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Smarandache-Soft Neutrosophic Near–Ring and Soft Neutrosophic(m,n) Bi-Ideals

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ABSTRACT

In this paper, we introduced Samarandache-2-algebraic structure of Soft Neutrosophic Near-ring namely Smarandache –Soft Neutrosophic Near-ring. A Samarandache-2-algebraic structure on a set N means a weak algebraic structure S_1 on N such that there exist a proper subset M of N, Which is embedded with a stronger algebraic structure S_2 ,stronger algebraic structure means satisfying more axioms, that is S_1<< S_2 ,by proper subset one can understand a subset different from the empty set, from the unit element if any , from the Whole set. We define Smarandache - Soft Neutrosophic Near-ring and obtain the some of its characterization through Generalized soft neutrosophic near-field and Generalized soft neutrosophic (r,m) bi-ideals.For basic concept of near-ring we refer to G.Pilz,for generalized near-field we refer T.Tamilchelvamand for soft neutrosophic algebraic structure we refer to MuhammedShabir ,MumtazAli,MunazzaNaz, and FlorentinSmarandache.

Introduction

In order that, New notions are introduced in algebra to better study the congruence in number theory by Florentinsmarandache [2] .By <proper subset> of a set A we consider a set P included in A, and different from A ,different from empty set, and from the unit element in A-if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures S_1 << S_2 if: both are defined on the same set;all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1 .

For example: Semi group<<Monoid<<group<< ring<<field, or Semi group<< to commutative semi group, ring<< unitary ring etc. They define a general special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where SM <<SN.Inaddition we have published [6,7,8,9].

Preliminaries

Definition 2.1

Let $\langle NUI \rangle$ be a neutrosophic near-ring and (F,A) be a soft set over $\langle NUI \rangle$.Then (F,A) is called soft neutrosophic near-ring if and only if F(a) is a neutrosophic sub near-ring of $\langle NUI \rangle$ for all $a \in A$.

Definition 2.2

Let $K(I) = \langle KUI \rangle$ be a neutrosophic near-field and let (F,A) be a soft set over $K(I)$. Then (F,A) is said to be soft neutrosophic near-field if and only if $F(a)$ is a neutrosophic sub near-field of $K(I)$ for all $a \in A$.

Definition 2.3

Let (F,A) be a soft neutrosophiczerosymmetric near-ring over $\langle N \cup I \rangle$, which contains a distributive element $F(a_1) \neq 0$ .Then (F,A) is a near-field if and only if for each

$F(a) \neq 0$ in (F,A),(F,A)F(a) = (F,A) .

Now we have introduced our basic concept, called Smarandache–Soft Neutrosophic–Near Ring.

Definition 2.4

A Soft neutrosophic –near ring is said to be Smarandache –soft neutrosophic –near ring, if a proper subset of it is a soft neutrosophic –near field with respect to the same induced operations.

Definition 2.5

Let (F,A) be a Smarandache - soft Neutrosophic near –ring over $\langle NUI \rangle$,then a subgroup (L_B,A) of ((F,A),+) is said to be a Soft Neutrosophic Bi –ideal of (F,A) if

$(L_B,A)(F,A)(L_B,A) \cap ((L_B,A)(F,A))^*(L_B,A) \subseteq (L_B,A)$

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Definition 2.6

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to said to have property (α) if F(x) (F,A) is a subgroup of ((F,A),+) for every F(x) in (F,A).

Definition 2.7

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be Soft Neutrosophic sub-commutative if F(x)(F,A) = (F,A)F(x) for every F(x) in (F,A).

Note that every soft neutrosophicsub-commutative near-ring is a soft neutrosophic near-ring with property (α) .

Definition 2.8

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be a soft nertrosophic s- near ring if F(a) in (F,A)F(a) for F(a) in (F,A).

Definition 2.9

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called soft neutrosophic left-bi-potent if (F,A)F(a) = (F,A)F(a)^2 for F(a) in (F,A).

Definition 2.10

An element F(a) in soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be soft neutrosophic regular if F(a) in F(a)(F,A)F(a).

Definition 2.11

A subgroup (H,A) of soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called (F,A)-subgroup if (F,A)(H,A) $\subseteq$ (H,A) and an invariant (F,A)-subgroup if in addition (H,A)(F,A) $\subseteq$ (H,A).

Definition 2.12

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called an (F,A) .S.I.softneutrosophic near-ring if every (F,A)-subgroup of (F,A)is invariant.

Definition2.13

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called a Generalized soft neutrosophic near-field (GSNNF) if for each F(a) in (F,A) there exists a unique F(b) in (F,A) such that F(a) = F(a)F(b)F(a) and F(b) = F(b)F(a)F(b).

Definition 2.14

For any subset (H,A) of a soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ we write Rad (H,A) = { H(x) in (H,A) / H(n) ^{n} in (H,A) for some positive integer n}.

Definition 2.15

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be soft neutrosophic left simple if (F,A)F(a) = (F,A) for every 0$\neq$ F(a) in (F,A).

Definition2.16

A soft neutrosophic bi-ideal (L_B,A) of ((F,A).+) is said to be a generalaized soft neutrosophic (m,n) bi-ideal if (L_B,A)^m(F,A)(L_B,A)^n$\subseteq$ (L_B,A), where m and n are positive integers.

Definition 2.17

A subgroup (H,A) of of ((F,A),+) is said to be a soft neutrosophic left (right) (F,A) – subgroup of (F,A) if (F,A)(H,A) $\subseteq$ (H,A) ((H,A)(F,A) $\subseteq$(H,A)).

Definition2.18

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be a soft neutrosophic two sided near ring if every left (right) (F,A) –subgroup is a right(left) (F,A)- Subgroup of (F,A).

Definition 2.19

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called soft neutrosophic L_B- regular near ring if F(a) in $(F(a))_r$ (F,A) $(F(a))_l$for every F(a) in (F,A).where $(F(a))_r$ $(F(a)_l)$ is the right (left) (F,A)-Subgroup generated by F(a) in (F,A).

Definition 2.20

A soft neutrosophic s - near-ring (F,A) over $\langle NUI \rangle$ is said to be a soft neutrosophic $\bar{s}$ -near ring if F(x) in F(x)(F,A) for F(x) in (F,A).

Definition 2.21

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called soft neutrosophic regular if for each F(a) in (F,A), F(a) = F(a)F(b)F(a) for some F(b) in (F,A) and(F,A) is called soft neutrosophic strongly regular if for each F(a) in (F,A), there exists F(b) in (F,A) such that F(a) = F(b)F(a)^2

Note that every soft neutrosophic strongly regular near-ring is always a soft neutrosophic regular near - ring.

Definition 2.22

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called soft neutrosophic reduced if it has no non-zero nilpotent elements.

Definition 2.23

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to have IFP (Insertion of factor property) if F(a) F(b) = 0 implies F(a)F(x)F(b) = 0 for all F(x) in (F,A).

Definition 2.24

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is called softneutrosophic left bi-potent if (F,A)F(a) = (F,A) F(a) ^{2} for F(a) in (F,A).

(E,A) denotes set of all idempotents in (F,A) over $\langle NUI \rangle$.

Definition 2.25

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ then C(F,A) = {F(n) in(F,A) F(n)F(x) = F(x)F(n) for all F(x) in (F,A)} is called the center of (F,A).

Definition 2.26

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be a soft neutrosophic $s_k(s'_k)$ near - ring is F(x) in (F,A)F(x)^k (F(x)^k(F,A)) for all F(x) in (F,A)

One can see that if (F,A) is a soft neutrosophic$S_k$near- ring ,then (F,A) is soft neutrosophic S_j –near ring for all j$\leq$ k.

Definition 2.27

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be soft neutrosophic p(r,m) near ring if F(a)^r (F,A) = (F,A)F(a)^{m}for each F(a) in (F,A) where r,m are positive integers.

Definition 2.28

A soft neutrosophic near-ring (F,A) over $\langle NUI \rangle$ is said to be soft neutrosophic $P_k(P'_k)$ near ring if F(x)^k(F,A) = F(x) (F,A)F(x) ((F,A)F(x)^k = F(x) (F,A) F(x)) for all F(x) in (F,A) and soft neutrosophic $P_k(r,m)\left(P'_k(r,m)\right)$ near-ring if F(x)^k(F,A) = F(x)^r (F,A)F(x)^m ((F,A)F(x)^k = F(x)^r (F,A)F(x)^m)for all F(x) in (F,A).

Definition 2.29

A soft neutrosophic sub-commutative near-ring (F,A) over $\langle NUI \rangle$ is said to soft neutrosophic stable if F(x)(F,A) = F(x)(F,A)F(x) for all F(x) in (F,A).

Equivalent Condition for Smarandache- Soft Neutrosophic Near- Field

Now we prove in the following theorem,13 equivalent condition for a smarandache- soft neutrosophic near-field.

Theorem 3.1

Let (F,A) be a Smarandache - soft neutrosophic near-ring over $\langle NUI \rangle$,there exists a proper subset (H,A) of (F,A),which is soft neutrosophiczerosymmetric,sub-commutative and soft neutrosophic s- near- ring .Then the following conditions are equivalent:

(i) (H,A) is generalized soft neutrosophic near field(GSNNF).
(ii) (L_B,A) = (L_B,A)(H,A)(L_B,A) for every soft neutrosophic bi-ideal (L_B,A) of (H,A).
(iii) (H,A) is regularand (H,A) is an (H,A).S.I soft neutrosophic near-ring.
(iv) For all (H,A)-subgroup (H_1,A) and (H_2,A) of (H,A),(H_1,A)$\cap$(H_2,A) = (H_1,A)(H_2,A).
(v) (H,A)H(x) $\cap$ (H,A)H(y) = (H,A)H(x)H(y) for all H(x),H(y) in (H,A).
(vi) (H,A) is soft neutrosophic left bi-potent.
(vii) (H,A) = Rad ((H,A)) for every (H,A)- subgroup (H,A) of (F,A).
(viii) (H,A) is soft neutrosophicstongly regular.
(ix) For H(a) in (H,A),H(a)(H,A)H(a) = (H,A)H(a) = (H,A)H(a)^2
(x) (H,A) is soft neutrosophic regular and (H,A) has no non-zero nilpotent element and the idempotents of (H,A) lie in centre.
(xi) (H,A) is soft neutrosophic regular and (H,A) contains a unique non-zero idempotent, which is also the right identity of (H,A)
(xii) (H,A) is soft neutrosophic left simple
(xiii) (H,A) is soft neutrosophic regular and (H,A) has no proper left (H,A) –subgroup.
(xiv) (H,A) is soft neutrosophic regular and (H,A) has no proper left ideal.

Proof

(i) Since (F,A) be a Samarandache - soft neutrosophic near ring over $\langle NUI \rangle$ .Then by definition,there exists aproper subset (H,A),which is soft neutrosophic near-field.

This implies (H,A) is a Generalized soft neutrosophic near-field(GSNNF).

(i) $\Rightarrow$(ii)

If (H,A) is GSNNF ,then (H,A) is soft neutrosophicregular.Since (H,A) is a soft neutrosophic s-near-ring,(L_B,A) = (L_B,A)(H,A)(L_B,A) for every soft neutrosophic bi-ideal (L_B,A) of (H,A)

(ii) $\Rightarrow$ (iii)

If (L_B,A) = (L_B,A)(H,A)(L_B,A) for every soft neutrosophic bi-ideal (L_B,A) of (H,A),then (H,A) is regular.Let (H_1,A) be any (H,A) – subgroup of (H,A).For every H_1(a) in(H_1,A),H_1(a)(H,A) = (H,A)H_1(a) and so H_1(a)(H,A) is a soft neutrosophic bi –ideal of (H,A).

Thus H_1(a)(H,A) = (H_1(a)(H,A))(H,A)((H,A)H_1(a)) $\subseteq$ (H_1(a)(H,A))H_1(a) $\subseteq$ (H,A)(H_1,A) $\subset$ (H_1,A).i.e.,(H_1,A)(H,A) $\subset$ (H_1,A) and so (H,A) is an (H,A).S.I soft neutrosophic near-ring.

(iii)$\Rightarrow$(iv)

Let H(x) in (H_1,A)$\cap$(H_2,A). Since (H,A) is soft neutrosophic regular.

H(x) = H(x)H(n)H(x) = H(x)(H(n)H(x)) in (H_1,A)(H_2,A).

i.e., (H_1,A) $\cap$(H_2,A) $\subseteq$ (H_1,A)(H_2,A).On the other hand let H(n) = H(y)H(z) in (H_1,A)(H_2,A) with H(y) in (H_1,A) and H(z) in (H_2,A).Clearly H(n) in (H_2,A).Since (H,A) is a (F,A).S.I soft neutrosophic near ring.H(n) = H(y)H(z) in (H,A)(H_2,A) $\subseteq$ (H_2,A) and so H(n) in (H_2,A).Thus (H_1,A) $\cap$(H_2,A) $\subseteq$ (H_1,A)(H_2,A).

(iv) $\Rightarrow$ (v)

For H(x),H(y) in (H,A) take (H_1,A) = (H,A)H(x) and (H_2,A) = (H,A)H(y) and hence by assumption.(H,A)H(x) $\cap$ (H,A)H(y) = (H,A)H(x)(H,A)H(y).

Now (H,A)H(x) = (H,A)H(x) $\cap$(H,A) = (H,A)H(x)(H,A).
which implies that (H,A)H(x)H(y) = (H,A)H(x)(H,A)H(y).
Thus (H,A)H(x) $\cap$ (H,A)H(y) = (H,A)H(x)H(y).

(v) ⇒(iv)

Since (H,A) is soft neutrosophic s - near-ring,H(a) in (H,A)H(a) = (H,A)H(a) $\cap$ (H,A)H(a) = (H,A)H(a)H(a) = (H,A)H(a)^2.i.e.,(H,A) is soft neutrosophic left bi-potent.

(vi)⇒(vii)

Clearly (H,A) $\subset$ Rad (H,A).Now let H(a) in Rad (H,A), then H(a)^n in (H,A) for some n.Also we have (H,A)H(a) = (H,A)H(a)^2 = ………………………………… =(H,A)H(a)^n in a left bi-potent soft neutrosophic near-ring .Since (H,A) is a soft neutrosophics-near-ring,H(a) in (H,A)H(a) = (H,A)H(a)^n.This gives that H(a) = H(b)H(a)n for some H(b) in (H,A) and hence H(a) in (H,A).i.e.,Rad ((H,A)) $\subseteq$ (H,A).

(vii)⇒(viii)

Let 0$\neq$H(a) in (H,A).Now H(a)^3 in (H,A)H(a)^2 so that
H(a) in Rad((H,A)H(a)^2) = (H,A)H(a)^2.i.e., (H,A) is soft neutrosophic strongly regular.

(viii) ⇒(ix)

If (H,A) is soft neutrosophic strongly regular,for each H(a) in (H,A),H(a) = H(x)H(a)^2 for someH(x) in(H,A).If H(z) in (H,A)H(a) then H(z) = H(y)H(a) = H(y)H(x)H(a)^2 in (H,A)H(a)^2.,

i.e.,(H,A)H(a) $\subseteq$ (H,A)H(a)^2 and so (H,A)H(a) = (H,A)H(a)^2.Since (H,A) is subcommutative,(H,A)H(a) = (H,A)H(a)^2 = ((H,A)H(a))H(a) = (H(a)(H,A))H(a) = H(a)(H,A)H(a).

(ix)⇒(x)

Assume that H(a)(H,A)H(a) = (H,A)H(a) = (H,A)H(a)^2 for every H(a) in (H,A).Since (H,A) is a soft neutrosophic s - near ring ,H(a) in (H,A)H(a) = H(a)(H,A)H(a) for H(a) in (H,A),i.e., (H,A) is soft neutrosophicregular.ForH(a) in (H,A) if H(a)^n = 0for some integer n$\geq$ 1,then H(a) in (H,A)H(a) = (H,A)H(a)^2,i.e.,H(a) = H(x)H(a)^2 for some H(x) in (H,A).

Hence H(a)$^{n-1}$ = H(a)H(a)$^{n-2}$ = (H(x)H(a)^2)H(a)$^{n-2}$ = H(x)H(a)^n = 0 continuing in this way we get that H(a) = 0.Hence (H,A) contains no non-zero nilpotent elements. IfH(e) is anidempotent in (H,A), then H(e) (H,A)H(e) = (H,A)H(e).
Let H(y) in (H,A).
Then
H(e)H(x)H(e) = H(y)H(e) for some H(x) in (H,A),and
H(e)H(y)H(e) = H(e)H(x)H(e) = H(y)H(e) = H(y)H(e)^2.i.e., (H(e)H(y) – H(y)H(e))H(e) = 0
(H(e)(H(e)H(y) – H(y)H(e)))^2 = H(e)(H(e)H(y) – H(y)H(e)) H(e)(H(e)H(y) – H(y)H(e)) = 0 and
(H(e)H(y)(H(e)H(y) – H(y)H(e)))^2
= H(e)H(y)(H(e)H(y) – H(y)H(e)) H(e)H(y)(H(e)H(y) – H(y)H(e)) = 0

Since (H,A) has no non-zero nilpotent elements. H(e)(H(e)H(y) – H(y)H(e)) = 0 and H(e)H(y)(H(e)H(y) – H(y)H(e)) = 0 ,which in turn imply that (H(e)H(y) – H(y)H(e))^2 = 0 and so H(e)H(y) – H(y)H(e) = 0, i.e., H(e)H(y) = H(y)H(e).

(x)⇒(xi)

Let H(e) be any non – zero idempotent. Suppose H(e)H(a) = 0 for some H(a) in (H,A).Since (H,A) is soft neutrosophic regular,(H,A)H(a) = (H,A) H(e) and so one can find H(y) in (H,A) such that H(a)^2 = H(y)H(e).

Now 0 = 0.H(A) = H(e)H(a) . H(a) = H(e)H(a)^2 = H(e)H(y)H(e) = H(y)H(e) = H(a)^2 and hence by assumption H(a) = 0.

Let H(n) in (H,A).Then (H(n)H(e) – H(n))H(e) = 0 and hence H(n)H(e) = H(n),i.e., H(e) is a right identity. Suppose H(e) and H(f) are two idempotents, then H(e) = H(e)H(f) = H(f)H(e) = H(f),i.e., (H,A) contains unique non-zero idempotent.

(xi) ⇒(xii)

Since (H,A) is soft neutrosophicregular,for H(a) $\neq$0 in (H,A) there exist H(x) in (H,A) such that H(a)H(x)H(a) = H(a).Let H(e) = H(x)H(a).Then H(a)H(e) = H(a),H(e) $\neq$ 0 and H(e)^2 = H(e).Now H(n)H(a) = (H,A)H(a)H(x)H(a) $\subseteq$ (H,A) H(x)H(a) = (H,A)H(e) $\subseteq$ (H,A)H(a) and so (H,A)H(a) = (H,A)H(e).By assumption H(e) is the right identity and hence (H,A)H(a) = (H,A)H(e) = (H,A),i.e., (H,A)H(a) = (H,A) for every H(a) in (H,A),i.e.,(H,A) is soft neutrosophic left simple.

(xii)⇒(xiii)

Clearly a soft neutrosophic left simple near-ring (H,A) contains no zero divisors.Given H(a) $\neq$ 0 in (H,A) there exists H(y) in (H,A) such that H(y)H(a) = H(a) and so (H(a)-H(a)H(y))H(a) = 0.This implies that H(a) = H(a)H(y) = H(y)H(a).

Now find H(x) in (H,A) such that H(x)H(a) = H(y).Then H(a)H(x)H(a) = H(a)H(y) = H(a).Hence (H,A) is soft neutrosophic regular.

Let (H_1,A) $\neq${0} is an (H,A)-subgroup of (H,A) and let H_1(x)($\neq$ 0) in (H_1,A).Since(H,A) is a soft neutrosophic s - near-ring,H_1(x) in H_1(x)(H,A) = (H,A) H_1(x) = (H,A)H_1(x)^2.Hence there exists H(y) in (H,A) such that H_1(x) = H(y)H_1(x)^2.

Let H(n) in (H,A) .Then H(n)H_1(x) = H(n)H(y)H_1(x)^2

which gives that H(n)H_1(x) = H(n)H(y)H_1(x)^2 = 0 and so((H(n)-H(n)H(y)H_1(x))H_1(x)) = 0.Since (H,A) contains no zero divisors H(n) – H(n)H(y)H_1(x) = 0,therefore H(n) = H(n)H(y)H_1(x) in H(n)H(y) (H_1,A) $\subseteq$ (H_1,A),i.e., (H,A) has no proper (H,A)-subgroup.

(xiii)⇒(xiv)

Since (H,A) is asoft neutrosophic zero-symmetric,every left ideal is a (H,A) – subgroupand so (H,A) has no proper left ideal.

Theorem 2

Let (F,A) be a Smarandache - soft neutrosophic near-ring over $\langle NUI \rangle$,there exists a proper subset (H,A) of (F,A),which is soft neutrosophic s_k- soft neutrosophc near-ring for k$\geq$2 .Then the following are equivalent:

(i) $(L_B,A) = (L_B,A)^r$ (H,A) $(L_B,A)^m$ for every Generalized soft neutrosophic (r,m) bi-ideal (L_B,A) of (H,A) and (H,A) is soft neutrosophicsubcommutative.
(ii) (H,A) is a $P'_k(1,1)$ soft neutrosophic near-ring.
(iii) (H,A) is a $P_k(r,m)$ soft neutrosophic near-ring for all r, m.
(iv) (H,A) is a softneutrosophic left bi-potent and (E,A) ⊆ C(H,A).
(v) (H,A) is soft neutrosophic s' and $P(1,2)$near-ring.
(vi) (H,A) is a generalized soft neutrosophic near field.
(vii) (H,A) is soft neutrosophic stable near-ring.
(viii) (H,A) is a $P(r,m)$ soft neutrosophic near-ring for all positive integers r and $m$and regular.
(ix) (H,A) is soft neutrosophic s', L_B-regular ,two saided soft neutrosophic near-ring with property (α).
(x) Let (H_1,A), (H_2,A) be two (H,A)-subgroups of (H,A).Then
a) $(H_1,A) \cap (H_2,A) = (H_1,A)(H_2,A)$
b) $(H_1,A)^2 = (H_1,A)$
c) $(H_1,A) \cap (H,A)(H_2,A) = (H_1,A)(H_2,A)$ and (H,A) is soft neutrosophic sub-commutative.

Proof

(i)⇒(ii)

Assume that $(L_B,A) = (L_B,A)^r$ (H,A) $(L_B,A)^m$ for every generalized softneutrosophic (r,m) bi-ideal (L_B,A) of (H,A).Then trivially $(L_B,A) = (L_B,A)(H,A)(L_B,A)$ for every softneutrosophic bi-ideal (L_B,A) of (H,A).Since (H,A) is a soft neutrosophic s_k-near-ring,(H,A) becomes a soft neutrosophic $\overline{s}$-near ring .Again by the assumption of soft neutrosophic sub-commutativity,(H,A) is with property (α) and so (H,A) is soft neutrosophic regular.Then it is clear that (H,A) is soft neutrosophic regular and idempotents lie in center.

Now for all H(n),H(x) in (H,A),$H(n)H(x)^k = (H(n)H(x))H(x)^{k-1} = (H(n)H(x)H(b)H(x))H(x)^{k-1}$.

Since H(x)H(b) and H(b)H(x) are idempotents,we have $H(n)H(x)^k = (H(x)H(b)H(n)H(x))H(x)^{k-1}$ in H(x)(H,A)H(x).From this we get that $(H,A)H(x)^k \subseteq H(x)(H,A)H(x)$.

On the other hand, $H(x)H(n)H(x) = (H(x)H(b)H(x))H(n)H(x) = H(x)H(n)H(b)H(x)^2 = (H(x)H(b)H(x))(H(n)H(b)H(x)^2) = H(x)H(n)H(b)^2H(x)^3$.

Repeating thisprocess,we get $H(x)H(n)H(x) = H(n)' H(x)^k$ in $(H,A)H(x)^k$ for all positive integrsk.Thus $(H,A)H(x)^k = H(x)(H,A)H(x)$ for all H(x) in (H,A) .

(ii)⇒(iii)

By assumption $(H,A)H(x)^k = H(x)(H,A)H(x)$ for all H(x) in(H,A).Since (H,A) is a soft neutrosophic s_k-nearring,(H,A)becomessoft neutrosophic strongly regular and so (H,A) has no non-zero nilpotent elements.

Let E(a)in (E,A) and H(n) in (H,A).Then (E(a)H(n) – E(a)H(n)E(a))E(a) = 0 and so (E(a)H(n) – E(a)H(n)E(a))E(a)H(n)E(a) = 0.Since (H,A) is a soft neutrosophic IFP near-ring,we get E(a)(E(a)H(n) – E(a)H(n)E(a)) = 0 and E(a)H(n)E(a) (E(a)H(n) – E(a)H(n)E(a)) = 0.From this E(a)H(n) = E(a)H(n)E(a) .Now E(a)(H,A) = E(a)(H,A)E(a) = (H,A)E(a) for all E(a) in (E,A).Then we have idempotents lie in center.Letr, m be any two positive integers.

For H(x) in (H,A),$H(x)^r(H,A)H(x)^m \subseteq H(x)(H,A)H(x) = (H,A)H(x)^k$.
That is ,$H(x)^r(H,A)H(x)^m \subseteq (H,A)H(x)^k$. Now let H(z) in $(H,A)H(x)^k$ (=H(x)(H,A)H(x)).
Then there exists H(y) in (H,A) such that
H(z) = H(x)H(y)H(x) = (H(x)H(a)H(x))H(y)(H(x)H(a)H(x))
= H(x)H(a) (H(x)H(y)H(a)) $H(a)H(x) = H(x)H(a))^r$ (H(x)H(y)H(x)) $(H(a)H(x))^r$ as H(x)H(a),H(a)H(x) in (E,A).
Thus $H(z) = H(z)^rH(a)^r H(x)H(y)H(x) H(a)^mH(x)^m$ in $H(x)^r(H,A)H(x)^m$.
That is ,$(H,A)H(x)^k \subseteq H(x)^r(H,A)H(x)^m$.Thisimples that (H,A) Soft neutrosophic $P_k(r,m)$ near ring.

(iii)⇒(iv)

Since (H,A) is soft neutrosophic s_k- near ring,H(x) in $(H,A)H(x)^k = H(x)^r(H,A)H(x)^m$
$H(x)(H(x)^{r-1}(H,A)H(x)^{m-1})H(x) \subseteq H(x)(H,A)H(x)$.
That is (H,A) is soft neutrosophicregular.Since (H,A) is a soft neutrosophic s_k and $P'_k(r,m)$ –near- ring,(H,A) is soft neutrosophic strongly regular hence (H,A) has no non-zero nilpotent element.This implies that (H,A) has IFP and so E(a)H(n) = E(a)H(n)E(a) for E(a) in (E,A).

Thus we get E(a)(H,A) = E(a)(H,A)E(a) = (H,A)E(a).Then (E,A) ⊆ C(H,A) and so (H,A) is soft neutrosophic regular and sub-commutative.

Thus $(H,A)H(x)^2$ = H(x) (H,A)H(x) ⊆ (H,A)H(x) . On the other hand

(H,A)H(x) = (H,A) H(x)H(a)H(x) = H(x)(H,A)H(a)H(x) ⊆ H(x)(H,A)H(x).Hence (H,A) is soft neutrsophic left bi-potent and (E,A) ⊆ C(H,A).

(iv)⇒(v)

Assume that (H,A) is soft neutrosophic left bi-potent and (E,A) ⊆ C(H,A).Since (H,A) is soft neutrosophic s-near ring,H(x) in (H,A) H(x) = (H,A)H(x)^2 , (H,A) is soft neutrosophic strongly regular and so (H,A) is soft neutrosophicregular.Then it is clear that ,(H,A) is soft neutrosophic regular and sub-commutative.

From thiswe get that H(x) (H,A) = (H,A)H(x)^2.That is (H,A) is soft neutrosophic P(1,2)near ring.Since (H,A) is softneutrosophic sub-commutative, (H,A) is soft neutrosophic s′-near-ring.

(v)⇒(iv)

By assumption ,H(x) in H(x) (H,A) = (H,A)H(x)^2 .That is ,(H,A) is soft neutrosophic strongly regular and so (H,A) is soft neutrosophicregular.Since (H,A) is soft neutrosophic P(1,2) near ring,for E(a) in (E,A),E(a)(H,A) = (H,A) E(a)^2 = (H,A)E(a) = E(a)(H,A)E(a).

Then (E,A) ⊆ C (H,A).Hence (H,A) is a generalized soft neutrosophic near field.

(vi)⇒(vii)

Since (H,A) is soft neutrosophic regular and sub-commutative near-ring. Now let H(y) in H(x)(H,A).Then

H(y) = H(x)H(a) = H(x)H(b)H(x)H(a) = H(x)H(a)H(b)H(x) in H(x)(H,A)H(x) ⊆ H(x) (H,A).

Thus H(x) (H,A) = H(x) (H,A)H(x) = (H,A)H(x).That is soft neutrosophic stable near ring.

(vii)⇒(viii)

Let (H,A) is soft neutrosophic stable near-ring .Then E(a) (H,A) = E(a) (H,A)E(a) = (H,A)E(a) for E(a) in (E,A).Then (E,A) ⊆ C(H,A).Since (H,A) is soft neutrosophic s_k-near ring,(H,A) becomes a soft neutrosophic s-near ring. Since (H,A) is soft neutrosophic stable and regular.Let r andm be any two positive integers.

Let H(a) in H(a)^r(H,A) .Now

H(a) = H(x)^{r}H(n) = (H(x)H(y)H(x))^{r}H(n) = H(x)^r(H(y)H(x))^{r}H(n) = H(x)^r (H(y)H(x)H(n)) =H(x)^{r}H(n) (H(y)H(x)) = H(x)^{r}H(n)(H(y)H(x))^m = H(x)^{r}H(n)H(y)^{m}H(x)^m = H(x)^{r}H(n)H(y)^m)H(x) in (H,A)H(x)^m.

That is ,H(x)^r(H,A) ⊆ (H,A)H(x)^m.In a similar fashion we can show that

(H,A)H(x)^m ⊆H(x)^r(H,A).Hence H(x)^r(H,A) = (H,A)H(x)^m

(viii)⇒(ix)

If (H,A) is soft neutrosophic$P(r,m)$near-ring,then E(a)^r(H,A) = (H,A)E(a)$^{m.}$

That is ,E(a)(H,A) = (H,A)E(a) and so E(a) (H,A)E(a) = E(a)((H,A)E(a)) = E(a)E(a) (H,A) = E(a) (H,A).

Therefore (E,A) ⊆ C(H,A).Since (H,A) is soft neutrosophic regular ,then (H,A) is soft neutrosophic sub-commutative and so (H,A) has property(α).

In case of a soft neutrosophic $\bar{s}$-near-ring with property (α),

H(x)$_r$= H(x) (H,A) = (H,A)H(x) = H(x)$_l$ and so (H,A) is soft neutrosophic twosided.Also every soft neutrosophic regular near-ring is soft neutrosophic L$_B$- regular.

(xi)⇒(x)

By assumption (H,A) is soft neutrosophic regular .If (H,A) is soft neutrosophic two sided ,then H(x) (H,A) = H(x)$_r$ = H(x)$_l$ = (H,A) H(x) , and hence (H,A) is soft neutrosophic sub- commutative.

Let (H$_1$,A) , (H$_2$,A) be two soft neutrosophic left (H,A)-subgroups of (H,A).

To prove a)

Let H(x) in (H$_1$,A) ∩ (H$_2$,A) .Since (H,A) is soft neutrosophic regular,

H(x) = H(x)H(a)H(x) = (H(x)H(a))H(x) in (H$_1$,A) (H,A) (H$_2$,A) ⊆(H$_1$,A) (H$_2$,A) .

i.e., (H$_1$,A) ∩ (H$_2$,A) ⊆(H$_1$,A) (H$_2$,A)⊆ (H$_1$,A) ∩ (H$_2$,A)

b) By taking (H$_2$,A) ∩ (H$_1$,A) in the above ,weget (H$_1$,A)^2= (H$_1$,A)

c) (H$_1$,A) ∩ (H,A) (H$_2$,A) ⊆ (H$_1$,A) ∩ (H$_2$,A) =(H$_1$,A) (H$_2$,A)⊆(H$_1$,A) ∩ (H,A) (H$_2$,A).

(xi)⇒(x)

Since (H,A) is a soft neutrosophic s-near ring ,

H(a) in (H,A)H(a) = (H,A)H(a) ∩ (H,A)H(a) = (H,A)H(a) (H,A)H(a) ⊆ (H,A)H(a)^2.

Therefore (H,A)is soft neutrosophic strongly regular and so (H,A) is soft neutrosophicregular.Since (H,A) is soft neutrosophic sub – commutative,then (E,A) ⊆ C(H,A).

Let L$_B$(x) in (L$_B$,A) .Since (H,A) is soft neutrosophic regular,

L$_B$(x) = L$_B$(x) L$_B$(y) L$_B$(x) = (L$_B$(x)H(a) L$_B$(x)) L$_B$(y)(L$_B$(x)H(a) L$_B$(x))

= L$_B$(x)H(x)(L$_B$(x) L$_B$(y) L$_B$(x))H(a) L$_B$(x)

= (L$_B$(x)H(a))^r(L$_B$(x) L$_B$(y) L$_B$(x))(H(a) L$_B$(x))^m

= (L$_B$(x))^r H(a)^r (L$_B$(x) L$_B$(y) L$_B$(x))H(a)^m L$_B$(x)^m in (L$_B$,A)^r(H,A)(L$_B$,A)^m

That is (L$_B$,A) ⊆ (L$_B$,A$^{)r}$(H,A)(L$_B$,A)^m.By definition of a generalized soft neutrosophic (r,m) bi-ideal, (L$_B$,A)^r(H,A)(L$_B$,A)^m ⊆ (L$_B$,A) .Hence (L$_B$,A) = (L$_B$,A)^r(H,A)(L$_B$,A)^m for every generalized soft neutrosophic (r,m) bi-ideal (L$_B$,A) of (H,A)

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