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# Dufour Effect on Unsteady MHD Flow through Porous Medium past an Impulsively Started Inclined Oscillating Plate with Variable Temperature and Mass Diffusion

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**ABSTRACT**
Dufour effect on unsteady MHD flow through porous medium past an impulsively started inclined oscillating plate with variable temperature and mass diffusion is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity profile is discussed with the help of graphs drawn for different parameters.



## Introduction

The study of hydromagnetic flow problems play important roles in different areas of science and technology, like biological science, petroleum engineering, chemical engineering, mechanical engineering, biomechanics, irrigation engineering and aerospace technology. Such problems frequently occur in petro-chemical industry, chemical vapour deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics. The influence of magnetic field on viscous, incompressible and electrically conducting fluid is of great importance in many applications such as magnetic material processing, glass manufacturing control processes and purification of crude oil. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill[1]. Sparrow and Husar[2] have investigated longitudinal vortices in natural convection flow on inclined plates. Datta and Jana[4] have considered oscillatory magneto hydrodynamic flow past a flat plate with Hall effects. Free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction was analyzed by Soundalgekar[3] which was further improved by Vajravelu and. Sastri [5]. Rajput and Kumar[10] have investigated MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Singh [12] have analyzed heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. MHD flow between two parallel plate with heat transfer was studied by Attia and katb[6]. Kesavaiah and Satyanarayana[14] have considered radiation absorption and Dufour effects to MHD flow in vertical surface. Free convective flow of visco-elastic fluid in a vertical channel with Dufour effect was investigated by Acharya et al [13]. Postelnicu [7] has considered influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects, heat mass transfer. Ibrahima[8] has worked on analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect. Soret and Dufour effects on natural convection flow past a vertical surface in a porous medium with variable surface temperature was studied by Hassan[9]. Makinde[11] has considered on MHD mixed convection with Soret and Dufour effects past a vertical plate embedded in a porous medium. Soret and Dufour effects on unsteady hydromagnetic free convective fluid flow past an infinite vertical porous plate in the presence of chemical reaction was analyzed by Murali et al[15]. We are considering Dufour effect on unsteady MHD flow through porous medium past an impulsively started inclined oscillating plate with variable temperature and mass diffusion. The results are shown with the help of graphs.

## Mathematical Analysis

In this paper we have considered MHD flow between two parallel electrically non conducting plates inclined at an angle $\alpha$ from vertical. x axis is taken along the plate and y normal to it. A transverse magnetic field $B_0$ of uniform strength is applied on the flow. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially it has been considered that the plate as well as the fluid is at the same temperature $T_\infty$ and the concentration level $C_\infty$ everywhere in the fluid is same in stationary condition. At time $t > 0$, the plate starts oscillating in its own plane with frequency $\omega$ and temperature of the plate is raised to $T_w$ and the concentration level near the plate is raised linearly with respect to time.

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The flow modal is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta Cos\alpha(T - T_\infty) + g\beta^* Cos\alpha(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon u}{K}, \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}, \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

With the corresponding initial and boundary conditions:

$$\left.\begin{array}{l} t \le 0: u = 0, T = T_\infty, C = C_\infty, \text{ for all } y, \\ t > 0: u = u_0 Cos\omega t, \\ T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon}, \text{ at } y = 0, \\ C = C_\infty + (C_w - C_\infty)\frac{u_0^2 t}{\upsilon}, \\ u \to 0, T \to T_\infty, C \to C_\infty, \text{ as } y \to \infty, \end{array}\right\} \tag{4}$$

Here

$U$ is the velocity of the fluid,
$g$-the acceleration due to gravity,
$\beta$- volumetric coefficient of thermal expansion,
t-time,
$T$-temperature of the fluid,
$B^*$-volumetric coefficient of concentration expansion,
$C$- species concentration in the fluid,
$\nu$- the kinematic viscosity,
$\rho$ - the density,
$C_p$- the specific heat at constant pressure,
$C_s$ -Concentration susceptibility,
$k$ - thermal conductivity of the fluid,
$D$ - the mass diffusion coefficient ,
$D_m$- the effective mass diffusivity rate,
$T_w$- temperature of the plate at y= 0,
$C_w$-species concentration at the plate y= 0,
$B_0$ - the uniform magnetic field, $\sigma$ - electrically conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2) and (3) into dimensionless form:

$$\left.\begin{array}{l} \bar{y} = \frac{yu_0}{\upsilon}, \bar{u} = \frac{u}{u_0}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, S_c = \frac{\upsilon}{D}, \mu = \rho\upsilon, \\ P_r = \frac{\mu C_p}{k}, G_r = \frac{g\beta\upsilon(T_w - T_\infty)}{u_0^3}, \bar{\omega} = \frac{\omega\upsilon}{u_0^2} M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \\ G_m = \frac{g\beta^*(C_w - C_\infty)}{u_0^3}, \bar{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \bar{t} = \frac{tu_0^2}{\upsilon}, \\ \bar{K} = \frac{u_0^2}{\upsilon^2}K, D_f = \frac{D_m K_T (C_w - C_\infty)}{\upsilon C_s C_p (T_w - T_\infty)}. \end{array}\right\} \tag{5}$$

where $\bar{u}$ -the dimensionless velocity, $\bar{t}$ -dimensionless time, $\theta$-the dimensionless temperature, $\bar{C}$ -the dimensionless concentration, $G_r$ -thermal Grashof number, $G_m$ - mass Grashof number, $\mu$ -the coefficient of viscosity, $P_r$ - the Prandtl number, $S_c$ - the Schmidt number, $D_f$ - dufour number, $K_T$ -Thermal diffusion ratio , $\bar{K}$ - is the permeability parameter, and M - the magnetic parameter .
Thus the model becomes

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + G_r Cos\alpha\theta + G_m Cos\alpha\bar{C} - M\bar{u} - \frac{1}{\bar{K}}\bar{u}, \tag{6}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{y}^2} + D_f \frac{\partial^2 \bar{C}}{\partial \bar{y}^2}, \tag{7}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}, \tag{8}$$

with the following boundary conditions:

$$\left.\begin{array}{l} \bar{t} \le 0: \bar{u} = 0, \theta = 0, \bar{C} = 0, \text{ for all } \bar{y}, \\ \bar{t} > 0: \bar{u} = Cos\omega t, \theta = \bar{t}, \bar{C} = \bar{t} \text{ at } \bar{y} = 0, \\ \bar{u} \to 0, \theta \to 0, \bar{C} \to 0 \text{ as } \bar{y} \to \infty. \end{array}\right\} \tag{9}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r Cos\alpha\theta + G_m Cos\alpha C - Mu - \frac{1}{K}u, \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + D_f \frac{\partial^2 C}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \tag{12}$$

with the following boundary conditions:

$$\left.\begin{array}{ll} t\le 0: u=0,\theta=0,C=0, & \text{for all y,} \\ t>0: u=Cos\omega t,\theta=t,C=t, & \text{at y=0,} \\ u\to 0,\theta\to 0,C\to 0 & \text{as } y\to\infty. \end{array}\right\} \quad (13)$$

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13), are solved by the usual Laplace - transform technique.

The solution obtained is as under:

$$u=\frac{e^{-it\omega}A_{13}}{4}+(e^{-\sqrt{A}y}(2A_1+2AA_2t+A_3\sqrt{A}y$$
$$+2A_2P_r)+2A_{16}A_4(1-P_r)).\{\frac{G_rCos\alpha}{4A^2}-$$
$$\frac{G_rD_fP_rS_cCos\alpha}{4A^2(S_c-P_r)}\}-\frac{G_rD_fP_rS_cCos\alpha}{4A^2(S_c-P_r)}$$
$$(e^{-\sqrt{A}y}(2A_1+2AA_2t+A_3\sqrt{A}y+2A_2S_c)$$
$$+2A_{17}A_5(1-S_c))+\frac{e^{-\sqrt{A}y}G_rCos\alpha}{4\sqrt{A}}(2\sqrt{A}t+$$
$$2A_{30}\sqrt{A}t-A_{31}y+e^{2\sqrt{A}y}A_{31}(2\sqrt{A}+y))$$
$$(yAA_{11}\sqrt{P_r}+A_6A_{16}\sqrt{\pi}-2A_{14}\sqrt{\pi}+2AA_{14}t\sqrt{\pi}$$
$$+A_7A_{16}\sqrt{\pi}P_r+2A_{14}P_r\sqrt{\pi})\{\frac{G_rD_fP_rS_cCos\alpha}{4A^2\sqrt{\pi}(S_c-P_r)}$$
$$-\frac{G_rCos\alpha}{4A^2\sqrt{\pi}}\}+\frac{G_rD_fP_rS_cCos\alpha}{4A^2\sqrt{\pi}(S_c-P_r)}(yAA_{12}\sqrt{S_c}+$$
$$A_8A_{17}\sqrt{\pi}-2A_{15}\sqrt{\pi}+2AA_{15}t\sqrt{\pi}+A_8A_{17}\sqrt{\pi}S_c$$
$$+2A_{15}S_c\sqrt{\pi})-G_mA_{10}Cos\alpha$$

$$\theta=A_{14}\left(t-\frac{y^2}{2}\right)-\frac{A_{29}y\sqrt{tP_r}}{\sqrt{\pi}}-\frac{D_fP_rS_c}{S_c-P_r}\{t\left(A_{14}+A_{15}\right)$$
$$-\frac{y^2(A_{14}P_r+A_{15}S_c)}{2}-\frac{y\sqrt{t}}{\sqrt{\pi}}(A_{29}\sqrt{P_r}+A_{28}\sqrt{S_c})\}$$

$$c=t[(1+2\eta^2S_c)erfc(\eta\sqrt{S_c})-\frac{2\eta\sqrt{S_c}}{\sqrt{\pi}}e^{-\eta^2S_c}]$$

The expressions for the constants involved in the above equations are given in the appendix.

**Result and Discussion**

The velocity and temperature profile for different parameters like mass Grashof number Gm, thermal Grashof number Gr, magnetic field parameter M, permeability parameter, Dufour mumber, Prandtl number Pr and time t is shown in figures 1 to 14. It is observed from figure 1, that velocity of fluid decreases when the angle of inclination ( $\alpha$ ) is increased. It is observed from figure 2, when the mass Grashof number is increased then the velocity is increased. From figure 3, it is deduced that when thermal Grashof number Gr is increased then the velocity is decreased. It is observed from figure 4, that the effect of increasing values of the parameter M results in decreasing u. It is deduced that when permeability parameter is increased then the velocity is increased (figure 5). If Dufour number is increased then the velocity is decreased (figure 6). It is deduced that when phase angle is increased then the velocity is decreased (figure 7). Further, it is observed that velocity is increased when Prandtl number is increased (figure 8). When the Schmidt number is increased then the velocity is increased (figure 9). Further, from figure 10, it is observed that velocities increase with time. It is observed that the temperature is increased when Dufour number, Prandtl number, Schmidt number and time are increased (figures 11,12,13,14).

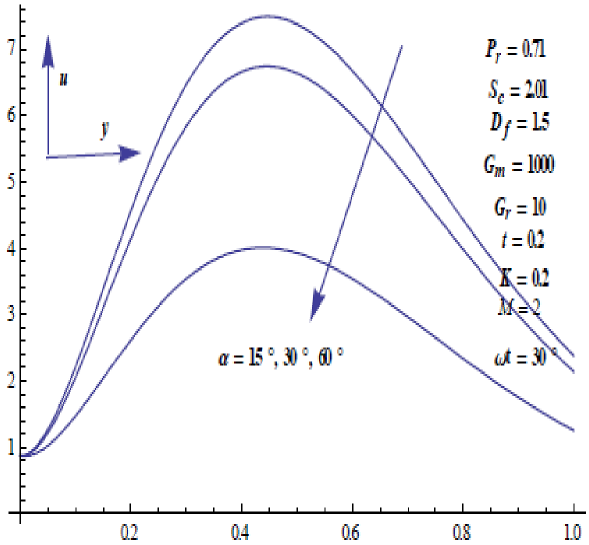


**Figure 1. Velocity profile for different values of $\alpha$**

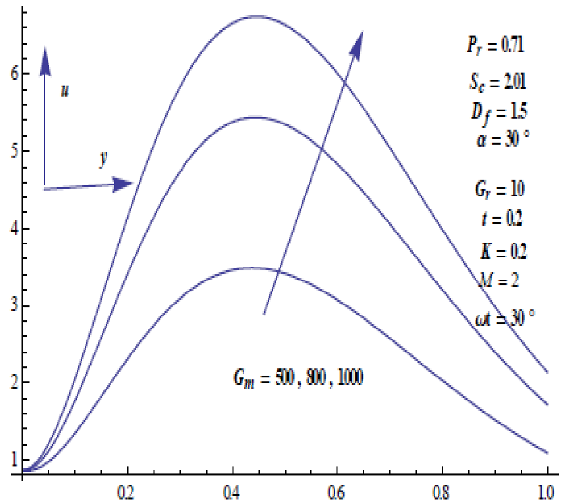


**Figure 2. Velocity profile for different values of Gm**

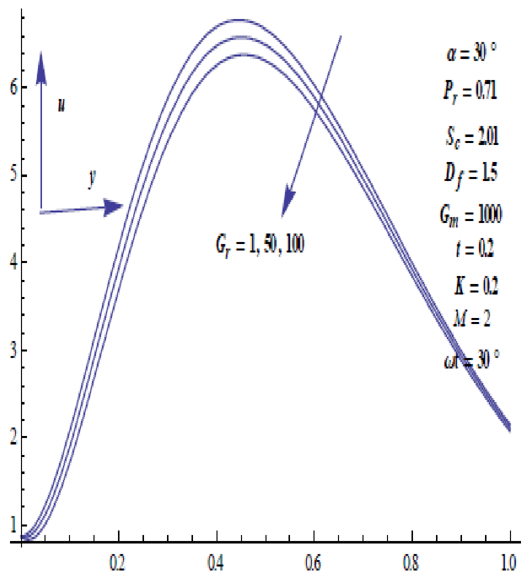


**Figure 3. Velocity profile for different values of Gr**

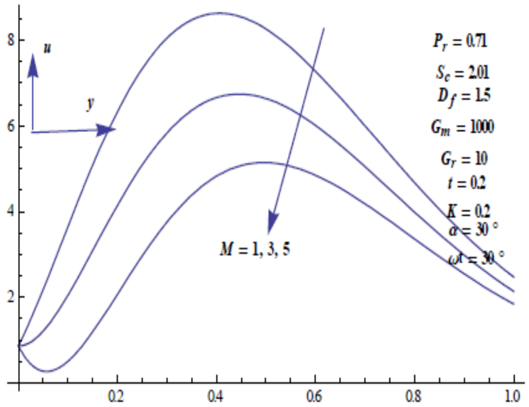


**Figure 4. Velocity profile for different values of M**

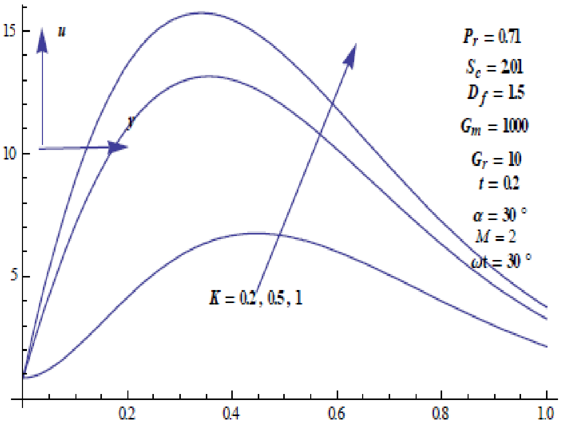


**Figure 5. Velocity profile for different values of K**

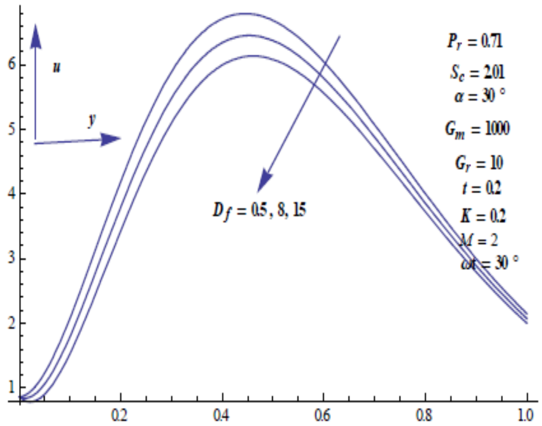


**Figure 6. Velocity profile for different values of Df**

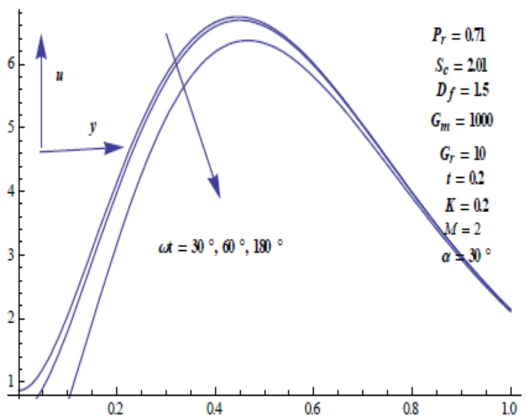


**Figure 7. Velocity profile for different values of $\omega t$**

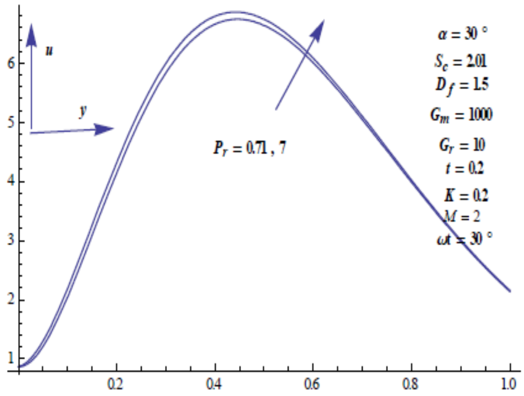


**Figure 8. Velocity profile for different values of Pr**

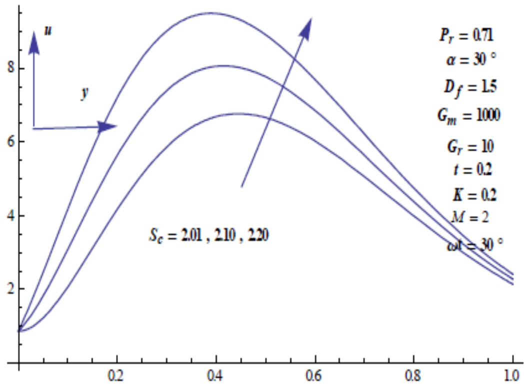


**Figure 9. Velocity profile for different values of Sc**

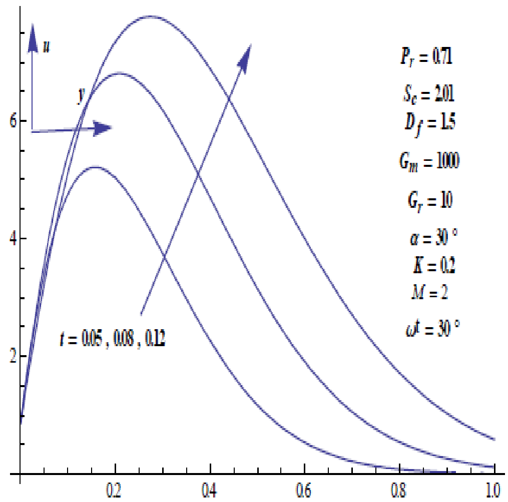


**Figure 10. Velocity profile for different values of t**

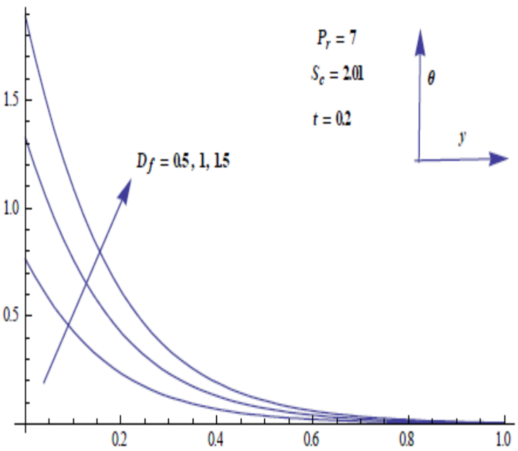


**Figure 11. Temperature profile for different values of Df**

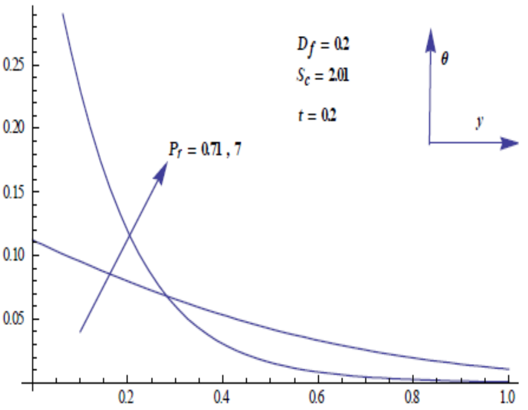


**Figure 12. Temperature profile for different values of Pr**

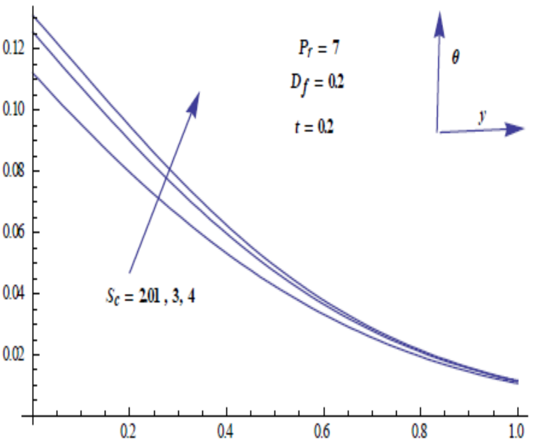


**Figure 13. Temperature profile for different values of Sc**

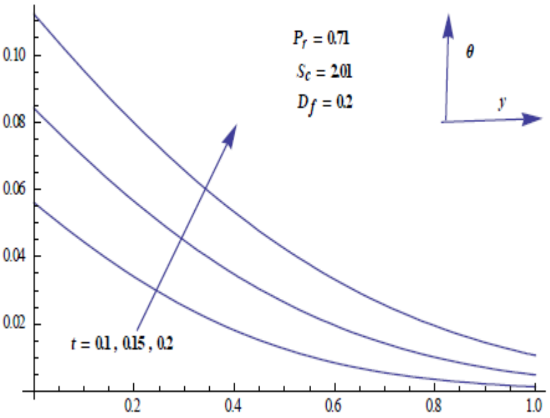


**Figure 14. Temperature profile for different values for t**

**Conclusion**

In this paper a theoretical analysis has been done to study the Dufour effect on unsteady MHD flow past an impulsively started inclined oscillating plate with variable temperature and mass diffusion. Solutions for the model have been derived by using Laplace - transform technique. Some conclusions of the study are as below:

- Velocity increases with the increase in mass Grashof Number, Prandtl number, Schmidt number, permeability parameter and time.
- Velocity decreases with the increase in the angle of inclination of plate, thermal Grashof number, the magnetic field, Dufour number and phase angle
- Temperature profile increases with the increase in Dufour number, Prandtl number , Schmidt number and time.

**Appendex**

$$A_1 = 1 + A_{18} + e^{2\sqrt{A}y}(1 - A_{19}),$$

$$A_2 = -A_1,$$

$$A_3 + A_1 = 2(1 + A_{18}),$$

$$A_4 = -1 + A_{20} - A_{26}(1 - A_{21}),$$

$$A_5 = A_4 - (1 + A_{21})(A_{27} - A_{26}),$$

$$A_6 = -1 + A_{22} - A_{26}(1 - A_{23}),$$

$$A_7 = -A_6,$$

$$A_8 = -1 + A_{24} - A_{27}(1 - A_{25}),$$

$$A_9 = -A_8,$$

$$A_{10} = -tA_{14} - \frac{y^2 A_{15} S_c}{2} - \frac{A_{28} y \sqrt{tS_c}}{\sqrt{\pi}},$$

$$A_{11} = 2A_{29}\sqrt{t} + yA_{14}\sqrt{P_r \pi},$$

$$A_{12} = 2A_{28}\sqrt{t} + yA_{15}\sqrt{S_c \pi},$$

$$A_{13} = A_{32} + A_{33} - e^{-z\sqrt{a+i\omega}} A_{34} - e^{-z\sqrt{a+i\omega}+2it\omega} A_{55},$$

$$A_{14} = -1 + erf\left[\frac{y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{15} = -1 + erf\left[\frac{y\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{16} = e^{\frac{At}{-1+P_r} - y\sqrt{\frac{AP_r}{-1+P_r}}},$$

$$A_{17} = e^{\frac{At}{-1+S_c} - y\sqrt{\frac{AS_c}{-1+S_c}}},$$

$$A_{18} = erf\left[\frac{2\sqrt{At} - y}{2\sqrt{t}}\right],$$

$$A_{19} = erf\left[\frac{2\sqrt{At} + y}{2\sqrt{t}}\right],$$

$$A_{20} = erf\left[\frac{y - 2t\sqrt{\frac{AP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{21} = erf\left[\frac{y + 2t\sqrt{\frac{AP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{22} = erf\left[\frac{2t\sqrt{\frac{A}{-1+P_r}} - y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{23} = erf\left[\frac{2t\sqrt{\frac{A}{-1+P_r}} + y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{24} = erf\left[\frac{2t\sqrt{\frac{A}{-1+S_c}} - y\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{25} = erf\left[\frac{2t\sqrt{\frac{A}{-1+S_c}} + y\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{26} =, e^{2y\sqrt{\frac{AP_r}{-1+P_r}}},$$

$$A_{27} =, e^{2y\sqrt{\frac{AS_c}{-1+S_c}}},$$

$$A_{28} =, e^{\frac{-y^2S_c}{4t}},$$

$$A_{29} =, e^{\frac{-y^2P_r}{4t}},$$

$$A_{30} = erf\left[\frac{2t\sqrt{A} - y}{2\sqrt{t}}\right],$$

$$A_{31} = erfc\left[\frac{2t\sqrt{A} + y}{2\sqrt{t}}\right],$$

$$A_{32} = e^{-y\sqrt{A+i\omega}} + e^{-y\sqrt{A-i\omega}},$$

$$A_{33} = e^{-y\sqrt{A+i\omega}+2it\omega} + e^{y\sqrt{A-i\omega}+2it\omega},$$

$$A_{34} = erf\left[\frac{y - 2t\sqrt{A-i\omega}}{2\sqrt{t}}\right] + erf\left[\frac{y + 2t\sqrt{A-i\omega}}{2\sqrt{t}}\right],$$

$$A_{35} = erf\left[\frac{y - 2t\sqrt{A+i\omega}}{2\sqrt{t}}\right] + erf\left[\frac{y + 2t\sqrt{A+i\omega}}{2\sqrt{t}}\right],$$

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