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MHD Flow past an Impulsively Started Vertical Plate with Variable Temperature and Constant Mass Diffusion in the Presence of Hall Current.

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ABSTRACT

In the present paper, MHD flow past an impulsively started vertical plate with variable temperature and constant mass diffusion in the presence of Hall current is studied. The fluid considered is an electrically conducting, absorbing-emitting radiation but a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profile and Skin friction. The velocity profile and Skin friction have been studied for different parameters like Schmidt number, Hall parameter, Magnetic parameter, Mass Grashof number, Thermal Grashof number, Prandtl number, and Time. The effect of parameters is shown graphically and the values of the skin-friction for different parameters have been tabulated.

Introduction

The study of MHD flow and Hall effect plays an important role in engineering and biological science. M. Rangamma, et al. [2] have studied Hall effect on unsteady MHD flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction. Rajput and Kumar [5] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Rajput and sahu [6] have studied Transient free convection MHD flow between two long vertical parallel plates with constant temperature and variable mass diffusion. Further, Sulochana, [1] have studied Hall Effects on unsteady MHD three dimensional flow through a porous medium in a rotating parallel plate channel with effect of inclined magnetic field. Kalita,et al [7] have studied unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. Manivannan et al.[8] have studied Mass transfer effect on vertical oscillating plate with heat flux. Ram, et al. [9] have studied Hall effects on heat and mass transfer flow through porous medium. Singh [4] have studied Heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. Rajput and sahu [3] have studied effects of chemical reactions on free convection MHD past an exponentially accelerated infinite vertical plate through a porous medium with variable temperature and mass diffusion. We are considering MHD flow past an impulsively started vertical plate with variable temperature and constant mass diffusion in the presence of Hall current.

The effect of Hall current on the velocity have been observed with the help of graphs, and the skin friction has been tabulated.

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Mathematical Analysis

We consider an unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate. The plate is electrically non-conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially, at time $t \leq 0$ the temperature of the fluid and the plates are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time t>0, temperature of the plate is raised to T_w and the concentration of the fluid is raised to C_w. Using the relation $\nabla \bullet B = 0$, for the magnetic field $\overline{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. Due to Hall effect, there will be two components of the momentum equation, which are as under. The usual assumptions have been taken in to consideration.

The fluid model is as under :

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(u + mw), \quad (1)$$

$$\frac{\partial w}{\partial t} = \upsilon \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(w - mu), \quad (2)$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2}, \quad (3)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The following boundary conditions have been assumed:

$$\left.\begin{array}{l} t \le 0 : u = 0, w = 0, C = C_\infty, T = T_\infty, \text{for all the values of y} \\ t > 0 : u = u_0, w = 0, C = C_w, T = T_\infty + (T_w - T_\infty)\dfrac{u_0{}^2 t}{\upsilon} \text{ at y} = 0 \\ u \to 0, w \to 0, C \to C_\infty, \mathrm{T} \to T_\infty \quad \text{as y} \to \infty \end{array}\right\} \tag{5}$$

Here u is the velocity of the fluid in x- direction, w - the velocity of the fluid in z- direction, m - Hall parameter, g – acceleration due to gravity, β - volumetric coefficient of thermal expansion, β^* - volumetric coefficient of concentration expansion, t- time, C_∞ - the concentration in the fluid far away from the plate, C - species concentration in the fluid , C_w - species concentration at the plate, D - mass diffusion, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate, T - the temperature of the fluid , k - the thermal conductivity, υ – the kinematic viscosity, ρ - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, and C_P - specific heat at constant pressure. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left.\begin{array}{l} \bar{u} = \dfrac{u}{u_0}, \bar{w} = \dfrac{w}{u_0}, \bar{y} = \dfrac{y u_0}{\upsilon}, Sc = \dfrac{\upsilon}{D}, \Pr = \dfrac{\mu C_P}{k}, \\ M = \dfrac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \bar{t} = \dfrac{t u_0{}^2}{\upsilon}, G_r = \dfrac{g\beta\upsilon(T_w - T_\infty)}{u_0{}^3}, \\ Gm = \dfrac{g\beta\upsilon(C_w - C_\infty)}{u_0{}^3}, \bar{C} = \dfrac{C - C_\infty}{C_w - C_\infty}, \theta = \dfrac{(T - T_\infty)}{(T_w - T_\infty)} \end{array}\right\} \tag{6}$$

Here the symbols used are:

$\bar{u}$ - dimensionless velocity, $\bar{w}$ - dimensionless velocity, θ - the dimensionless temperature, $\bar{C}$ - the dimensionless concentration, G_r - thermal Grashof number, G_m - mass Grashof number, μ - the coefficient of viscosity, $\Pr$ - the Prandtl number, Sc - the Schmidt number, M - the magnetic parameter.

The dimensionless forms of equations (1), (2), (3) and (4) are as follows

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + G_r\theta + G_m\bar{C} - \frac{M(\bar{u} + m\bar{w})}{(1+m^2)}, \tag{7}$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} = \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{M(\bar{w} - m\bar{u})}{(1+m^2)}, \tag{8}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{y}^2}, \tag{10}$$

with the corresponding boundary conditions:

$$\begin{array}{l} \bar{t} \le 0, \bar{u} = 0, \bar{C} = 0, \theta = 0, \vec{w} = 0\text{ , } for\ all\ value\ of\ \bar{y} \\ \bar{t} > 0, \bar{u} = 1, \bar{w} = 0, \theta = \bar{t}, C = 1\ at\ \bar{y} = 0 \\ \bar{u} \to 0, \bar{C} \to 0, \theta \to 0, \bar{w} \to 0\ as\ \bar{y} \to \infty \end{array} \tag{11}$$

Dropping the bars and combining the equations (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\theta + GmC - \left(\frac{M}{1+m^2}(1 - mi)\right)q \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{13}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{14}$$

where $\mathrm{q = u + iw}$, with corresponding boundary conditions

$$\left.\begin{array}{l} t \le 0 : q = 0, \theta = 0, C = 0, \text{for all value of y} \\ t > 0 : q = 1, w = 0, \theta = t, C = 1, \text{at y} = 0 \\ q \to 0, C \to 0, \theta \to 0, w \to 0 \text{ as y} \to \infty \end{array}\right\} \tag{15}$$

$$\begin{aligned} q = {} & \frac{1}{2}e^{-\sqrt{a}\,y}A_1 + \frac{1}{4a^2}\,y\,Gr\{A_{13}(1 - \Pr - at)\} + \sqrt{a}\,e^{-\sqrt{a}y}(A_1 - e^{2\sqrt{a}y}) \\ & + B_{13}\{A_{14}(1 - \Pr)\} \\ & + \frac{1}{2a}Gm\left(-e^{-\sqrt{a}y}A_1 + e^{\frac{at}{-1+Sc}\sqrt{\frac{aSc}{-1+Sc}}}(1 + B_{11} + e^{2\sqrt{\frac{aSc}{-1+Sc}}}B_{12})\right) - \\ & \frac{1}{2a^2\sqrt{\pi}}Gr\sqrt{\Pr}y\left(\begin{array}{l} -B_{14}\{-1 + \sqrt{\Pr} + at\} + a\left(\begin{array}{l} 2e^{\frac{-\Pr y^2}{4t}}\sqrt{t} - \\ \sqrt{\pi}\sqrt{\Pr}y\{1 - Erf[\dfrac{\sqrt{\Pr}y}{2\sqrt{t}}]\} \end{array}\right) \\ + \dfrac{1}{y}e^{\frac{at}{-1+\Pr}\sqrt{\frac{a}{-1+\Pr}}\sqrt{\Pr}y}\sqrt{\pi}B_{17}\left(\dfrac{\Pr - 1}{\sqrt{\Pr}}\right) \end{array}\right) \\ & - \frac{1}{2a}Gm\left(-2Erfc[\frac{\sqrt{Sc}y}{2\sqrt{t}}] + e^{\frac{at}{-1+Sc}\sqrt{\frac{a}{-1+Sc}}\sqrt{Sc}y}\left(1 + B_{18} + e^{2\sqrt{\frac{aSc}{-1+Sc}}y}\right)\right) \end{aligned} \tag{16}$$

$$\theta = \left[\left(t + \frac{\Pr y^2}{2} \right) Erf\left(\frac{\sqrt{\Pr} y}{2\sqrt{t}} \right) - e^{-\frac{y^2}{4t}\Pr} \frac{\sqrt{\Pr t} y}{\sqrt{\pi}} \right] \quad (17)$$

$$C = Erfc\left[\frac{\sqrt{Sc} y}{2\sqrt{t}} \right] \quad (18)$$

Skin Friction

The dimensionless skin friction at the plate is given by $\left(\frac{dq}{dy}\right)_{y=0}$.

Separating real and imaginary part in $\left(\frac{dq}{dy}\right)_{y=0}$, the dimensionless skin – friction component $\tau_x = \left(\frac{du}{dy}\right)_{y=0}$ and $\tau_z = \left(\frac{dw}{dy}\right)_{y=0}$ can be computed.

Table1. Skin friction

m	Gr	Gm	M	Sc	Pr	t	τ_x	τ_z
0.5	10	10	2	2.01	0.71	0.2	0.7024	0.2289
0.5	10	10	2	2.01	7	0.2	0.5309	0.2246
0.5	20	10	2	2.01	0.71	0.2	1.0553	0.2346
0.5	30	10	2	2.01	0.71	0.2	1.4082	0.2404
1	10	10	2	2.01	0.71	0.2	0.8737	0.2971
5	10	10	2	2.01	0.71	0.2	1.1647	0.1215
0.5	10	20	2	2.01	0.71	0.2	2.7015	0.2699
0.5	10	30	2	2.01	0.71	0.2	4.7005	0.3109
0.5	10	10	3	2.01	0.71	0.2	0.4728	0.3266
0.5	10	10	4	2.01	0.71	0.2	0.2524	0.4149
0.5	10	10	2	5	0.71	0.2	0.2133	0.2110
0.5	10	10	2	10	0.71	0.2	0.1141	0.2020
0.5	10	10	2	2.01	0.71	0.15	0.1847	0.1920
0.5	10	10	2	2.01	0.71	0.25	1.1430	0.2642

Result and Discussions

The numerical values of velocity and skin friction are computed for different parameters like thermal Grashof number Gr, mass Grashof number Gm, magnetic field parameter M, Hall parameter m, Prandtl number Pr, Schmidt number Sc and time t. The values of the main parameters considered are

Gr = 10, 20, 30
M = 2, 3, 4
m = 1, 5
Gm = 10, 20, 30
Pr = 0.71, 7
Sc = 2.01, 5, 10
t = 0.15, 0.2, 0.25.

Figures 1, 2, 6 and 7 show that primary velocity increases when m, Gm, Gr, and t are increased. Figures 3, 4 and 5 show that primary velocity decreases when M, Sc and Pr are increased. And figures 9, 10, 12 and 14 show that the secondary velocity increases when Gm, M, Gr, and t are increased. Figures 8, 11 and 13 show that secondary velocity decreases when m, and Sc are increased.

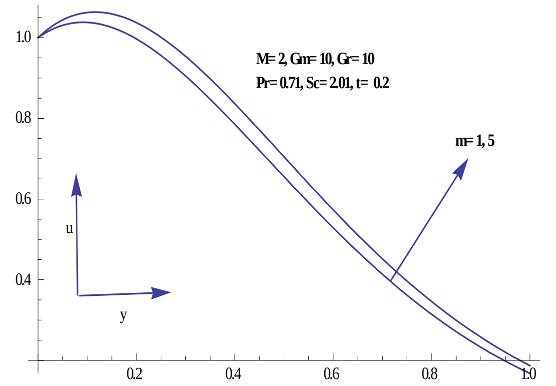

Figure 1. Velocity profile for different values of m.

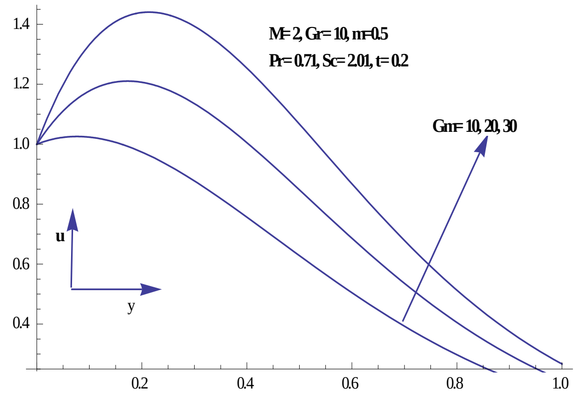

Figure 2. Velocity profile u for different values of Gm.

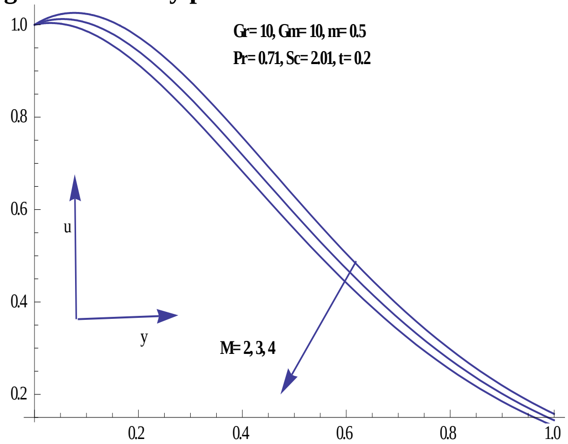

Figure 3. Velocity profile u for different values of M.

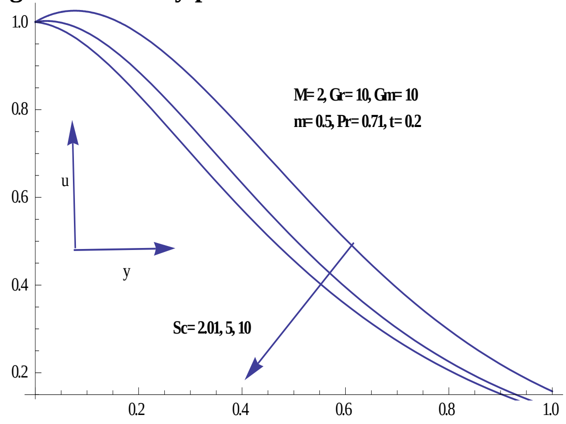

Figure 4. Velocity profiles u for different values of Sc.

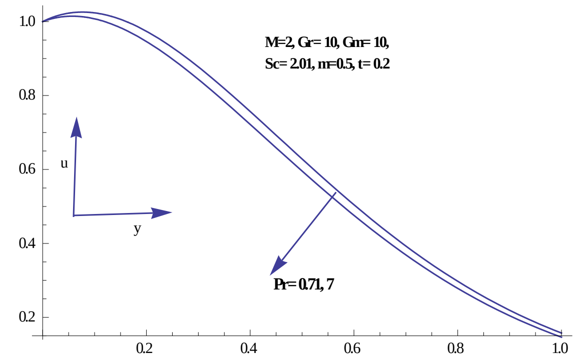

Figure 5. Velocity profile u for different values of Pr.

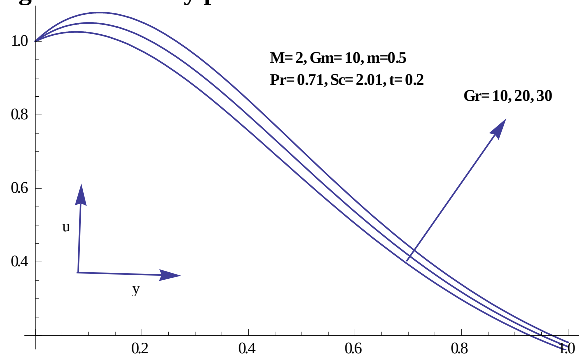

Figure 6. Velocity profile u for different values of Gr.

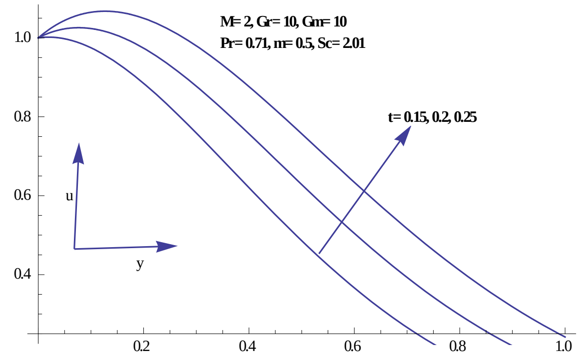

Figure 7. Velocity Profile u for different values for t.

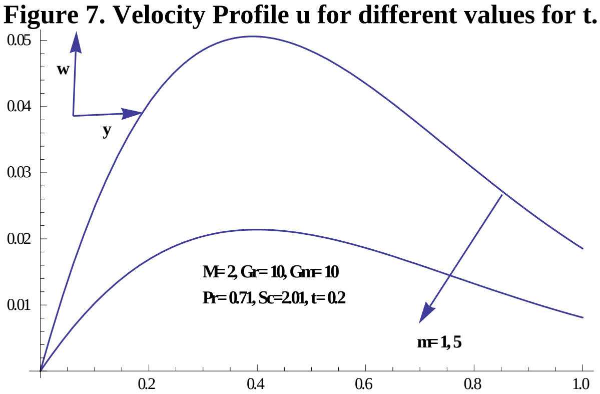

Figure 8. Velocity profile w for different values of m.

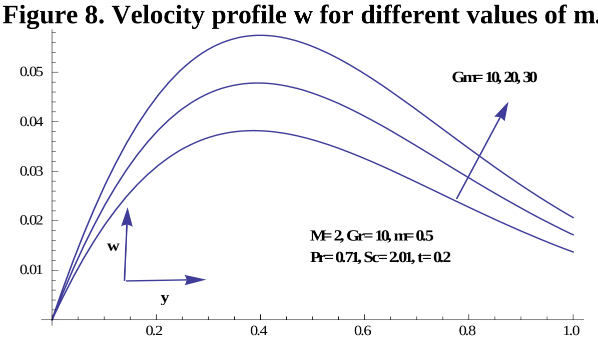

Figure 9. Velocity profile w for different values of Gm.

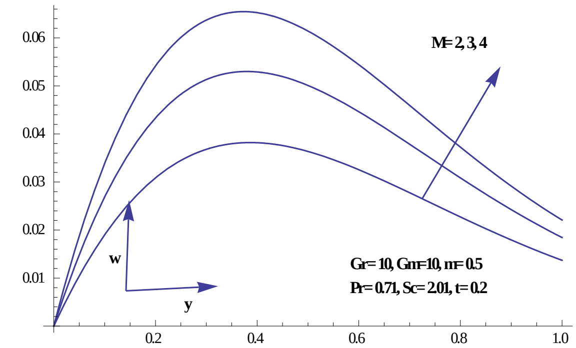

Figure 10. Velocity profile w for different values of M.

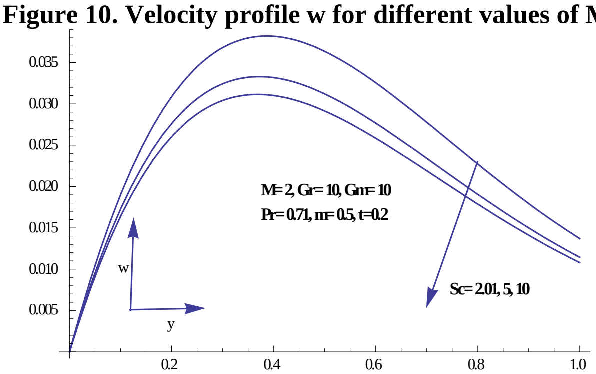

Figure 11. Velocity profile w for different values of Sc.

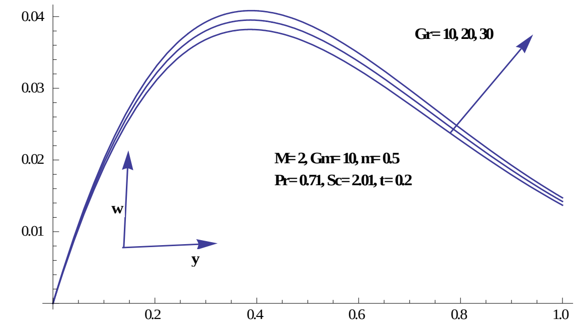

Figure 12. Velocity profile w for different values of Gr.

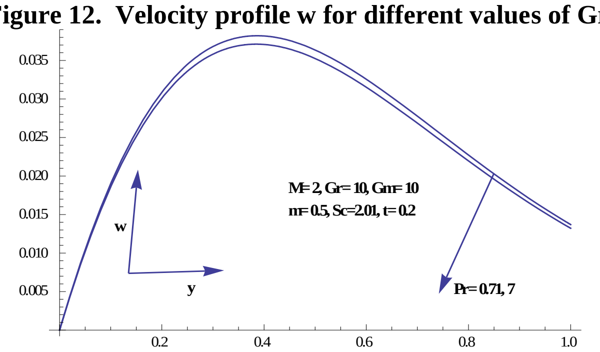

Figure 13. Velocity profile w for different values of Pr.

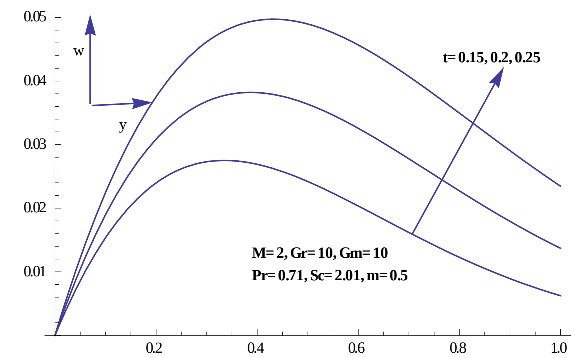

Figure 14. Velocity profile w for different values of t.

Conclusion

Some conclusions of the study are as under

1. Primary Velocity increases with the increase in thermal Grashof number, Hall parameter, mass Grashof number and time.
2. Primary velocity decreases with the increase in magnetic field parameter, Prandtl number and Schmidt number.
3. Secondary velocity increases with increase in thermal Grashof number, mass Grashof number, time, and magnetic field parameter.
4. Secondary velocity decreases with the increase in Hall parameter, Prandtl number and Schmidt number.
5. τ_x decreases with increase in Schmidt number, Prandtl number, magnetic field parameter, and it increases with thermal Grashof number, mass Grashof number, Hall parameter and time. τ_z increases with increase in thermal Grashof number, mass Grashof number, time, and magnetic field parameter, and it decreases when Prandtl number, hall parameter and Schmidt number are increased.

Appendex

$$A_1=\left(\left(1+Erf\left[\frac{2\sqrt{a}t-y}{2\sqrt{t}}\right]\right)+e^{2\sqrt{a}y}Erfc\left[\frac{2\sqrt{a}t+y}{2\sqrt{t}}\right]\right)$$

$$A_{11}=1+Erf\left[\frac{2\sqrt{a}t-y}{2\sqrt{t}}\right] \quad A_{12}=e^{2\sqrt{a}y}Erfc\left[\frac{2\sqrt{a}t+y}{2\sqrt{t}}\right]$$

$$A_{13}=\frac{2e^{-\sqrt{a}y}\left(1+e^{2\sqrt{a}y}+A_{11}-A_{12}\right)}{y}$$

$$A_{14}=\frac{2e^{-\sqrt{a}y}\left(-1-e^{2\sqrt{a}y}-A_{11}+A_{12}\right)}{y}$$

$$B_1=\frac{Efr\left[2\sqrt{\frac{a\,\mathrm{Pr}}{-1+\mathrm{Pr}}}t-y\right]}{2\sqrt{t}} \quad B_2=\frac{Efr\left[2\sqrt{\frac{a\,\mathrm{Pr}}{-1+\mathrm{Pr}}}t+y\right]}{2\sqrt{t}}$$

$$B_{11}=\frac{Efr\left[2\sqrt{\frac{aSc}{-1+Sc}}t-y\right]}{2\sqrt{t}}$$

$$B_{12}=\frac{Efr\left[2\sqrt{\frac{aSc}{-1+Sc}}t+y\right]}{2\sqrt{t}} \quad B_{13}=\left(-1-e^{\frac{2\sqrt{a\mathrm{Pr}}}{-1+\mathrm{Pr}}y}-B_1+e^{\frac{2\sqrt{a\mathrm{Pr}}}{-1+\mathrm{Pr}}y}B_2\right)$$

$$B_{14}=\frac{2\sqrt{\pi}\left(-1+Erf\left[\frac{\sqrt{\mathrm{Pr}}y}{2\sqrt{t}}\right]\right)}{\sqrt{\mathrm{Pr}}y}$$

$$B_{15}=\frac{Efr\left[2\sqrt{\frac{a}{-1+\mathrm{Pr}}}t-\sqrt{\mathrm{Pr}}y\right]}{2\sqrt{t}},$$

$$B_{16}=\frac{Efr\left[2\sqrt{\frac{a}{-1+\mathrm{Pr}}}t+\sqrt{\mathrm{Pr}}y\right]}{2\sqrt{t}},$$

$$B_{17}=\left(1+e^{2\sqrt{\frac{a}{-1+\mathrm{Pr}}}\sqrt{\mathrm{Pr}}y}+B_{15}-e^{2\sqrt{\frac{a}{-1+\mathrm{Pr}}}\sqrt{\mathrm{Pr}}}B_{16}\right)$$

$$B_{18}=Erfc\frac{\left[2\sqrt{\frac{a}{1+Sc}}t-\sqrt{Sc}y\right]}{2\sqrt{t}},\; B_{19}=Erfc\frac{\left[2\sqrt{\frac{a}{1+Sc}}t+\sqrt{Sc}y\right]}{2\sqrt{t}},$$

$$a=\frac{M}{1+m^2}\left(1-im\right),$$

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