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Average Co–Isolated Locating Domination Number on Trees

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ABSTRACT

Let G (V, E) be a simple, finite, undirected connected graph. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices v, $w \in V – S$, $N(v) \cap S \neq N(w) \cap S$. A locating dominating set $S \subseteq V$ is called a co – isolated locating dominating set, if there exists atleast one isolated vertex in <V – S >. γ_{Dcild} is the number of minimum co – isolated locating dominating set of a graph G. The co – isolated locating domination number γ_{cild} is the minimum cardinality of a co – isolated locating dominating set. In this paper average co – isolated domination number $\gamma_{agcild}(G)$ is defined and, γ_{cild} and γ_{agcild} are obtained for binomial trees, binary trees, ternary trees and complete c – ary trees. Also the bounds for $\gamma_{agcild}(G)$ are found.

Introduction

Let G = (V, E) be a simple graph of order p. For $v \in V(G)$, the neighborhood $N_G(v)$ (or simply N(v)) of v is the set of all vertices adjacent to v in G. A graph which contains no cycles is said to be acyclic (forest). A tree is a connected acyclic graph. The concept of domination in graphs was introduced by Ore [9]. A non – empty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in V(G) – S is adjacent to some vertex in S. A special case of dominating set S is called a locating dominating set. It was defined by D. F. Rall and P. J. Slater in [10]. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices v, $w \in V(G) – S$, $N_G(v) \cap S$, $N_G(w) \cap S$ are distinct . The locating dominating number of G is defined as the minimum number of vertices in a locating dominating set in G. A locating dominating set $S \subseteq V(G)$ is called a co–isolated locating dominating set , if <V – S> contains atleast one isolated vertex. The minimum cardinality of a co – isolated locating dominating set is called the co – isolated locating domination number $\gamma_{cild}(G)$. γ_{Dcild} is the number of minimum co – isolated locating dominating sets of a graph G. Henning[5] introduced the concept of average independence and average domination number. The average domination number $\gamma_{ag}(G)$ is defined as $\frac{1}{|V(G)|}\sum_{v\in V(G)} \gamma_v(G)$, where $\gamma_v(G)$ is the minimum cardinality of a dominating set that contains v. The average co–isolated locating domination number $\gamma_{agcild}(G)$ defined as $\frac{1}{|V(G)|}\sum_{v\in V(G)} \gamma_{vcild}(G)$, where $\gamma_{vcild}(G)$ is the minimum cardinality of a γ_{cild} – set that contains v. In this paper γ_{cild} and γ_{agcild} are obtained for binomial trees, binary trees, ternary trees and complete c – ary trees. Also the bounds for $\gamma_{agcild}(G)$ are found.

Prior Results

The following results are obtained in [6], [7] & [8]

Theorem 2.1 [6]

For a path P_p on p vertices,

$\gamma_{cild}(P_p) = \left\lfloor \frac{2p+4}{5} \right\rfloor$, $p \geq 3$.

Theorem 2.2 [6]

If C_p ($p \geq 3$) is a cycle on p vertices, then $\gamma_{cild}(C_p) = \left\lceil \frac{2p}{5} \right\rceil$.

Theorem 2.3[8]

For any integer $n \geq 1$, $\gamma_{Dcild}(P_{5n}) = 1$.

Theorem 2.4[8]

For any integer n,

$\gamma_{Dcild}(P_{5n+1}) = \frac{(n+3)(n^2+9n+2)}{6}$, if $n \geq 1$

$\gamma_{Dcild}(P_{5n+2}) = n + 2$, if $n \geq 1$

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$\gamma_{Dcild}(P_{5n+3}) = \dfrac{(5n^2+51n-44)}{2}$, if $n \geq 4$

$\gamma_{Dcild}(P_{5n+4}) = \dfrac{(n^2+7n+8)}{2}$, if $n \geq 1$

Theorem 2.5[7]

For $n \geq 1$,

$\gamma_{Dcild}(C_{5n}) = 4$, if $n \equiv 0, 2, 4 \pmod 5$

$\gamma_{Dcild}(C_{5n}) = 10n - 2$, if $n \equiv 1, 3 \pmod 5$.

Main Results

In the following, co–isolated locating domination number for binomial trees is found.

Definition 3.1

A binomial tree of order $k \geq 0$ with root R is the tree B_k defined as follows.

(1) If $k = 0$, then $B_k = B_0 = \{R\}$. That is, the binomial tree of order zero consists of a single vertex R.

(2) If $k \geq 0$ then $B_k = \{R, B_0, B_1, \ldots, B_{k-1}\}$. That is, the binomial tree of order $k \geq 0$ comprises the root and k binomial subtrees $B_0, B_1, \ldots, B_{k-1}$.

The first five binomial subtrees are given in Figure 3.1.

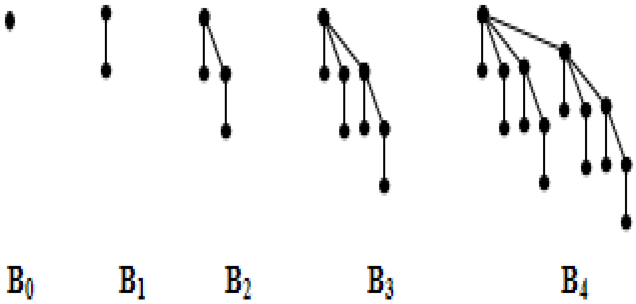

Figure 3.1

Theorem 3.1

For a binomial tree B_k, $\gamma_{cild}(B_k) = 2^{k-1}$, where $k \geq 0$.

Proof

In a binomial tree B_k, each support has exactly one leaf (a vertex of degree 1) and there are no intermediate vertices of degree 2. By the definition of co–isolated locating dominating set, γ_{cild} – sets of B_k contain either all the leaves or its corresponding support. If S is a γ_{cild} – set of B_k, then $N(u) \cap S \neq N(v) \cap S$ for $u, v \in V(B_k) - S$. Hence $\gamma_{cild}(B_k)$ is the number of leaves of B_k. Let n_k be the number of leaves of B_k. For $k = 1$, the number n_1 of leaves of B_1 is $1 = 2^{1-1} = 2^0$. For $k = 2$, the number n_2 of leaves of B_2 is $2 = 2^1$. Proceeding like this, the number of leaves in B_k is given by $n_k = 2^k$, for all $k \geq 1$. Hence $\gamma_{cild}(B_k) = 2^{k-1}$.

Definition 3.2

A tree in which one vertex (called the root) is distinguished from all the others is called a rooted tree. In a rooted tree, the level (depth) of a vertex v is the length of the unique path from the root to v. Hence the root is at level 0. The height of a rooted tree is the length of a longest path from the root. If the vertex u immediately precedes the vertex v on the path from the root to v, then u is the parent of v and v is a child of u. Vertices having the same parent are called siblings. A vertex v is said to be descendant of a vertex u (u is then said to be an ancestor of v), if u is on the unique path from the root to v. If, in addition, $v \neq u$, then v is a proper descendant of u (u is a proper ancestor of v).

Definition 3.3

A leaf in a rooted tree is any vertex having no children. An internal vertex of a rooted tree is any vertex that is not a leaf.

Definition 3.4

An m – ary tree ($m \geq 2$) is a rooted tree in which each vertex has less than or equal to m children. When m = 2, 3, the corresponding m – ary trees are called as binary tree and ternary tree respectively.

In the following, co–isolated locating domination number for binary trees is found.

Theorem 3.2

$$\gamma_{cild}(B_t) = \begin{cases} \dfrac{2^2\times((2^3)^k - 1)}{2^3 - 1}; & t = 3k \\ \dfrac{((2^3)^k - 1)}{2^3 - 1}; & t = 3k+1 \\ \dfrac{2^1\times((2^3)^k - 1)}{2^3 - 1}; & t = 3k+2 \end{cases}$$

Proof

Let B_t denote a binary tree formed at the depth t. Let B'_t set of vertices in the t[th] stage. Then B'_1 = {the root}, $B'_1 = \{v_{11}\}$, $V(B_1) = B'_1$ and $|B'_1| = 1$. Since each parent has two children, $B'_2 = \{v_{21}, v_{22}\}$ and $V(B_2) = B'_1 \cup B'_2$ and $|B'_2| = 2^1 = 2$. Similarly $B'_3 = \{v_{31}, v_{32}, v_{33}, v_{34}\}$ and $V(B_3) = B'_1 \cup B'_2 \cup B'_3$ and $|B'_3| = 2^2 = 4$. Proceeding in this way, the Binary tree B_t is obtained by $B'_t = \{v_{t1}, v_{t2}, \ldots, v_{t(2^{t-1})}\}$; $V(B_3) = B'_1 \cup B'_2 \cup \ldots \cup B'_t$ and $|B'_t| = 2^{t-1}$. Let S_t be the γ_{cild} – set of B_t. From the definition of the co – isolated locating dominating set, it is observed that S_t contains 2^{t-2} vertices on the t[th] stage (that is, one child form each parent) and 2^{t-2} vertices on the (t – 1)[th] stage (that is, all the parents) no vertices on the (t – 2)[th] stage and so on.

Let t take the values 3k, 3k +1, 3k+2 and the theorem is proved by the method of induction on k.

Case 1 t = 3k

If k = 1, then a γ_{cild} – set S_3 of B_3 is given by $S_3 = \{v_{31}, v_{32}, v_{21}, v_{22}\}$ and hence $\gamma_{cild}(B_3) = 4$.

If k = 2, a γ_{cild}–set S_6 of B_6 is given by

$S_6 = \{v_{61}, v_{63}, v_{65}, \ldots, v_{6(2^5-1)}, v_{51}, v_{52}, \ldots, v_{5(2^4)}\} \cup S_3$ and hence $\gamma_{cild}(B_6) = 2^4 + 2^4 + 2^2 = 2^2(2^3 + 1)$.

Similarly if k = 3, then a γ_{cild} – set S_9 of B_9 is given by

$S_9 = \{v_{91}, v_{93}, \ldots, v_{9(2^8-1)}, v_{81}, v_{82}, \ldots, v_{87}\} \cup S_6$ and hence $\gamma_{cild}(B_9) = 2^7 + 2^7 + 2^5 + 2^2 = 2^8 + 2^5 + 2^2 = 2^2((2^3)^2 + 2^3 + 1)$.

By induction hypothesis

if k = k – 1, $\gamma_{cild}(B_{3(k-1)}) = 2^2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2})$. Therefore, $\gamma_{cild}(B_{3k}) = \gamma_{cild}(B_{3(k-1)}) + 2^{3k-2} + 2^{3k-2} = 2^2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2}) + 2^{3k-1} = 2^2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2} + (2^3)^{k-1}) = \frac{2^2 \times ((2^3)^k - 1)}{2^3 - 1}$.

Case 2 t = 3k + 1

If k = 1, then a γ_{cild} -set S_4 of B_4 is given by

$S_4 = \{v_{41}, v_{43}, v_{45}, v_{47}, v_{31}, v_{32}, v_{33}, v_{34}, v_{11}\}$ and $\gamma_{cild}(B_4) = 2^2 + 2^2 + 1 = 2^3 + 1$.

If k = 2, then a γ_{cild}–set S_7 of B_7 is given by

$S_7 = \{v_{71}, v_{73}, \ldots, v_{7(2^6-1)}, v_{61}, v_{62}, \ldots, v_{6(2^5)}\} \cup S_4$ and $\gamma_{cild}(B_7) = 2^5 + 2^5 + 2^3 + 1 = 2^6 + 2^3 + 1 = 1 + 2^3 + (2^3)^2$.

By induction hypothesis if k = k – 1, $\gamma_{cild}(B_{3(k-1)+1}) = 1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-1}$ and $\gamma_{cild}(B_{3k+1}) = \gamma_{cild}(B_{3(k-1)+1}) + (2^3)^{k-1} + (2^3)^{k-1} = 1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-1} + (2^3)^k = \frac{((2^3)^k - 1)}{2^3 - 1}$.

Case 3 t = 3k + 2

If k = 1, then a γ_{cild} – set S_5 of B_5 is given by

$S_5 = \{v_{51}, v_{53}, \ldots, v_{5(2^4-1)}, v_{41}, v_{42}, \ldots, v_{48}, v_{21}, v_{22}\}$ and $\gamma_{cild}(B_5) = 2^3 + 2^3 + 2^1 = 2(1 + 2^3)$.

Similarly if k = 2, then a γ_{cild} – set S_8 of B_8 is given by

$S_8 = \{v_{81}, v_{83}, \ldots, v_{8(2^7-1)}, v_{71}, v_{72}, \ldots, v_{7,(2^6)}\} \cup S_5$ and $\gamma_{cild}(B_8) = 2^6 + 2^6 + 2^4 + 2^2 = 2^7 + 2^4 + 2 = 2(1 + 2^3 + (2^3)^2)$.

By induction hypothesis if k = k – 1, then $\gamma_{cild}(B_{3(k-1)+2}) = 2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-1})$ Therefore, $\gamma_{cild}(B_{3k+2}) = \gamma_{cild}(B_{3(k-1)+2}) + 2^{3k} + 2^{3k} = 2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-1} + (2^3)^k) = \frac{2^1 \times ((2^3)^k - 1)}{2^3 - 1}$

Remark 3.1

For a ternary tree T_t, in which each parent has 3 children with the depth t, a co – isolated locating dominating set contains two children from each parent in the t[th] stage and all the parents in the (t – 1)[th] stage and no vertices on the (t – 2)[th] stage and so on.

Therefore $\gamma_{cild}(T_t)$ is given by

$$\gamma_{cild}(T_t) = \begin{cases} \frac{3^2 \times ((3^3)^k - 1)}{3^3 - 1}; t = 3k \\ \frac{((3^3)^k - 1)}{3^3 - 1}; t = 3k + 1 \\ \frac{3^1 \times ((3^3)^k - 1)}{3^3 - 1}; t = 3k + 2 \end{cases}$$

Definition 3.5

A complete c –ary tree is an c – ary tree in which each internal vertex has exactly c children and all leaves have the same depth.

Remark 3.2

For a complete c – ary binary tree B_c, in which each parent has c children with the depth t, then $\gamma_{cild}(B_c)$ is given by

$$\gamma_{cild}(B_c) = \begin{cases} \frac{c^2 \times ((c^3)^k - 1)}{c^3 - 1}; t = 3k \\ \frac{((c^3)^k - 1)}{c^3 - 1}; t = 3k + 1 \\ \frac{c^1 \times ((c^3)^k - 1)}{c^3 - 1}; t = 3k + 2 \end{cases}$$

In the following, bounds on γ_{agcild} are obtained.

Theorem 3.3

For any connected graph G, $\gamma_{cild}(G) \leq \gamma_{agcild}(G) \leq \gamma_{cild}(G) + 1$.

Proof

Let G be a connected graph with |V(G)| = p. Let $S_1, S_2, \ldots, S_r$, S be γ_{cild} – sets of G with $|S_1| = |S_2| = \ldots = |S_r| = |S| = \gamma_{cild}(G)$ and $S_1 \neq S_2 \neq \ldots \neq S_r$. Hence $\gamma_{vcild}(G) = |S|$, for every $v \in S_i$; i = 1, 2, …, r. If there exists a vertex u which does not belong to S_i, for i = 1, 2, …, r, then $\gamma_{cild}(G) = |S| + 1$, since including the vertex u to any γ_{cild} – set also forms a γ_{cild} – set of G and let the number of these vertices be k. Let A be the set of vertices which does not belong to any of the γ_{cild} – set of G with |A| = p – k. By definition,

$$\gamma_{agcild}(G) = \frac{1}{|V(G)|} \sum_{v \in V(G)} \gamma_{vcild}(G)$$

$$= \frac{1}{p} \left(\sum_{v_i \in S_i} \gamma_{vcild}(G) + \sum_{v_i \notin S_i} \gamma_{vcild}(G) \right)$$

$$= \frac{1}{p} (|S| \times k + |A| (|S| + 1))$$

$$= \frac{1}{p} (|S| \times k + (p - k) \quad (|S| + 1))$$

$$= \frac{1}{p} \times p \times |S| + \frac{(p-k)}{p}$$

$$\leq |S| + 1, \text{ since } p - k < p.$$

Hence, $\gamma_{agcild}(G) + 1 \leq \gamma_{cild}(G)$.

In the following, average co–isolated locating domination number for binomial trees, binary trees, ternary trees and complete n – ary trees are obtained.

Observation 3.1

There are 3 possibilities for γ_{agcild} – sets of G.

(1) Each vertex of G belongs to any one of γ_{cild} – sets of G and in this case, $\gamma_{cild}(G) = \gamma_{agcild}(G)$. This is illustrated in Example 3.1.

Example 3.1

Let G ≅ C_7 and $V(C_7) = \{v_1, v_2, \ldots, v_7\}$. Let S_r (r = 1, 2, 3) be γ_{cild} – sets of C_7 with $S_1 = \{v_1, v_4, v_6\}$; $S_2 = \{v_2, v_5, v_7\}$ and $S_3 = \{v_1, v_3, v_6\}$. Therefore, $|S_r| = 3$, r = 1, 2, 3. Also all the vertices belong to any one of the γ_{cild} – sets of C_7. Hence, $\gamma_{cild}(G) = \gamma_{agcild}(G) = 3$.

(2) If there exists exactly one γ_{cild} – set S of G, then $\gamma_{agcild}(G) = |S| + \frac{|V(G)| - |S|}{|V(G)|}$. This is illustrated in Example 3.2.

Example 3.2

Let G ≅ P_5 and $V(P_5) = \{v_1, v_2, v_3, v_4, v_5\}$. Then S = {$v_2, v_4$} is the only γ_{cild} – set of G. $\gamma_{agcild}(G) = \frac{2\times2+(5-2)\times(2+1)}{5} = 2 + \frac{3}{5} = |S| + \frac{|V(G)| - |S|}{|V(G)|}$.

(3) If there exists more than one γ_{cild} – sets of G and if there exists atleast one vertex that does not belong to any of the γ_{cild} – sets of G, then $\gamma_{agcild}(G) = |S| + \frac{p-k}{p}$ where |V(G)| = p; S is a γ_{cild} – set of G and k is the number of vertices belonging to any one of the γ_{cild} – sets. This is illustrated in Example 3.3.

Example 3.3

Let G ≅ P_9 and $V(P_9) = \{v_1, v_2, \ldots, v_9\}$. Let S_r; r = 1, 2, 3; be a γ_{cild} – set of P_9 with $S_1 = \{v_1, v_3, v_6, v_8\}$; $S_2 = \{v_2, v_4, v_7, v_9\}$ and $S_3 = \{v_1, v_4, v_6, v_8\}$. Therefore, $|S_r| = 3$, r = 1, 2, 3 and v_5 does not belong to any γ_{cild} – sets of P_9. Hence, $\gamma_{agcild}(P_9) = \frac{8 \times 4 + 1 \times 9}{9} = 4 + \frac{1}{9}$.

Observation 3.2

Let S be a γ_{cild} – set of a connected graph G. If A denotes the set of vertices which does not belong to any of the γ_{cild} – sets of G, then $\gamma_{agcild}(G) = |S| + \frac{|A|}{|V(G)|}$.

Remark 3.3

Let S be a γ_{cild} – set of the binomial tree B_k. Then if S_1 is a set obtained from S by removing the supports from S and including the corresponding leaves, then S_1 will also be a γ_{cild} – set of B_k. Similarly all the roots can be included in any one of the γ_{cild} – sets of B_k.

Therefore $\gamma_{agcild}(B_k) = \gamma_{cild}(B_k)$.

Theorem 3.4

Let A_t denote the set of vertices which does not belong to any of the γ_{cild} – sets of binary tree B_t, then

$$|A_t| = \begin{cases} \frac{(2^3)^k - 1}{2^3 - 1}; t = 3k \\ \frac{2 \times (2^3)^k - 1}{2^3 - 1}; t = 3k+1 \\ \frac{2^2 \times (2^3)^k - 1}{2^3 - 1}; t = 3k+2 \end{cases}$$

Proof

Let B_t denote a binary tree formed at the depth t and B'_t denote the number of vertices at the t^th^ stage. Let S_t be the γ_{cild} – set of B_t. Let A_t denote the number of vertices which does not belong to any of the γ_{cild} – sets of B_t. Let the children in B_t be denoted by v_{ij}.

There exist exactly two γ_{cild} – sets of B_t, one having the child with the second suffix j even and the other having the second suffix j odd. From the definition of the co – isolated locating dominating set and from the structure of the binary tree B_t, it is observed that the set S_t contains 2^{t-2} vertices on the t^th^ stage (That is, one γ_{cild} – set contains one child from each parent having even second suffix and the other γ_{cild} – set contains child having odd second suffix) and 2^{t-2} vertices on $(t-1)^{th}$ stage (That is, all the parents) and no vertices in the $(t-2)^{th}$ stage and so on.

Let the values of t be 3k, 3k+1, 3k+2. This theorem is proved by the method of induction on k.

Case 1 t = 3k

If k = 1, then the set A_3 of B_3 is given by $A_3 = B'_1$; $|A_3| = 2^0 = 1$.

If k = 2, then the set A_6 of B_6 is given by $A_6 = B'_4 \cup A_3$; $|A_6| = 2^3 + 1$.

Similarly if k = 3, then the set A_9 of B_9 is given by $A_9 = A_{3(3)} = B'_7 \cup A_6$ and $|A_9| = 2^6 + 2^3 + 1 = 1 + 2^3 + (2^3)^2$.

By induction hypothesis if k = k – 1, then $|A_{3(k-1)}| = 1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2}$.

Now for k, $A_{3k} = B'_{3(k-1)+1} \cup A_{3(k-1)}$

$|A_{3k}| = 1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2} + (2^3)^{k-1} = \frac{(2^3)^k - 1}{2^3 - 1}$.

Case 2 t = 3k + 1

If k = 1, then the set A_4 of B_4 is given by $A_4 = B'_2$;

$|A_4| = 2^1 = 2$.

If k = 2, then the set A_7 of B_7 is given by $A_7 = B'_5 \cup A_4$;

$|A_7| = 2^4 + 2 = 2(1 + 2^3)$.

Similarly if k = 3, then the set A_{10} of B_{10} is given by

$A_{10} = A_{3(3)+1} = B'_8 \cup A_7$ and $|A_{10}| = 2^7 + 2^4 + 1 = 2(1 + 2^3 + (2^3)^2)$.

By induction hypothesis if k = k – 1, then

$|A_{3(k-1)+1}| = 2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2})$.

Now for k, $|A_{3k+1}| = 2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2} + (2^3)^{k-1}) = 2 \times \frac{(2^3)^k - 1}{2^3 - 1}$.

Case 3 t = 3k + 2

If k = 1, then the set A_5 of B_5 is given by $A_5 = B'_3$;

$|A_5| = 2^2 = 4$.

If k = 2, then the set A_8 of B_8 is given by $A_8 = B'_6 \cup A_5$;

$|A_8| = 2^5 + 2^2 = 2^2(1 + 2^3)$.

Similarly if k = 3, then the set A_{11} of B_{11} is given by

$A_{11} = A_{3(3)+2} = B'_9 \cup A_8$ and

$|A_{11}| = 2^8 + 2^5 + 2^2 = 2^2(1 + 2^3 + (2^3)^2)$.

By induction hypothesis if k = k – 1, then for

$|A_{3(k-1)+2}| = 2^2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2})$.

Now for k, $|A_{3k+2}| = 2^2(1 + 2^3 + (2^3)^2 + \ldots + (2^3)^{k-2} + (2^3)^{k-1}) = 2^2 \times \frac{(2^3)^k - 1}{2^3 - 1}$.

Remark 3.4

The average co – isolated locating domination number for the binary tree B_t is given by

$\gamma_{agcild}(B_t) = \gamma_{cild}(B_t) + \frac{A_t}{|V(B_t)|}$ where $\gamma_{cild}(B_t)$ and $|A_t|$ are given in Theorem 3.2 and Theorem 3.4.

Remark 3.5

For a ternary tree T_t, if A_{tr} denote the set of vertices which does not belong to any of the γ_{cild} – sets of the ternary tree T_t, then

$$A_{tr} = \begin{cases} \dfrac{(3^3)^k - 1}{3^3 - 1}; t = 3k \\ \dfrac{3 \times (3^3)^k - 1}{3^3 - 1}; t = 3k+1 \\ \dfrac{3^2 \times (3^3)^k - 1}{3^3 - 1}; t = 3k+2 \end{cases}$$

Remark 3.6

For a complete c – ary tree B_c, if A_c denote the set of vertices which does not belong to any of the γ_{cild} – sets of the tree B_c the set A_c is given by

$$A_c = \begin{cases} \dfrac{(c^3)^k - 1}{c^3 - 1}; t = 3k \\ \dfrac{c \times (c^3)^k - 1}{c^3 - 1}; t = 3k+1 \\ \dfrac{c^2 \times (c^3)^k - 1}{c^3 - 1}; t = 3k+2 \end{cases}$$

Conclusion

In this paper, the average co – isolated locating domination number is defined and $\gamma_{cild}(G)$ and $\gamma_{agcild}(G)$ are obtained for binomial trees, binary trees, ternary trees and complete c – ary trees. Also the bounds for $\gamma_{agcild}(G)$ are found. $\gamma_{agcild}(G)$ can be related with other parameters of G.

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