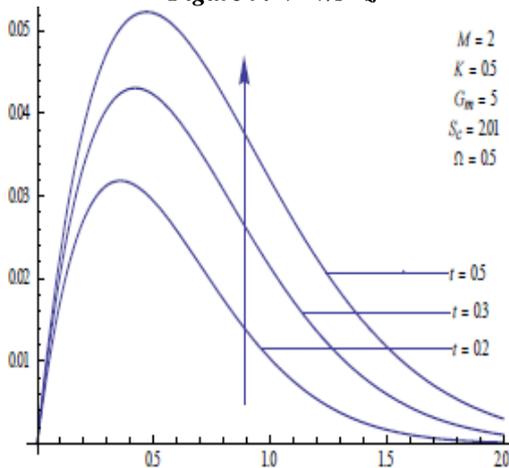


Figure 10. $v/vs z$

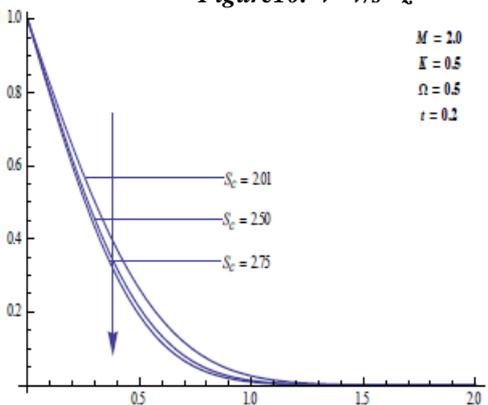


Figure 11. $c/vs z$

Table 1. (Skin friction for different parameters)

G_m	K	M	Ω	t	S_c	τ_x	τ_y
05	0.5	2	0.5	0.2	2.01	1.2178	0.2201
25	0.5	2	0.5	0.2	2.01	2.5525	0.3074
50	0.5	2	0.5	0.2	2.01	7.2653	0.4164
05	0.2	2	0.5	0.2	2.01	1.8283	0.1891
05	0.3	2	0.5	0.2	2.01	1.5006	0.2052
05	0.4	2	0.5	0.2	2.01	1.3262	0.2143
05	0.5	2	0.5	0.2	2.01	1.2178	0.2201
05	0.5	4	0.5	0.2	2.01	1.6349	0.1918
05	0.5	7	0.5	0.2	2.01	2.1894	0.1731
05	0.5	2	2.5	0.2	2.01	1.2178	0.2201
05	0.5	2	4.5	0.2	2.01	1.3582	1.0755
05	0.5	2	0.5	0.2	2.01	1.6563	1.8383
05	0.5	2	0.5	0.2	2.01	1.2178	0.2201
05	0.5	2	0.5	0.4	2.01	0.8208	0.2791
05	0.5	2	0.5	0.6	2.01	0.6333	0.3126
05	0.5	2	0.5	0.2	2.01	1.2178	0.2201
05	0.5	2	0.5	0.2	2.50	1.2718	0.2176
05	0.5	2	0.5	0.2	2.75	1.2952	0.2166

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