Available online at www.elixirpublishers.com (Elixir International Journal)

Advanced Mathematics

Elixir Adv. Math. 92 (2016) 38743-38748

ISSN: 2229-712X

Hamilton Equations on a Contact 5-Manifolds

Zeki KASAP

ARTICLE INFO

Article history:
Received: 18 January 2016;
Received in revised form:
1 March 2016;
Accepted: 4 March 2016;

Keywords
Contact Manifold,
Mechanical System,
Dynamic Equation,
Hamiltonian Formalism.

ABSTRACT

It is well known that a dynamical system is a concept in mathematics where a fixed rule describes how a point in a geometrical space depends on time. A mathematical model is a precise representation of a system's dynamics used to answer questions via analysis and simulation. Mathematica models allow us to reason about a system and make predictions about who a system will behave. Contact geometry is the odd-dimensional analogue of symplectic geometry. It is close to symplectic geometry and like the latter it originated in questions of classical and analytical mechanics. If contact geometry is considered as a symplectic geometry, it has broad applications in mathematical physics, geometrical optics, classical mechanics, analytical mechanics, mechanical systems, thermodynamics, geometric quantization and applied mathematics such as control theory. It is well known fact that one way of solving problems in classical mechanics occur with the help of the Hamilton equations. Hamiltonian method is particularly important because of its utility in formulating quantum mechanics. In this study, Hamilton equations as representive the object motion were found on a contact 5-manifolds. Also, implicit solutions of the differential equations found in this study are solved by Maple computation program.

Introduction

Dynamical systems are mathematical objects used to model physical phenomena whose state changes over time that its can be viewed in two different ways: the internal and the external view. The prototype (mechanical) problem is describing the motion of the planets. It is natural to give a complete characterization of the motion of all planets that this involves careful analysis of the effects of gravitational pull and the relative positions of the planets in a system for mechanical problems.

Classical mechanics, under the influence of specified force laws, is the investigation of the motion of dynamical systems of particles in Euclidean three-dimensional space. Also, the motion's evolution determined by Newton's second law that is a differential equation. So, mechanical problems are to determine the positions of all the particles at all times for given certain laws determining physical forces, some boundary conditions on the positions of the particles at some particular times. Classical mechanics of a system of point particles and rigid object is usually divided into statics, kinematics and dynamics.

Classical field theory utilizes traditionally the language of Hamiltonian dynamics. Hamiltonian mechanics is a theory developed as a reformulation of classical mechanics. Also, this theory has extended to time-dependent classical mechanics.
Contact geometry has been seen to underlay many physical phenomena and be related to many other mathematical structures. Contact geometry is in many ways an odd-dimensional counterpart of symplectic geometry such that it belongs to the even-dimensional world. Both contact and symplectic geometry are motivated by the mathematical formalism of classical and analytical mechanics. Besides, one can consider either the even-dimensional phase space of a mechanical system or the odd-dimensional extended phase space that includes the time variable.

A mathematical model is a precise representation of a system's dynamics used to answer questions via analysis and simulation. The mathematical models choose depends on lots of questions, so there may be multiple models for physical systems in the space.

In this study, the movements for moving objects modeling Hamilton equations to be found on the space defined on contact 5-manifolds. Also, the graphics of the path taken by the object that will be drawn when the angle changes. *Bellettini* obtained almost complex structures J that satisfy, for any vector v in the horizontal distribution, $d\alpha(v,Jv)=0$ such that a contact manifold is (M^5,α) [1]. *Janssens* and *Vanhecke* determined an orthogonal decomposition of the vector space of some curvature tensors on a co-Hermitian real vector space [2]. *Chaubey* studied some geometrical properties of almost contact metric manifolds equipped with semi-symmetric non-metric connection [3]. *Kodama* classified the local structure of complex contact manifolds of dimension three with Legendrian vector fields [4]. *Piercey* defined contact manifolds and identify simple examples and basic properties [5]. *Doubrov* and *Komrakov* submitted the complete classification of all real Lie algebras of contact vector fields on the first jet space of one-dimensional submanifolds in the plane [6]. *Attarchi* and *Rezaii* submitted that a comprehensive study of contact and Sasakian structures on the indicatrix bundle of Finslerian warped product manifolds is reconstructed [7]. *Kashiwara* showed that the existence of the stack of micro-differential modules on an arbitrary contact manifold, although he cannot expect the global existence of the ring of micro-differential operators [8]. *Manev* and *Gribachev* examined the main classes of almost contact manifolds with B-metric [9]. *Iglesias-Ponte* and *Wade* gave simple characterizations of contact 1-forms in terms of

E-mail address: zekikasap@hotmail.com

Dirac structures [10]. *Manev* and *Ivanova* examined that the canonical-type connection on the almost contact manifolds with B-metric is constructed [11]. *Etnyre* showed any almost contact structure on a 5-manifold is homotopic to a contact structure [12]. *Sekiya* introduced generalized almost contact structures which admit the B-field transformations on odd dimensional manifolds [13]. *Dwivedi* et al proved that a (k,μ)-manifold with vanishing Endo curvature tensor is a Sasakian manifold [14]. *Malek* and *Balgeshir* introduced the notion of slant submanifold of an almost contact metric 3-structure manifold [15]. *Davidov* observed that the CR-structures on the twistor space are induced by almost contact metric structures [16]. *Kasap* and *Tekkoyun* examined Lagrangian and Hamiltonian formalism for mechanical systems using para/pseudo-Kähler manifolds, representing an interesting multidisciplinary field of research [17]. *Kasap* obtained that the Weyl-Euler-Lagrange and Weyl-Hamilton equations on $\mathbb{R}^{2n}_n$ such that is a model of tangent manifolds of constant W-sectional curvature [18]. *Tekkoyun* showed paracomplex analogue of Euler-Lagrange and Hamiltonian equations [19].

Preliminaries

Definition 1

A real dynamical system, real-time dynamical system, continuous time dynamical system, or flow is a triple (T,M,Φ) where $T=(a,b)$ is a monoid, written additively, M a manifold locally diffeomorphic to a Banach space and $\Phi(t,x)$ is a function $\Phi(t,x):U\subset T\times M\rightarrow M$ with

$$I(x)=\{t\in T: (t,x)\in U\},$$
$$\Phi(0,x)=x,\ \Phi(t_2,\Phi(t_1,x))=\Phi(t_1+t_2,x), \qquad (1)$$

for $t_1,t_2,t_1+t_2\in I(x)$. The function $\Phi(t,x)$ is called the evolution function of the dynamical system: it associates to every point in the set M a unique image, depending on the variable t, called the evolution parameter. M is called phase space or state space, while the variable x is called initial state of the system. If the manifold M is locally diffeomorphic to $\mathbb{R}^n$, the dynamical system is finite-dimensional; if not, the dynamical system is infinite-dimensional.

Definition 2

In three dimensions, the vector from the origin to the point with cartesian coordinates (x,y,z) can be written as [20] :

$$r=xi+yj+zk=x\left(\frac{\partial}{\partial x}\right)+y\left(\frac{\partial}{\partial y}\right)+z\left(\frac{\partial}{\partial z}\right). \qquad (2)$$

Definition 3

Let M be a manifold of odd dimension (2n+1). A contact structure is a maximally non-integrable hyperplane field $\xi=\ker\alpha\subset TM$, that is, the defining 1-form α is required to satisfy $\alpha\wedge(d\alpha)^n\neq 0$ (meaning that it vanishes nowhere). Such a 1-form α is called a contact form. The pair (M,ξ) is called a contact manifold.

Definition 4

Symplectic geometry is a branch of differential geometry and differential topology that studies symplectic manifolds; that is, differentiable manifolds equipped with a closed, nondegenerate 2-form.

Symplectic geometry has its origins in the Hamiltonian formulation of classical mechanics where the phase space of certain classical systems takes on the structure of a symplectic manifold.

Definition 5

Let V be a vector space. Let $\omega:V\times V\rightarrow\mathbb{R}$ be a skew-symmetric, bilinear 2-form, $\omega\in\Lambda^2V^*$. The form ω is nondegenerate if for every $v\in V$, $\omega(v,u)=0$, $\forall u\in V\Rightarrow v=0$. Note that since ω is skew-symmetric $\omega(u,v)=-\omega(v,u)$, hence $\omega(v,v)=0$.

Definition 6

Let M^{2n} be an even-dimensional manifold. A symplectic structure on M^{2n} is a closed nondegenerate differential 2-form ω on M^{2n}: (1) $d\omega=0$ is closed, (2) $\forall x\in M$, $\exists\xi\in T_xM$, if $\omega(\xi,\eta)=0$, $\forall\eta\in T_xM$, then $\xi=0$ (nondegenerate).

The pair (M,ω) a symplectic manifold. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds, in the Hamiltonian formulation of classical mechanics, which provides one of the major motivations for the field. The set of all possible configurations of a system is modelled as a manifold, and this manifold's cotangent bundle describes the phase space of the system.

Example 1

An almost complex symplectic manifold is standard Euclidean space $(\mathbb{R}^{2n},\omega_0)$ with its standard almost complex structure J_0 obtained from the usual identification with $\mathbb{C}^n$. Thus, one sets $z_j=x_{2j-1}+ix_{2j}$ for j=1,...,n and defines J_0 by

$$J_0(\partial_{2j-1})=\partial_{2j},\quad J_0(\partial_{2j})=-\partial_{2j-1} \qquad (3)$$

where $\partial j=\partial/\partial x_j$ is the standard basis of $T_x\mathbb{R}^{2n}$ [21].

Lemma 1

Let M be a smooth manifold. If M admits a complex structure A , then M admits an almost complex structure J. Let $\dim_{\mathbb{C}}M=m$ and (z,U) be any holomorphic chart inducing a coordinate frame $\partial x_1,\partial y_1,...,\partial x_m,\partial y_m$. Then J is given locally as

$$Jp(\partial x_i|p)=\partial y_i|p\ ,\quad Jp(\partial y_i|p)=-\partial x_i|p, \qquad (4)$$

where $1\leq i\leq m$ and $p\in U$ [22].

Definition 7

A pseudo J-holomorphic curve is a smooth map from a Riemannian surface into an almost complex manifold such that satisfies the Cauchy-Riemann equation [21].

Definition 8

Let M be a differentiable manifold of dimension (2n+1), and suppose J is a differentiable vector bundle isomorphism $J:TM\rightarrow TM$ such that $J_x:T_xM\rightarrow T_xM$ is a (almost) complex structure for T_xM, i.e. $J^2=J\circ J=-I$ where I is the identity (unit) operator on V. Then J is called an (almost) complex structure for the differentiable manifold M. A manifold with a fixed (almost) complex structure is called an (almost) complex manifold.

Definition 9

An almost complex structure on a differentiable manifold M^{2n} is a differentiable endomorphism on the tangent bundle $J:T_{\mathbb{R}}M\rightarrow T_{\mathbb{R}}M$ with $J^2=-Id$. A differentiable manifold with some fixed almost complex structure is called an almost complex manifold.

A celebrated theorem of Newlander and Nirenberg [23] says that an almost complex structure is a complex structure if and only if its Nijenhuis tensor or torsion vanishes.

Theorem 1

The almost complex structure J on M is integrable if and only if the tensor N_J vanishes identically, where N_J is defined on two vector fields X and Y by

$$N_J[X,Y]=[JX,JY]-J[X,JY]-J[JX,Y]-[X,Y]. \quad (5)$$

The tensor (2,1) is called the Nijenhuis tensor (5). We say that J is torsion free if $N_J=0$. Complex Nijenhuis tensor of an almost complex manifold (M,J) is given by (5).

Complex Structures on Contact 5-Manifolds

A 5-manifold is a 5-dimensional topological manifold, possibly with a piecewise linear or smooth structure. Contact geometry is the study of a geometric structure on smooth manifolds given by a hyperplane distribution in the tangent bundle and specified by a one-form.

Definition 10

Assume that, on a contact 5-manifold (M^5,α), given a horizontal 2-form ω is given, that satisfies $\omega\wedge d\alpha=0$ and $\omega\wedge\omega\neq 0$.
Here it should be understood ω is horizontal. Decompose $\omega=\omega_+ + \omega_-$, where ω_+ is the self-dual part and ω_- is the anti self-dual part and $\omega\wedge d\alpha=\omega_+\wedge d\alpha+\omega_-\wedge d\alpha$. The notation ‖ ‖ denotes here the standard norm for differential forms coming from the metric on the manifold and $\omega_+=((\sqrt{2})/(\|\omega_+\|))\omega_+$. We can choose an orthonormal basis for P∈M of the form $\{e_1=X, e_2=IX, e_3=Y, e_4=IY\}$ and denote by $\{e^1,e^2,e^3,e^4\}$ the dual basis of orthonormal one-forms. Then dα has the form $e^1\wedge e^2+e^3\wedge e^4$. The forms $e^1\wedge e^2+e^3\wedge e^4$, $e^1\wedge e^3+e^4\wedge e^2$ and $e^1\wedge e^4+e^2\wedge e^3$ are an orthonormal basis for $\wedge_+^2$. The fact that ω_+ is orthogonal to dα implies that $\omega_+=a(e^1\wedge e^3+e^4\wedge e^2)+b(e^1\wedge e^4+e^2\wedge e^3)$ and $\|\omega_+\|^2=2(a^2+b^2)$, therefore $\omega_+=\cos\theta(e^1\wedge e^3+e^4\wedge e^2)+\sin\theta(e^1\wedge e^2+e^3\wedge e^4)$ for some θ depending on the chosen point,

$$\cos\theta=a/\sqrt{a^2+b^2}\,,\ \sin\theta=b/\sqrt{a^2+b^2}\,. \quad (6)$$

Proposition 1

Then the explicit expression J are, any point v∈P, there exist local coordinates (x_1,x_2,x_3,x_4,θ) centered at P,

$$J(e_1)=\cos\theta e_3+\sin\theta e_4,\ J(e_2)=-\cos\theta e_4+\sin\theta e_3,$$
$$J(e_3)=-\cos\theta e_1-\sin\theta e_2,\ J(e_4)=\cos\theta e_2-\sin\theta e_1. \quad (7)$$

Proposition 2

The dual form J* of the above J is as follows:

$$J^*(dx_1)=\cos\theta dx_3+\sin\theta dx_4,\ J^*(dx_2)=-\cos\theta dx_4+\sin\theta dx_3,$$
$$J^*(dx_3)=-\cos\theta dx_1-\sin\theta dx_2,\ J^*(dx_4)=\cos\theta dx_2-\sin\theta dx_1, \quad (8)$$

and an easy computation shows that $d\alpha(v,J(v))=0$ for any v∈P. The above structures (7) have been taken from [1].

Proof

Instead of J conformal structure representing the structure of J* will be used and $e_i=dx_i$. J* denote the structure of the holomorphic property:

$$J^{*2}(dx_1)=\cos\theta J^*(dx_3)+\sin\theta\ J^*(dx_4)=-dx_1, \quad (9)$$

and similar manner it is shown that

$$J^{*2}(dx_i)=-dx_i,\ i=1,...,4. \quad (10)$$

As can be seen from (9) and (10) $J^{*2}=-I$ are the complex structures.

Hamiltonian Mechanical System

It is well-known that a Hamiltonian space has been certified as an excellent model for some important problems in relativity, gauge theory and electromagnetism. Hamilton's equations can be easily shown to be equivalent to Newton's equations. Also, Hamiltonian gives a model for both the gravitational and electromagnetic field in a very natural blending of the geometrical structures of the space with the characteristic properties of these physical fields and Hamiltonian dynamics is used as a model for field theory, quantum physics, optimal control, biology and fluid dynamics.

Lemma 2. The closed 2-form on a vector field and 1-form reduction function on the phase space defined of a mechanical system is equal to the differential of the energy function 1-form of the Lagrangian and the Hamiltonian mechanical systems [24,25].

Theorem 2

If α and β are 1-forms, then $\alpha\wedge\beta$ is a 2-forms.

Definitions 11

Let M is the configuration manifold and its cotangent manifold T*M. By a symplectic form we mean a 2-form Φ on T*M such that
(i) Φ is closed , that is, $d\Phi=0$; (ii) for each z∈ T*M, $\Phi: T^*M\times T^*M\to\mathbb{R}$ is weakly nondegenerate. If Φz in (ii) is nondegenerate, we speak of a strong symplectic form. If (ii) is dropped we refer to Φ as a presymplectic form. Let (T*M,Φ) be a symplectic manifold. A vector field $X_H:T^*M\to T^*M$ is called Hamiltonian if there is a C^1 function $H:T^*M\to\mathbb{R}$ such that dynamical equation is determined by

$$i_{X_H}\Phi=dH. \quad (11)$$

We can say that X_H is locally Hamiltonian vector field if $i_{X_H}\Phi$ is closed and where Φ shows the canonical symplectic form so that $\Phi=-d\Omega$, $\Omega=J^*(\omega)$, J* a dual of J, ω a 1-form on T*M. The trio (T^*M,Φ,X_H) is named Hamiltonian system which it is defined on the cotangent bundle T*M [26,27].

Definitions 12

The vector field X on T*M given by $i_X\omega=dH$ is called the geodesic flow of the metric g.

Definitions 13

If $\gamma:(a,b)\to T^*M$ is an integral curve of the geodesic flow, then the curve p(γ) in M is called a geodesic.
Recall from elementary physics that momentum of a particle, pi, is defined in terms of its velocity qi by $p_i=m_i\dot{q}_i$. In fact, the more general definition of conjugate momentum, valid for any set of coordinates, is given in terms of the Lagrangian:

$$p_i=\partial L/(\partial\dot{q}_i),\ \dot{p}_i=\partial L/(\partial q_i). \quad (12)$$

Note that these two definitions are equivalent for Cartesian variables. In terms of cartesian momenta, the kinetic energy is given by $T=\sum_{i=1}^{n}p_i^n/2m_i$. Then, the Hamiltonian, which is defined to be the sum, H=T+V, expressed as a function of positions and momenta, will be given by

$$H(q_i,p_i)=\sum_{i=1}^{n}p_i^n/2m_i+V(q_1,...,q_n), \quad (13)$$

where $p=p_1,...,p_n$. The function H is equal to the total energy of the system. In terms of the Hamiltonian, the equations of motion of a system are given by Hamilton's equations:

$$\dot{q}_i = \partial H/(\partial p_i),\ \dot{p}_i = \partial H/(\partial q_i). \tag{14}$$

Hamilton Equations

Now, we, using Lemma 2 and (11), present Hamilton equations and Hamiltonian mechanical systems for quantum and classical mechanics constructed on a contact 5-manifold.

Proposition 3

Let (T*M,J*,ω) be on a contact 5-manifold. Suppose that the complex structures, a Liouville form and a 1-form on a contact 5-manifold are shown by J*, Ω and ω, respectively. Let a 1-form ω be as follows:

$$\omega=(1/2)[x_1dx_1+x_2dx_2+x_3dx_3+x_4dx_4]. \tag{15}$$

Then, we obtain the Liouville form as follows:

$$\Omega=-J^*(\omega)=(1/2)[x_1(\cos\theta dx_3+\sin\theta dx_4)+x_2(-\cos\theta dx_4+\sin\theta dx_3)$$
$$+x_3(-\cos\theta dx_1-\sin\theta dx_2)+x_4(\cos\theta dx_2-\sin\theta dx_1)]. \tag{16}$$

It is well known that if Φ is a closed on a contact 5-manifold, then Φ is a symplectic structure on (T*M,J*,ω). Therefore the 2-form Φ=-dΩ indicates the canonical symplectic form and derived from the 1-form Ω to find to mechanical equations. Also, dH is

$$dH = \sum_{i=1}^{4}\left(\partial H/(\partial x_i)\right)dx_i \tag{17}$$

Then the 2-form Φ is calculated as below:

$$\Phi=-d\Omega$$
$$=-(1/2)[((dx_1)/(dx_1))(\cos\theta)dx_1\wedge dx_3+\sin\theta dx_1\wedge dx_4)$$
$$+((dx_2)/(dx_2))\ (-\cos\theta dx_2\wedge dx_4+\sin\theta dx_2\wedge dx_3)$$
$$+((dx_3)/(dx_3))(-\cos\theta dx_3\wedge dx_1-\sin\theta dx_3\wedge dx_2)+((dx_4)/(dx_4))$$
$$(\cos\theta dx_4\wedge dx_2-\sin\theta dx_4\wedge dx_1)].$$

$$=(1/2)[\cos\theta dx_3\wedge dx_1+\sin\theta dx_4\wedge dx_1)+(\cos\theta dx_4\wedge dx_2$$
$$+\sin\theta dx_3\wedge dx_2)+(-\cos\theta dx_1\wedge dx_3-\sin\theta dx_2\wedge dx_3)$$
$$+(\cos\theta dx_2\wedge dx_4-\sin\theta dx_1\wedge dx_4)]. \tag{18}$$

Take a vector field X_H so that called to be Hamiltonian vector field associated with Hamiltonian energy H and determined by

$$X_H = \sum_{i=1}^{4} X^i \partial/(\partial x_i) \tag{19}$$

$\Phi(X_H)$ will be calculated using Φ and X_H. Calculations use external product feature. These properties are

$$f\wedge g=-g\wedge f,\ f\wedge g(v)=f(v)g-g(v)f,\ dx_i(\partial/(\partial x_i))=1,\ dx_i(\partial/(\partial x_k))=0. \tag{20}$$

We have

$$i_{X_H}\Phi=\Phi(X_H)$$
$$=(1/2)[-X^1\cos\theta dx_3-X^1\sin\theta dx_4-X^1\cos\theta dx_3-$$
$$X^1\sin\theta dx_4+X^2\cos\theta dx_4-X^2\sin\theta dx_3-X^2\sin\theta dx_3+X^2\cos\theta dx_4$$
$$+X^3\cos\theta dx_1+X^3\sin\theta dx_2+X^3\cos\theta dx_1+X^3\sin\theta dx_2+X^4\sin\theta dx_1$$
$$-X^4\cos\theta dx_2-X^4\cos\theta dx_2+X^4\sin\theta dx_1]. \tag{21}$$

Furthermore, the differential of Hamiltonian energy H is obtained by

$$dH=((\partial H)/(\partial x_1))dx_1+((\partial H)/(\partial x_2))dx_2+((\partial H)/(\partial x_3))dx_3$$
$$+((\partial H)/(\partial x_4))x_4. \tag{22}$$

X^1, X^2, X^3, X^4 are obtained using the $i_{X_H}\Phi=dH$ the following equations:

$$\cos\theta X^3+\sin\theta X^4=((\partial H)/(\partial x_1)),$$
$$\sin\theta X^3-\cos\theta X^4=((\partial H)/(\partial x_2)),$$
$$-\cos\theta X^1-\sin\theta X^2=((\partial H)/(\partial x_3)),$$
$$-\sin\theta X^1+\cos\theta X^2=((\partial H)/(\partial x_4)). \tag{23}$$

They are

$$X^1=-\cos\theta((\partial H)/(\partial x_3))-\sin\theta((\partial H)/(\partial x_4)),$$
$$X^2=-\sin\theta((\partial H)/(\partial x_3))+\cos\theta((\partial H)/(\partial x_4)),$$
$$X^3=\cos\theta((\partial H)/(\partial x_1))+\sin\theta((\partial H)/(\partial x_2)),$$
$$X^4=\sin\theta((\partial H)/(\partial x_1))-\cos\theta((\partial H)/(\partial x_2)). \tag{24}$$

Consider the curve and its velocity vector fields; $\alpha(t):I\subset\mathbb{R}\to M$, I is an index set.

$$\dot{\alpha}(t) = \partial\alpha/\partial t = \sum_{i=1}^{4} dx_i/(dt)\left(\partial/(\partial x_i)\right), \tag{25}$$

such that an integral curve of the Hamiltonian vector field X_H,

$$X_H(\alpha(t))=(\partial/(\partial t))(\alpha),\ t\in I. \tag{26}$$

This equations are as follows:

$$[-\cos\theta((\partial H)/(\partial x_3))-\sin\theta((\partial H)/(\partial x_4))](\partial/(\partial x_1))$$
$$+[-\sin\theta((\partial H)/(\partial x_3))+\cos\theta((\partial H)/(\partial x_4))](\partial/(\partial x_2))$$
$$+[\cos\theta((\partial H)/(\partial x_1))+\sin\theta((\partial H)/(\partial x_2))](\partial/(\partial x_3))$$
$$+[\sin\theta((\partial H)/(\partial x_1))-\cos\theta((\partial H)/(\partial x_2))](\partial/(\partial x_4))$$
$$+[-\cos\theta((\partial H)/(\partial x_3))-\sin\theta((\partial H)/(\partial x_4))](\partial/(\partial x_1))$$
$$+[-\sin\theta((\partial H)/(\partial x_3))+\cos\theta((\partial H)/(\partial x_4))](\partial/(\partial x_2))$$
$$+[\cos\theta((\partial H)/(\partial x_1))+\sin\theta((\partial H)/(\partial x_2))](\partial/(\partial x_3))$$
$$+[\sin\theta((\partial H)/(\partial x_1))-\cos\theta((\partial H)/(\partial x_2))](\partial/(\partial x_4))$$
$$=((dx_1)/(dt))(\partial/(\partial x_1))+((dx_2)/(dt))(\partial/(\partial x_2))$$
$$+((dx_3)/(dt))(\partial/(\partial x_3))+((dx_4)/(dt))(\partial/(\partial x_4)). \tag{27}$$

Then, if the same term in this equation together equalized on both sides, we find the following equations;

$$\text{(dif1)}\quad ((dx_1)/(dt))=-\cos\theta((\partial H)/(\partial x_3))-\sin\theta((\partial H)/(\partial x_4)),$$
$$\text{(dif2)}\quad ((dx_2)/(dt))=-\sin\theta((\partial H)/(\partial x_3))+\cos\theta((\partial H)/(\partial x_4)),$$
$$\text{(dif3)}\quad ((dx_3)/(dt))=\cos\theta((\partial H)/(\partial x_1))+\sin\theta((\partial H)/(\partial x_2)),$$
$$\text{(dif4)}\quad ((dx_4)/(dt))=\sin\theta((\partial H)/(\partial x_1))-\cos\theta((\partial H)/(\partial x_2)). \tag{28}$$

Hence, the equations introduced in (28) are named Hamilton equations on a contact 5-manifold (T*M,J*,ω) and then the triple (T^*M,Φ,X_H) is said to be a Hamiltonian mechanical system on a contact 5-manifold.

Equations Solving with Computer

The solution of Hamilton's equations of motion will yield a trajectory in terms of positions and momenta as functions of time. Hamilton's equations can be used to determine the equations of motion of a system in any set of coordinates for a dynamical system. There are two classes of definitions for a dynamical system: one is motivated by differential equations and the other is motivated by measure theoretical in flavor. If the system can be solved, given an initial point it is possible to determine all its future positions, a collection of points known as a trajectory or orbit. Before the advent of computers, finding an orbit required sophisticated mathematical techniques and could be accomplished only for a small class of dynamical systems.

Nowadays, modeling and solving of difficult mechanical problem has become easier by computer programs.

It is well-known that an electromagnetic field is a physical field produced by electrically charged objects. How the movement of objects in electrical, magnetically and gravitational fields force is very important. For instance, on a weather map, the surface wind velocity is defined by assigning a vector to each point on a map. So, each vector represents the speed and direction of the movement of air at that point. The location of each object in space represented by three dimensions in physical space. These three dimensions can be labeled by a combination of three chosen from the terms length, width, height, depth, mass, density and breadth. These found (28) are partial differential equations system on a contact 5-manifolds and it dissolved with Maple computation program. The First, implicit function at (28) will be selected as a special. After, the graph of the equation (28) has been drawn for the route of the movement of objects in the electromagnetic field.

Example 2

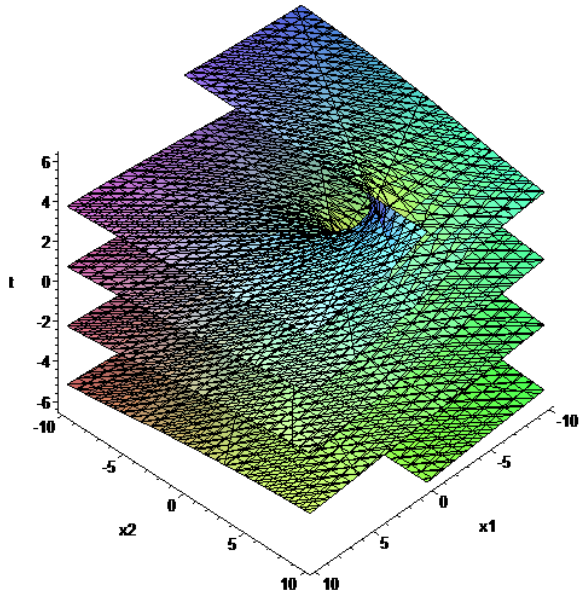
Here, we'll make implicit with the Maple program solution of the above equations (28).

For $x_1(t)=\sin(t)$, $x_2(t)=\cos(t)$, $x_3(t)=\sin(t)$, $x_3(t)=\cos(t)$ and $\theta=0$;

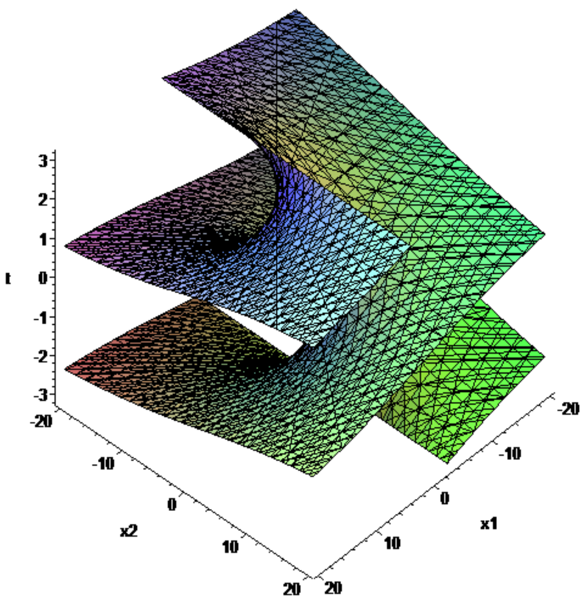
$$H(x_1,x_2,x_3,x_4,t)=(x_2-x_4+F_1(t))*\sin(t)+\cos(t)*(x_1-x_3). \qquad (29)$$

It found that (29) will be plotted with a special selection of closed function of graph (29):

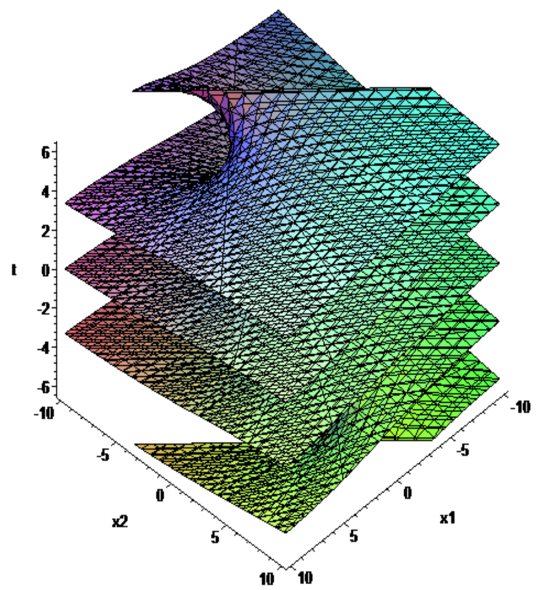
θ=Pi



θ=0



θ=Pi/4



Discussion and Conclusions

A classical field theory explains the study of how one or more physical fields interact with matter which is used quantum and classical mechanics of physics branches. Also, it explains the study of how one or more physical fields interact with matter which is used quantum and classical mechanics of physics branches. A classical field theory is just a mechanical system with a continuous set of degrees of freedom that of electromagnetism deals with electric and magnetic fields and their interaction with each other and with charges and currents. An electromagnetic field is a physical field produced by electrically charged objects. How the movement of objects in electrical, magnetically and gravitational fields force is very important. For example, on a weather map, the surface wind velocity is defined by assigning a vector to each point on a map. So, we said that each vector represents the speed and direction of the movement of air at this point. In this study, Hamilton equations (28) raised on a contact 5-manifold for mechanical systems such that they could be used in modelling the problems in various physical, relativistic and mechanical areas. In addition, in the equations implicit solutions (29) were found using Maple computation program for changing angles. It shows us how to act on time. The Hamilton mechanical equations (28) derived on a contact 5-manifold may be suggested to deal with problems in electrical, magnetically and gravitational fields for the path of movement (30) of defined space moving objects [26,28,29].

Acknowledgements

This work was supported by the agency BAP of Pamukkale University. Also, it is presented as a poster at "13th Geometry Symposium, 27-30 July 2015, Yildiz Technical University, Istanbul, TURKEY".

References

[1] C. Bellettini, Almost complex structures and calibrated integral cycles in contact 5-manifolds, Advances in Calculus of Variations, Vol.6, Issue 3, (2013), 339-374.

[2] D. Janssens and L. Vanhecke, Almost contact structures and curvature tensors, Kodai Math. J., 4, (1981), 1-27.

[3] S.K. Chaubey, Almost contact metric manifolds admitting semi-symmetric non-metric connection, Bulletin of Mathematical Analysis and Applications, Vol.3, Issue 2, (2011), 252-260.

[4] H. Kodama, Complex contact three manifolds with Legendrian vector fields, Proc. Japan Acad. Ser. A Math. Sci., 78, Number 4, (2002), 51-54.

[5] V.I. Piercey, Contact Geometry, University of Arizona, (2008).

[6] B.M. Doubrov and B.P. Komrakov, Contact Lie algebras of vector fields on the plane, Geometry & Topology, 3 (1999), 1-20.

[7] H. Attarchi and M.M. Rezaii, Contact structure on the indicatrix bundle of Finslerian warped product manifolds, http://arxiv.org/pdf/1106.2663.pdf, (2011), 1-15.

[8] M. Kashiwara, Quantization of contact manifolds, Publ. Res. Inst. Math., 32 (1), (1996), 1-7.

[9] M. Manev and K. Gribachev, Conformally invariant tensors on almost contact manifolds with B-Metric, Serdica, 20, No.2, (1994), 133-147.

[10] D. Iglesias-Ponte and A. Wade, Contact manifolds and generalized complex structures, http://arxiv.org/abs/math/0404519v2, (2004), 1-12.

[11] M. Manev and M. Ivanova, Canonical-type connection on almost contact manifolds with B-metric, Ann. Glob. Anal. Geom., 43, (2013), 397-408.

[12] J.B. Etnyre, Contact structures on 5-manifolds, http://arxiv.org/pdf/1210.5208v2.pdf, (2013), 1-18.

[13] K. Sekiya, Generalized almost contact structures and generalized sasakian structures, arxiv.org/abs/1212.6064, (2012), 1-16.

[14] M.K. Dwivedi, J-B. Jun and M.M. Tripathi, On endo curvature tensor of a contact metric manifold, Tamkang Journal of Mathematics, 39, (2), (2008), 177-186.

[15] F. Malek and M.B.K. Balgeshir, Slant submanifolds of almost contact metric 3-Structure manifolds, Mediterranean Journal of Mathematics, 10, (2013), 1023-1033.

[16] J. Davidov, On the twistor space of a 5-manifold with an irreducible SO(3)-structure, Mediterranean Journal of Mathematics, Volume 13, Issue 1, (2016), 413-442.

[16] Z. Kasap and M. Tekkoyun, Mechanical systems on almost para/pseudo-Kähler-Weyl manifolds, IJGMMP, Vol.10, No.5, (2013), 1-8.

[17] Z. Kasap, Weyl-mechanical systems on tangent manifolds of constant W-sectional curvature, IJGMMP, Vol.10, No.10, (2013), 1-13.

[18] M. Tekkoyun, On para-Euler-Lagrange and para-Hamiltonian equations, Phys. Lett. A, 340, (2005), 7-11.

[19] D.J. Griffiths, Introduction to Electrodynamics. Prentice Hall. ISBN 0-13-805326-X, (1999).

[20] D. McDu and D. Salamon, J-Holomorphic Curves and Quantum Cohomology, (1995).

[21] N. Nowaczyk, J. Niediek and M. Firsching, Basics of Complex Manifolds, (2009).

[22] N. Nowaczyk, J. Niediek and M. Firsching, Basics of Complex Manifolds, (2009).

[23] A. Newlander and L. Nirenberg, Complex analytic coordinates in almost complex manifolds, Ann. of Math., 65, (1957), 391-404.

[24] J. Klein, Escapes Variationnels et Mécanique, Ann. Inst. Fourier, Grenoble, 12, (1962).

[25] A. Ibord, The Geometry of Dynamics, Extracta Mathematicable, Vol.11, Num.1, (1996), 80-105.

[26] M. de Leon and P.R. Rodrigues, Methods of Differential Geometry in Analytical Mechanics, North-Hol. Math. St., 152, Elsevier Sc. Pub. Com. Inc., Amsterdam, (1989).

[27] R. Abraham, J.E. Marsden and T. Ratiu, Manifolds Tensor Analysis and Applications, Springer, (2001).

[28] B. Thidé, Electromagnetic Field Theory, (2012).

[29] R.G. Martín, Electromagnetic Field Theory for Physicists and Engineers: Fundamentals and Applications, (2007).