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# Metric Dimension of Fuzzy Complete Graph and Metric Dimension of Total Graph and Subdivision Graph of Some Graphs

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**ABSTRACT**

Let $G=(V,E,\mu)$ be a fuzzy graph. Let $\tilde{M}$ be a subset of $V$. $\tilde{M}$ is said to be a fuzzy metric basis of $G$ if for every pair of vertices $x,y\in V-\tilde{M}$, there exists a vertex $w\in\tilde{M}$ such that $\tilde{d}(w,x)\neq\tilde{d}(w,y)$ The number of elements in $\tilde{M}$ is said to be fuzzy metric dimension (FMD) of $G$ and is denoted by $\tilde{\beta}(G)$. In this paper, we investigate the bounds for the fuzzy metric dimension of complete fuzzy graph and the bounds for total fuzzy star graph. Next we find the exact values of fuzzy metric dimension of Total graph of fuzzy path and fuzzy cycles and subdivision graph of fuzzy paths, fuzzy cycles and fuzzy star graph.



## 1. Introduction

Graphs can be assigned to study various concepts of navigation in space. Each vertex of the graph denotes a place where the work is to be done, and edges denote the connections between the places. The problem of minimum number of machines (or robots) to be placed at certain vertices to trace each and every vertex uniquely is a classical one. This problem can be solved by a graph structural framework in which the navigating agent moves from one vertex to another. The places or vertices of a graph where we place the machines are called landmarks. The minimum number of machines required to locate uniquely each and every vertex is termed as the metric dimension, with the set of all minimum possible number of landmarks constituting a metric basis. All the graphs considered here are finite, connected, undirected and without multiple edges. We use standard terminology. The terms not defined here may be found in [1], [2].

The metric dimension was first studied by Harary and Melter, and independently by Slater [3, 4]. The basic parameter for this topic is the distance between two vertices in a graph. For any two distinct vertices $u,v\in V(G)$, the distance $d_G(u,v)$ between $u$ and $v$ is the length of a shortest $(u,v)-$ path in $G$.

Fuzzy graph theory is now finding numerous Applications in modern science and technology especially in the fields of information theory, neural network, expert systems and cluster analysis, medical diagnosis, etc... In 2012, B. Praba, P. Venugopal and N. Padmapriya [9] were introduced the concept of finding the fuzzy metric dimension in graphs.

Rosenfeld [5] has obtained the fuzzy analogues of several basic graph- theoretic concepts like bridges, paths, cycles, trees, and connectedness and established some of the properties.

The concept of fuzzy star graph was studied in [6] Fuzzy Subdivision Graph and Fuzzy Total Graph are defined in [7]. This paper is structured as follows: In section 2, we give the necessary definitions that are required to study the forth coming sections. In section 3, we discussed the fuzzy metric dimension of complete fuzzy graph. Fuzzy metric dimension of total graph and subdivision graph of paths, cycles and stars are discussed in sections 4. The final section gives the conclusion.

## 2. Prior Results

A fuzzy graph $G$ is a 2-tuple $(V,E)$ where $V$ is a non empty set of vertices $\{v_1,v_2,...,v_n\}$ and $E$ is the nonempty finite set of edges such that $\mu:V\rightarrow[0,1]$ and $\sigma:V\times V\rightarrow[0,1]$ where

$$\sigma(v_i,v_j)=\min\left(\mu(v_i),\mu(v_j)\right)\ for\ i\neq j$$
$$=0 \text{ for } i=j$$

For any $v\in V$, if $\mu(v)>0$ then we call $v$ as an active vertex. If $\mu(v)=0$ then we call v as an inactive vertex. In this paper, we assume that all the vertices as active vertices. We use the notation $e_{ij}$ to denote the edge connecting the vertices $v_i$ and $v_j$ .The weight of the edge $e_{ij}$ given by $\sigma(v_i,v_j)$ and is denoted by $w(e_{ij})$.

A fuzzy path from a vertex $v_i$ to a vertex $v_j$ in a fuzzy graph is a sequence of distinct vertices and edges starting from $v_i$ and ending at $v_j$ .This is denoted by $p(v_i,v_j)=p$ . If $v_i$ and $v_j$ coincide in a fuzzy path p then we call this sequence as a fuzzy cycle.

Let $P_{ij}$ be the set of all fuzzy paths p from $v_i$ to $v_j$ . For $v_i$ , $v_j\in V$, we define the fuzzy set $\mu_{ij}:P_{ij}\rightarrow[0,1]$ by

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$\mu_{ij}(p) = \min_{e \in p}(w(e))$ where $p \in P_{ij}$ . Here $\mu_{ij}(p)$ is called the weight of the path $p$. The fuzzy path $p \in P_{ij}$ forwhich $\mu_{ij}(p)$ is minimum, is called as a fuzzy shortest path (FSP) between $v_i$ and $v_j$. The weight of this FSP is denoted by $d^*(v_i, v_j)$. Thus, d* can be viewed as a fuzzy set, $d^* : V \times V \to [0,1]$ where $d^*(v_i, v_j) = \min p \in P_{ij}(\mu_{ij}(p))$ and $d^*(v_i, v_i) = 0$

**Remark**

For any two fuzzy shortest path $p$ and $q$ between $v_i$ and $v_j$, we consider the path with lesser number of intermediate vertices. The 3-tuple $(V, d^*, t)$ [8] is defined as

$$\tilde{d}(v_i, v_j, t) = \frac{t}{t + d^*(v_i, v_j)}$$

Where t is the number of intermediate vertices in the shortest path from which d* is calculated [8]. We denote the number of intermediate vertices between $v_i$ and $v_j$ in FSP as $N(v_i, v_j)$ and $\tilde{d}(v_i, v_j, t)$ as $\tilde{d}(v_i, v_j)$.

Let $G = (V, E, \mu)$ be a fuzzy graph. Let $\tilde{M}$ be a subset of $V$. $\tilde{M}$ is said to be a fuzzy metric basis of $G$ if for every pair of vertices $x, y \in V - \tilde{M}$, there exists a vertex $w \in \tilde{M}$ such that $\tilde{d}(w, x) \neq \tilde{d}(w, y)$. The number of elements in $\tilde{M}$ is said to be fuzzy metric dimension [9] (FMD) of $G$ and is denoted by $\tilde{\beta}(G)$. The elements in $\tilde{M}$ are called as source vertices.

A star in fuzzy graph consist of two node sets $V$ and $U$ with $|V| = 1$ and $|U| > 1$, such that $\mu(v, u_i) > 0$ and $\mu(u_i, u_{i+1}) = 0$, $1 \le i \le n$. It is denoted by $S_{1,n}$.

Let $G : (\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$ i.e. let $G^*$ be $(V, E)$.The vertex and edges of G are taken together as vertex set, of the pair $sd(G) : (\sigma_{sd}, \mu_{sd})$. In $sd(G)$, each edge '$e$' in $G$ is replaced by a new vertex and that vertex is made as a neighbor of those vertices which lie on '$e$' in $G$. Here $\sigma_{sd}$ is a fuzzy subset defined on $V \cup E$ as

$\sigma_{sd}(u) = \sigma(u)$ if $u \in V$

$= \mu(u)$ if $u \in E$

The fuzzy relation $\mu_{sd}$ on $V \cup E$ is defined as

$\mu_{sd}(u, e) = \sigma_{sd}(u) \Lambda \sigma_{sd}(e)$ if $u \in V$, $e \in E$ and $u$ lies on $e$.

$= 0$ otherwise.

As $\mu_{sd}(u, e) = \sigma_{sd}(u) \Lambda \sigma_{sd}(e)$ for all $u$, $e$ in $V \cup E$, $\mu_{sd}(u, e)$ is a fuzzy relation on $\sigma_{sd}$ and hence the pair $sd(G) : (\sigma_{sd}, \mu_{sd})$ is a fuzzy graph. This pair is termed as Subdivision fuzzy graph of $G$.

Let $G : (\sigma, \mu)$ be a fuzzy graph with its underlying set $V$ and crisp graph $G^* : (\sigma^*, \mu^*)$. The pair $T(G) : (\sigma_T, \mu_T)$ of $G$ is defined as follows. Let the vertex set of $T(G)$ be $V \cup E$. The fuzzy subset $\sigma_T$ is defined on $V \cup E$ as,

$\sigma_T = \sigma(u)$ if $u \in V$

$= \mu(e)$ if $e \in E$

The fuzzy relation $\mu_T$ is defined as,

$\mu_T(u, v) = \mu(u, v)$ if $u, v \in V$.

$\mu_T(u, e) = \sigma(u) \Lambda \mu(e)$ if $u \in V$, $e \in E$ and the node 'u' lies on the edge 'e'

$= 0$ otherwise.

$\mu_T(e_i, e_j) = \mu(e_i) \Lambda \mu(e_j)$ if the edges $e_i$ and $e_j$ have a node in common between them,

$= 0$ otherwise.

By the definition, $\mu_T(u, v) \le \sigma_T(u) \Lambda \sigma_T(v)$ for all u, v in $V \cup E$. Hence $\mu_T$ is a fuzzy relation on the fuzzy subset $\sigma_T$. Hence the pair $T(G) : (\sigma_T, \mu_T)$ is a fuzzy graph, and is termed as Total fuzzy graph of $G$.

A fuzzy graph $G : (\sigma, \mu)$ is said to be complete if $\mu(x, y) = \sigma(x) \Lambda \sigma(y)$ for all x and y.

**Theorem 2.1[8]**

d* is a metric.

**Theorem 2.2[9]**

If $G$ is a path then $\tilde{\beta}(G) = 1$.

**Theorem 2.3[9]**

If $P_n$ is a path on n vertices and $v_k$ is an intermediate vertex in $P_n$, $v_i$ and $v_j$ are two vertices on either side of $v_k$ then $\tilde{d}(v_k, v_i) = \tilde{d}(v_k, v_j)$ if and only if

$$\frac{N(v_k, v_i)}{N(v_k, v_j)} = \frac{d^*(v_k, v_i)}{d^*(v_k, v_j)}.$$

**Theorem 2.4[9]**

Let $P_n$ be a path on $n$ vertices and $v_k$ is an intermediate vertex in $P_n$, If $v_i$ and $v_j$ are two vertices on either side of $v_k$ such that $N(v_k, v_i) = N(v_k, v_j)$ then $\tilde{d}(v_k, v_i) = \tilde{d}(v_k, v_j)$ if and only if $d^*(v_k, v_i) = d^*(v_k, v_j)$.

## 3. Fuzzy Metric dimension of Complete Fuzzy Graph

In this section, we determine the fuzzy metric dimension of complete fuzzy graph.

**Theorem: 3.1**

For any $n \ge 2$, let $K_{2n}$ be a complete fuzzy graph on 2n vertices. Then $n \le \tilde{\beta}(K_{2n}) \le 2n - 1$.

**Proof**

Let $K_{2n}$ be a complete fuzzy graph with $2n$ vertices and $2nC_2$ edges. Label the vertices as follows: $0, 1, 2, ..., 2n-1$. We know that, for any $n \ge 2$, $K_{2n}$ is decomposable into Hamiltonian fuzzy path. Consider the following ways to find the metric dimension of complete fuzzy graph $K_{2n}$ $(n \ge 2)$.

If $n = 2$, $K_4$ is decomposable into two ways, that is, two Hamiltonian fuzzy path of length three otherwise three Hamiltonian fuzzy path of length two. Since number of edges of $K_4$ is six, then $\tilde{\beta}(K_4) = 2 or 3$ since the metric dimension of fuzzy path is one.

If $n = 3$, $K_6$ is decomposable into two ways, that is three Hamiltonian fuzzy path of length five or five Hamiltonian fuzzy path of length three since number of edges of $K_6$ is fifteen then $\tilde{\beta}(K_6) = 3 or 5$ since metric dimension of fuzzy path is one.

If $n=4$, $K_8$ is decomposable into two ways, four Hamiltonian fuzzy path of length seven or seven Hamiltonian fuzzy path of length four since number of edges of $K_8$ is twenty eight then $\tilde{\beta}(K_8)=4 \, or \, 7$ which depends upon the Hamiltonian path decomposition since $\tilde{\beta}(P_n)=1$, $n \ge 2$.

Continuing this process, we get, for any $n \ge 2$, $K_{2n}$ is decomposable into two ways of Hamiltonian fuzzy path

(i) $n$ Hamiltonian fuzzy path of length $2n-1$ (or)

(ii) $2n-1$ Hamiltonian fuzzy path of length $n$

then cardinality of minimum metric basis is $n$ and cardinality of maximum metric basis is $2n-1$. Hence $n \le \tilde{\beta}(K_{2n}) \le 2n-1$.

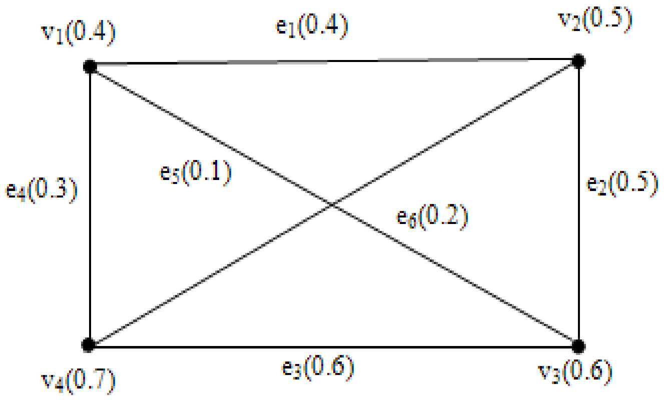


**Fig 1. Complete graph $K_4$**

**Theorem: 3.2**

For any $n \ge 1$, let $K_{2n+1}$ be a complete fuzzy graph on 2n+1 vertices. Then $n \le \tilde{\beta}(K_{2n+1}) \le 2n$.

**Proof**

Let $K_{2n+1}$ be a complete fuzzy graph with 2n+1vertices and $2n+1C_2$ edges. Label the vertices as $m, 0, 1, 2, ..., 2n-1$. We know that, for any $n \ge 1$, fuzzy graph $K_{2n+1}$ is decomposable into $n$ Hamiltonian fuzzy cycles $C_{2n+1}$. Above fuzzy graph forms the Hamiltonian fuzzy cycles as follows.

$C^1 : m, 0, 2n-1, 1, 2n-2, 2, 2n-3, ..., n-1, n, m.$

$C^2 : m, 1, 0, 2, 2n-1, 3, 2n-2, ..., n, n+1, m.$

$C^3 : m, 2, 1, 3, 0, 4, 2n-1, ..., n+1, n+2, m.$

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$C^n : m, n-1, n-2, n, n-3, n+1, n-4, ..., 2n-2, 2n-1, m.$

In $K_{2n+1}$, let $C_{2n+1}$ be an odd Hamiltonian fuzzy cycle with vertices $v_1, v_2, ..., v_{2n+1}$. If we fix $v_1$ as a source vertex, $v_{n+1}$ and $v_{n+2}$ are two diametrically opposite vertices of $v_1$. $d(v_1,v_{n+1}) = r = d(v_1,v_{n+2})$, where r = n = radius of $C_{2n+1}$. Let $p_1$ be the path $v_1, v_2, v_3, ..., v_{n+1}$. and $p_2$ be the path $v_1, v_{2n+1}, v_{2n}, v_{2n-1}, ..., v_{n+2}, v_{n+1}$.

If $n=1$, $K_3$ is decomposable into one Hamiltonian fuzzy cycle $c_3$. In $c_3$, $v_1$ as a source vertex. Let $v_2$ and $v_3$ be the two vertices on $c_3$. If both $v_2$ and $v_3$ have the same FSP (fuzzy shortest path)from source vertex then $v_1, v_2, v_3$ will be lie in same path then $\tilde{\beta}(K_3)=1$.Otherwise, we include another source vertex $v_2$ or $v_3$ then $\tilde{M}=\{v_1, v_2 (or) v_3\}$ hence $\tilde{\beta}(K_3) \le 2$.

If $n=2$, $K_5$ is decomposable into two Hamiltonian fuzzy cycle of length five. that is, $K_5 = C_{n1} \cup C_{n2}$ which had the same characterization mentioned for $K_3$ hence $2 \le \tilde{\beta}(K_5) \le 4$.

If $n=3$, $K_7$ is decomposable into three Hamiltonian fuzzy cycle $C_{n1}$, $C_{n2}$ and $C_{n3}$ of length seven, the lower value of the each cycle is one and the upper value of the each cycle is two then $3 \le \tilde{\beta}(K_7) \le 6$.

In general, $n \le \tilde{\beta}(K_{2n+1}) \le 2n$ since $K_{2n+1}$ is decomposable into $n$ edge-disjoint Hamiltonian fuzzy cycle of length $2n+1$.

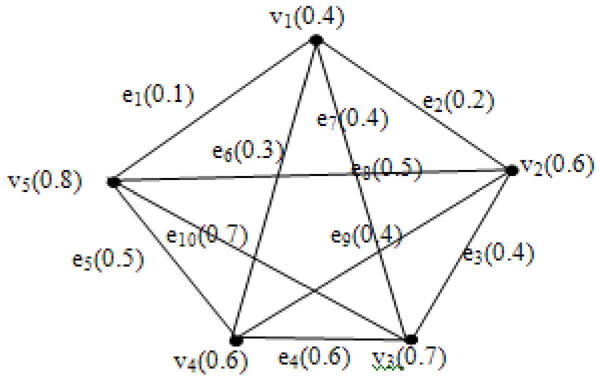


**Fig 2. Complete graph $K_5$**

## 4. Fuzzy Metric dimension of Total Graph and Subdivision Graph of Path, Cycle and Star in Fuzzy Graph

Following three theorems deals with the fuzzy metric dimension of total graph of fuzzy path, fuzzy cycle and fuzzy star graph.

**Theorem: 4.1**

If $T(P_n)$ is a Total fuzzy graph of path $P_n$ then $\tilde{\beta}\left[T(P_n)\right]=2.$

**Proof**

Let $v_1, v_2, ..., v_n$ and $e_1, e_2, ..., e_{n-1}$ be the vertices and edges of fuzzy path respectively. $T(P_n)=(\sigma_T, \mu_T)$ be a total fuzzy graph of path with $2n-1$ vertices $v_1, e_1, v_2, e_2, ..., e_{n-1}, v_n$. and $q_T = 4n-5$. We take $T(P_n)$ as the union of two fuzzy paths, that is, $T(P_n)=P_1 \cup P_2$ where

$P_1 = v_1, e_1, v_2, e_2, ..., e_{n-2}, v_{n-1}, v_n, e_{n-1}.$

$P_2 = e_1, e_2, ..., e_{n-2}, e_{n-1}, v_{n-1}, v_{n-2}, ..., v_2, v_1.$

Fix $v_1$, as a source vertex. If two vertices $v_i \, or \, e_j \in P_1$ and $v_j \, or \, e_i \in P_2$ such that fuzzy shortest path for $v_i \, or \, e_j$ is through $P_2$ and fuzzy shortest path for $v_j \; or \, e_i$ is through $P_1$ then $\tilde{d}\left(v_1, v_i \; or \, e_j\right) = \tilde{d}\left(v_1, v_j \; or \, e_i\right)$ if and only if $N\left(v_1, v_i \; or \, e_j\right) = N\left(v_1, v_j \; or \, e_i\right)$ this implies $\tilde{\beta}\,[T(P_n) \neq 1]$

Include $e_1$ as another source vertex so that $N\left(e_1, v_i \; or \, e_j\right) \neq N\left(e_1, v_j \; or \, e_i\right)$, $\tilde{d}\left(e_1, v_i \; or \, e_j\right) \neq \tilde{d}\left(e_1, v_j \; or \, e_i\right)$

$\tilde{M} = \{v_1, e_1\}$ ,Hence $\tilde{\beta}\,[T(P_n)\,] = 2$.

**Theorem: 4.2**

If $T(C_n)$ is a Total fuzzy graph of Cycle then $\tilde{\beta}\left[T\left(C_n\right)\right] = m$, where $m = 2, 3, 4$.

**Proof**

Let $v_1, v_2, ..., v_n$ and $e_1, e_2, ..., e_n$ be the vertices and edges of fuzzy cycle respectively. Let $T(C_n) = (\sigma_T, \mu_T)$ be a Total fuzzy graph of cycle with $2n$ vertices $v_1, e_1, v_2, e_2, ..., e_{n-1}, v_n, e_n$ and $q_T = 4n$, we will Separate $T(C_n)$ as the union of two fuzzy even cycle,that is $T(C_n) = C_{n1} \cup C_{n2}$ where

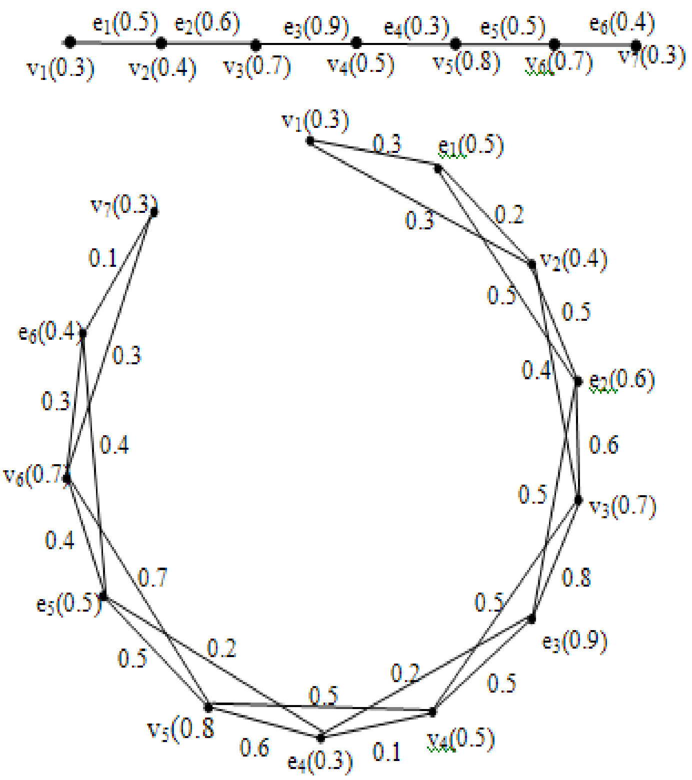


**Fig 3. Fuzzy Path and its Total Fuzzy Graph**

$C_{n1} = e_1, e_2, e_3, ..., e_{n-1}, e_n, v_n, v_{n-1}, ..., v_1, e_1$.

$C_{n2} = v_2, e_2, v_3, ..., v_{n-2}, e_{n-2}, v_{n-1}, e_{n-1}$, $v_n, v_1, e_n, e_1, v_2$.

In $C_{n1}$, Fix $e_1$ as a source vertex then $v_n$ is the diametrically opposite vertex of $e_1$.Let $P_1$ be the path $e_1, e_2, e_3, ..., e_{n-1}, e_n, v_n$ and $P_2$ be the path $v_n, v_{n-1}, ..., v_1, e_1$. In $C_{n2}$, Fix $v_2$ as a source vertex. If $n$ is even then $v_{\frac{n}{2}+2}$ is a diametrically opposite vertex of $v_2$ .If $n$ is odd then $e_{\frac{n+1}{2}+1}$ is a diametrically opposite vertex of $v_2$.

Let $P_3$ be the path $v_2, e_2, v_3, e_3, ..., e_{n-3}, v_{n-2}, ..., v_{\frac{n}{2}+2}$ when $n$ is even or $v_2, e_2, v_3, e_3, ..., e_{n-3}, v_{n-2}, ..., e_{\frac{n+1}{2}+1}$ when $n$ is odd and $P_4$ be the path $v_2, e_1, e_n, v_1, v_n, e_{n-1}, v_{n-1}, e_{n-2}, v_{n-2}, ..., v_{\frac{n}{2}+2}$. When $n$ is even or $v_2, e_1, e_n, v_1, v_n, e_{n-1}, v_{n-1}, e_{n-2}, v_{n-2}, ..., e_{\frac{n+1}{2}+1}$. when $n$ is odd

**Case: 1**

**Sub case: i**

In $C_{n1}$, Let $v_i \; or \, e_j$ and $v_j \; or \, e_i$ be two vertices on $C_{n1}$, both $v_i \; or \, e_j$ and $v_j \; or \, e_i \in P_1$ (or $P_2$), If both $v_i \; or \, e_j$ and $v_j \; or \, e_i$ have the same FSP from $e_1$ then $e_1, v_i \; or \, e_j$ and $v_j \; or \, e_i$ will be in same path then $\tilde{\beta}(C_{n1}) = 1$.

**Sub case: ii**

In $C_{n2}$ , Let $v_k \; or \, e_l$ and $v_l \; or \, e_k$ be two vertices on $C_{n2}$, both $v_k \; or \, e_l$ and $v_l \; or \, e_k \in P_3$ (or $P_4$), If both $v_k \; or \, e_l$ and $v_l \; or \, e_k$ have the same FSP from $v_2$ then $v_2$, $v_k \; or \, e_l$ and $v_l \; or \, e_k$ will be in same path then $\tilde{\beta}(C_{n2}) = 1$, $\tilde{\beta}[T(C_n)] = \tilde{\beta}[C_{n1} \cup C_{n2}]$, $\tilde{M} = \{e_1, v_2\}$ by sub cases i and ii then $\tilde{\beta}\,[T(C_n)\,] = 2$.

**Case: 2**

**Sub case: i**

In $C_{n1}$, If the two vertices $v_i \; or \, e_j$ and $v_j \; or \, e_i$ belongs to either $P_1$ or $P_2$ by sub case (i) in case 1 we get $\tilde{\beta}(C_{n1}) = 1$.

**Sub case: ii**

In $C_{n2}$, If $v_k \; or \, e_l \in p_3$ and $v_l \; or \, e_k \in p_4$ such that the FSP for $v_k \; or \, e_l$ is through $P_4$ and FSP for $v_l \; or \, e_k$ is through $P_3$ then $\tilde{d}\left(v_2, v_k \; or \, e_l\right) = \tilde{d}\left(v_2, v_l \; or \, e_k\right)$ if and only if $N\left(v_2, v_k \; or \, e_l\right) = N\left(v_2, v_l \; or \, e_k\right)$ this implies $\tilde{\beta}\,(C_{n2}) \neq 1$, Include $e_2$ as a another source vertex so that $N\left(e_2, v_k or \, e_l\right) \neq N\left(e_2, v_k \; or \, v_l\right)$, $\tilde{d}\left(e_2, v_k or \, e_l\right) \neq \tilde{d}\left(e_2, v_k \; or \, v_l\right)$, then $\tilde{M} = \{v_2, e_2\}$

$\tilde{\beta}\,(C_{n2}) = 2$.

$\tilde{\beta}[T(C_n)] = \tilde{\beta}[C_{n1} \cup C_{n2}]$

$\tilde{M} = \{e_1, v_2, e_2\}$ by sub cases i and ii

Hence $\tilde{\beta}\,[T(C_n)] = 3$.

**Case: 3**

**Sub case: i**

In $C_{n1}$, If $v_i \; or \, e_j \in P_1$ and $v_j \; or \, e_i \in P_2$ such that the FSP for $v_i \; or \, e_j$ is through $P_2$ and FSP for $v_j \; or \, e_i$ is through $P_1$

then $\tilde{d}(e_1, v_i \, or \, e_j) = \tilde{d}(e_1, v_j \, or \, e_i)$ if and only if $N(e_1, v_i \, or \, e_j) = N(e_1, v_j \, or \, e_i)$

This implies that $\tilde{\beta}(C_{n1}) \neq 1$.

Include $v_1$ as another source vertex so that $N(v_1, v_k \, or \, e_l) \neq N(v_1, v_l \, or \, e_k)$, $\tilde{d}(v_1, v_k \, or \, e_l) \neq \tilde{d}(v_1, v_l \, or \, e_k)$ then $\tilde{M} = \{e_1, v_1\}$, $\tilde{\beta}(C_{n1}) = 2$.

**Sub case: (ii)**

Similar to the sub case(ii) in case:2

We get $\tilde{M} = \{v_2, e_2\}$, $\tilde{\beta}[T(C_n)] = \tilde{\beta}[C_{n1} \cup C_{n2}]$

$\tilde{M} = \{e_1, v_1, v_2, e_2\}$ bysub cases (i) and (ii)

Hence $\tilde{\beta}[T(C_n)] = 4$.

**Theorem:4.3**

For any $n \geq 2$, let $T(K_{1,n})$ be a Total Fuzzy star graph then

$$\frac{3n}{2} \leq \tilde{\beta}[T(K_{1,n})] \leq 2n \quad \text{, when } n \text{ is even}$$

$$\left\lfloor \frac{3n}{2} \right\rfloor \leq \tilde{\beta}[T(K_{1,n})] \leq 2n \quad \text{, when } n \text{ is odd}$$

**Proof**

Let $v, v_1, v_2, ..., v_n$ and $e_1, e_2, ..., e_n$ be the vertices and edges of fuzzy star graph respectively. By the definition of total graph $V = [T(K_{1,n})] = \{v\} \cup \{e_i / (1 \leq i \leq n)\} \cup \{v_i / (1 \leq i \leq n)\}$ in which the vertices $e_1, e_2, ..., e_n$ induces a clique of order (say $K_n$) $n$. Also the vertex $v$ is adjacent with $v_i$ $(1 \leq i \leq n)$, $e_i$ is adjacent with $v_i$ and $v$ $(1 \leq i \leq n)$. Let us find the metric dimension of total fuzzy star graph by the following ways.

If $n = 2$, $T(K_{1,2})$ can be decomposed into two ways by cycles $C_{n1}$ and $C_{n2}$ of length three and Path $P$ of length one. $C_{n1}$ contains the vertices $v, e_1, v_1$. $C_{n2}$ contains the vertices $v_2, e_2, v$ and $P$ contains the vertices $e_1, e_2$.

In $C_{n1}$, $v_1$ as a source vertex of $C_{n1}$. If both e$_1$ and v have the same fuzzy shortest path from source vertex thenv$_1$, $e_1$ and v will lie in the same path then $\tilde{\beta}(C_{n1}) = 1$,

Otherwise we include another source vertex $v$ then $\tilde{M}_1 = \{v, v_1\}$, and $\tilde{\beta}(C_{n1}) = 2$. Similarly, in $C_{n2}$, take $v_2$ as a source vertex, we get $\tilde{M}(C_{n2}) = \{v_2\}$ or $\{v_2, v_1\}$ and $\tilde{M}(P) = \{e_1\}$,then $\tilde{\beta}(P) = 1$ since metric dimension of path is one. $\tilde{M} = \{\tilde{M}_1 \cup \tilde{M}_2 \cup e_1\} = \{v_1, v_2, v, e_1\}$.

Hence $(3 \leq \tilde{\beta}[T(K_{1,2})] \leq 4)$.

If $n = 3$, $T(K_{1,3})$ can be decomposed into $C_{n1}$, $C_{n2}$ and $C_{n3}$ of length three and $K_3$. $C_{n1}$ contains the vertices $v, e_1, v_1$, $C_{n2}$ contains the vertices $v_2, e_2, v$ and $C_{n3}$ contains the vertices $v_3, e_3, v$ and $K_3$ contains the vertices $e_1, e_2, e_3$. In $T(K_{1,3}) = \{C_{n1} \cup C_{n2} \cup C_{n3}\} \cup K_3$, each cycle have the same characterization as mentioned in $T(K_{1,2})$. Therefore,

$$\left|\tilde{M} = \{v_1, v_2, v_3\}\right| \leq \tilde{\beta}\{C_{n1} \cup C_{n2} \cup C_{n3}\} \leq \left|\tilde{M} = \{v_1, v_2, v_3, v\}\right|$$

$3 \leq \tilde{\beta}\{C_{n1} \cup C_{n2} \cup C_{n3}\} \leq 4$, and $(1 \leq \tilde{\beta}(K_3) \leq 2)$.

Hence, $4 \leq \tilde{\beta}[T(K_{1,3})] \leq 6$.

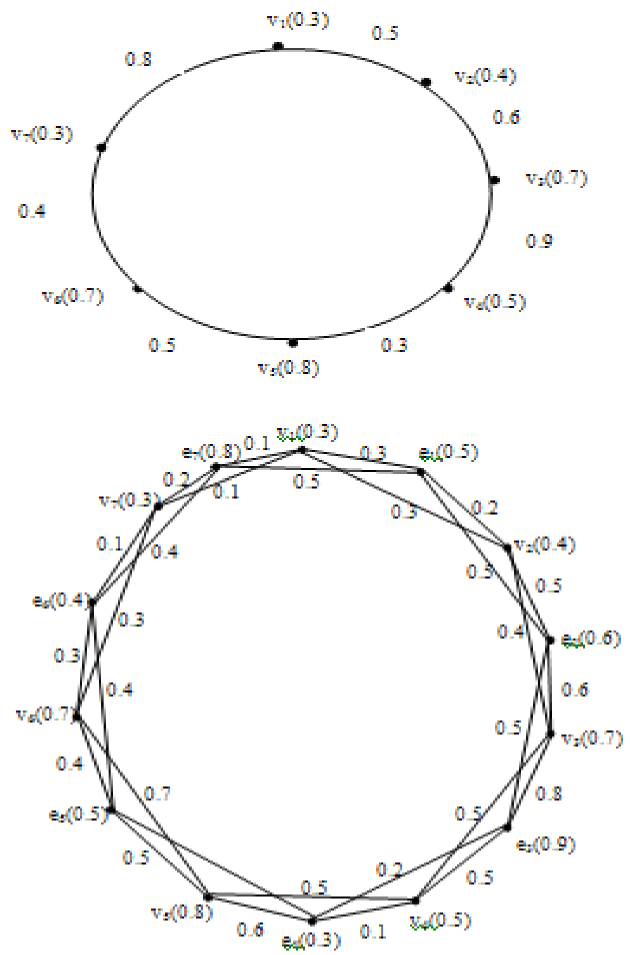


**Fig4. Fuzzy Cycle Graph and its Total Fuzzy Cycle Graph**

If $n = 4$, $T(K_{1,4})$ can be decomposed into four cycles $C_{n1}$, $C_{n2}$ $C_{n3}$, $C_{n4}$ and $K_4$ such that the vertices of $C_{n1}$ are $v, e_1, v_1$; the vertices of $C_{n2}$ are $v_2, e_2, v$; the vertices of $C_{n3}$ are $v_3, e_3, v$; the vertices of $C_{n4}$ are $v_4, e_4, v$ and complete graph with four vertices $e_1, e_2, e_3, e_4$. In $T(K_{1,4}) = \{C_{n1} \cup C_{n2} \cup C_{n3} \cup C_{n4}\} \cup K_4$, $\{C_{n1} \cup C_{n2} \cup C_{n3} \cup C_{n4}\}$ has the same characterization as mentioned for $T(K_{1,2})$. Therefore,

$$\left|\tilde{M} = \{v_1, v_2, v_3, v_4\}\right| \leq \tilde{\beta}\{C_{n1} \cup C_{n2} \cup C_{n3} \cup C_{n4}\} \leq \tilde{M} = \left|\{v_1, v_2, v_3, v_4, v\}\right|$$

Also, $4 \leq \tilde{\beta}\{C_{n1} \cup C_{n2} \cup C_{n3} \cup C_{n4}\} \leq 5$.

Hence $6 \leq \tilde{\beta}[T(K_{1,4})] \leq 8$, since $(2 \leq \tilde{\beta}(K_4) \leq 3)$.

In general, $T(K_{1,n})$ can be decomposed into $n$ cycles $C_{n1}$, $C_{n2}, \ldots, C_{nm}$ and $K_n$ such that the vertices of $C_{n1}$ are $v, e_1, v_1$; the vertices of $C_{n1}$ are $v_2, e_2, v$; the vertices of $C_{n1}$ are $v_m, e_m, v$ and the complete graph of $n$ vertices are $e_1, e_2, e_3, ..., e_n$, In

$T(K_{1,n}) = \{C_{n1} \cup C_{n2} \cup C_{n3}, ..., \cup C_{nm}\} \cup K_n$, $\{C_{n1} \cup C_{n2} \cup C_{n3}, ..., \cup C_{nm}\}$

Each cycle have the same characterization which mentioned for $T(K_{1,2})$ therefore

$$\tilde{M} = |\{v_1, v_2, v_3, ..., v_m\}| \le \tilde{\beta}\{C_{n1} \cup C_{n2} \cup C_{n3}, ..., \cup C_{nm}\} \le \tilde{M} = |\{v_1, v_2, v_3, ..., v_m, v\}|$$

then $n \le \tilde{\beta}\{C_{n1} \cup C_{n2} \cup C_{n3}, ..., \cup C_{nm}\} \le n+1$

hence $\left\lfloor \frac{3n}{2} \right\rfloor \le \tilde{\beta}[T(K_{1,n})] \le 2n$ when $n$ is odd since $(n \le \tilde{\beta}(K_{2n+1}) \le 2n)$

$\frac{3n}{2} \le \tilde{\beta}[T(K_{1,n})] \le 2n$ when $n$ is even since $(n \le \tilde{\beta}(K_{2n}) \le 2n-1)$

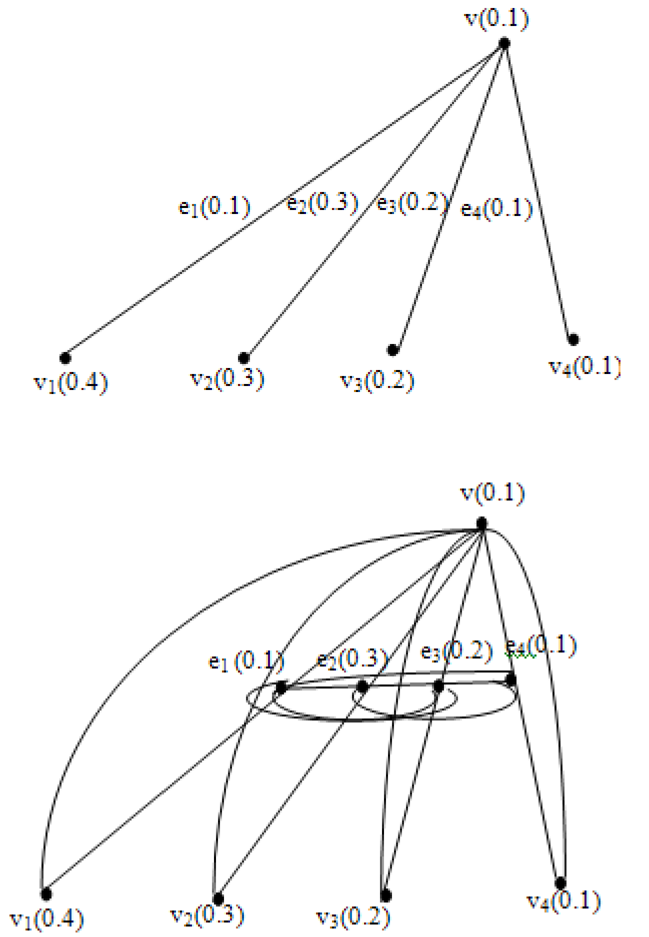


**Fig 5. Fuzzy Star Graph and its Total Fuzzy Star Graph**

Following theorem give the exact value of the subdivision of fuzzy star graph.

**Theorem: 4.4**

For any $n \ge 2$, $S(K_{1,n})$ be a Subdivision fuzzy star graph then

$\tilde{\beta}[S(K_{1,n})] = \frac{n}{2}$, when $n$ is even.

$\tilde{\beta}[S(K_{1,n})] = \left\lceil \frac{n}{2} \right\rceil$, when $n$ is odd.

**Proof**

Let $K_{1,n}$ be a fuzzy star graph with $n+1$ vertices and $n$ edges, $S(K_{1,n})$ be a Subdivision of fuzzy star graph with $2n+1$ vertices and $2n$ edges.

For any $n \ge 2$, $S(K_{1,n})$ can be decomposed into $\frac{n}{2}$ fuzzy path of length four when $n$ is even and $S(K_{1,n})$ can be decomposed into $\left\lfloor \frac{n}{2} \right\rfloor$ fuzzy path of length four and path of length two. Therefore,

$\tilde{\beta}[S(K_{1,n})] = \frac{n}{2}$, when $n$ is even, $\tilde{\beta}[S(K_{1,n})] = \left\lceil \frac{n}{2} \right\rceil$, when $n$ is odd.

**Theorem:4.5**

If $S(P_n)$ is a Subdivision of fuzzy path then $\tilde{\beta}[S(P_n)] = \tilde{\beta}[P_{2n-1}] = 1$

**Theorem:4.6**

If $S(C_n)$ is a Subdivision of fuzzy cycle then $\tilde{\beta}[S(C_n)] = \tilde{\beta}[C_{2n}] \le 2$

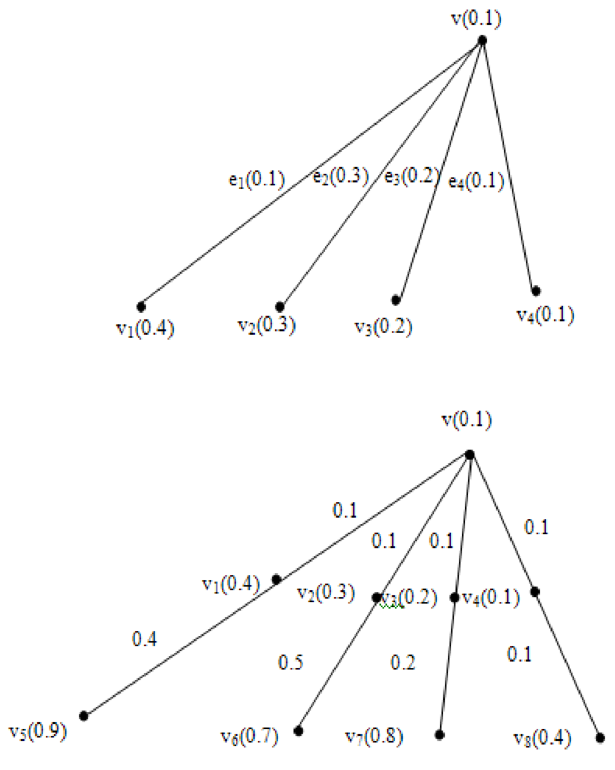


**Fig 6. Fuzzy Star Graph and its Subdivision Graph**

**Conclusion**

In this paper, we have obtained the bounds for the fuzzy metric dimension of complete fuzzy graph and total fuzzy star graph. Also, we have found the exact values of fuzzy metric dimension of Total graph of fuzzy path and fuzzy cycles and subdivision graph of fuzzy paths, fuzzy cycles and fuzzy star graph. In the future work, it is decided to study the fuzzy metric dimension of star fuzzy graphs and fuzzy metric dimension of Middle graph of some graphs.

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