Available online at www.elixirpublishers.com (Elixir International Journal)

Discrete Mathematics

Elixir Dis. Math. 93 (2016) 39435-39442

Elixir ISSN: 2229-712X

The Eccentric Dominating Graph $ED_mG^{abc}(G)$ of a graph G

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ARTICLE INFO

Article history:

Received: 11 February 2016;

Received in revised form:

2 April 2016;

Accepted: 7 April 2016;

Keywords

Eccentric dominating set,

Minimum eccentric dominating set,

Eccentric dominating graph.

ABSTRACT

The eccentric dominating graph $ED_mG^{abc}(G)$ of a graph G is obtained from G with vertex set $V' = V \cup S$, where $V = V(G)$ and S is the set of all γ_{ed}-sets of G. Two elements in V' are said to satisfy property 'a' if $u, v \in V$ and are adjacent in G. Two elements in V' are said to satisfy property 'b' if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in V' are said to satisfy property 'c' if $u \in V$, $v = D \in S$ such that $u \in D$. Two elements in V' are said to satisfy property 'd' if $u, v \in V$ and there exists $D \in S$ such that $u, v \in D$. A graph having vertex set V' and any two elements in V' are adjacent if and only if they satisfy any one of the property a, b, c is denoted by $ED_mG^{abc}(G)$. In this paper $ED_mG^{abc}(G)$ of some families of graphs and some basic properties of $ED_mG^{abc}(G)$ are studied. Also, we have discussed the eccentricity properties of $ED_mG^{abc}(G)$, and we have characterized graphs G for which $ED_mG^{abc}(G)$ is complete or a tree.

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Introduction

Graphs discussed in this paper are undirected and simple. For graph theoretic terminology refer to Harary[5], Buckley and Harary[3]. For a graph, let V(G) and E(G) denotes its vertex and edge set respectively. A graph with p vertices and q edges is called a (p, q) graph.

The concept of distance in graph plays a dominant role in the study of structural properties of graphs in various angles using related concept of eccentricity of vertices in graphs. The length of any shortest path between any two vertices u and v of a connected graph G is called the *distance between u and v* and it is denoted by $d_G(u, v)$. The distance between two vertices in different components of a disconnected graph is defined to be ∞. For a connected graph G, The *eccentricity* e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v) : u \in V\}$.The *radius* rad(G) is the minimum eccentricity of the vertices, whereas the *diameter* diam(G) is the maximum eccentricity. If these two are equal in a graph, that graph is called *self-centered* graph with radius r and is called an *r self-centered graph*. For any connected graph G, $rad(G) \leq diam(G) \leq 2rad(G)$. v is a central vertex if e(v) = r(G). The *center* C(G) is the set of all central vertices. For a vertex v, each vertex at a distance e(v) from v is an *eccentric vertex of v*.

A graph G is *connected* if every two of its vertices are connected, otherwise G is *disconnected*. The *vertex connectivity* or simply *connectivity* $\kappa(G)$ of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph. The *edge connectivity* $\lambda(G)$ of a graph G is the minimum number of edges whose removal from G results in a disconnected or trivial graph. A set S of vertices of G is *independent* if no two vertices in S are adjacent. The *independence number* $\beta_o(G)$ of G is the maximum cardinality of an independent set.

The concept of domination in graphs was introduced by Ore [11]. The concept of domination in graphs is originated from the chess games theory and that paved the way to the development of the study of various domination parameters and its relation to various other graph parameters. For details on $\gamma(G)$, refer to [4, 12].

A set $D \subseteq V$ is said to be a *dominating set* in G, if every vertex in V–D is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the *domination number* and is denoted by $\gamma(G)$.

Janakiraman, Bhanumathi and Muthammai [6] introduced and studied the concept of eccentric dominating set. A set $D \subseteq V(G)$ is an *eccentric dominating set* if D is a dominating set of G and for every $v \in V–D$, there exists at least one eccentric vertex of v in D. The minimum cardinality of an eccentric dominating set is called the *eccentric domination number* and is denoted by $\gamma_{ed}(G)$. An eccentric dominating set with cardinality $\gamma_{ed}(G)$ is known as *minimum eccentric dominating set.*

If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. An eccentric dominating set D is a *minimal eccentric dominating set* if no proper subset $D'' \subseteq D$ is an eccentric dominating set.

A partition of V(G) is called *eccentric domatic* if all its classes are eccentric dominating sets in G. The maximum number of classes of an eccentric domatic partition of V(G) is called the eccentric domatic number of G and is denoted by $d_{ed}(G)$.

A vertex v is said to be good [14] if there is a γ-set of G containing v. If there is no γ-set of G containing v, then v is said to be a bad vertex.

In this manner, we define ed-good and ed-bad vertices as follows: Let $u \in V(G)$. u is said to be ed-*good* if u is contained in a γ_{ed}-set of G. u is said to be ed-*bad* if there exists no γ_{ed}-set of G containing u.

In [13], Walikar, Acharya and et al., defined $\gamma_D(G)$ as the total number of minimum dominating sets in a graph G.

In [1], we have defined $\gamma_{ED}(G)$ as the total number of minimum eccentric dominating sets in a graph G.

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In [7, 8, 9, 10], Kulli, Janakiram and Niranjan introduced the following concepts in the field of domination theory.

The *minimal dominating graph MD(G)*[8] of a graph G is the intersection graph defined on the family of all minimal dominating sets of vertices of G. The *vertex minimal dominating graph* $M_vD(G)$[9] of a graph G with $V(M_vD(G)) = V' = V \cup S$, where S is the collection of all minimal dominating sets of G with two vertices u, v ∈ V′ are adjacent if either they are adjacent in G or v = D is a minimal dominating set of G containing u.

The *dominating graph D(G)*[10] of a graph G = (V, E) is a graph with $V(D(G)) = V \cup S$, where S is the set of all minimal dominating sets of G and with two vertices u, v ∈ V(D(G)) are adjacent if u ∈ V and v = D is a minimal dominating set of G containing u.

In [2], we have defined and studied the dominating graph $DG^{abc}(G)$ of a graph G.

In this paper, we define $ED_mG^{abc}(G)$ with property a, b and c. We find $ED_mG^{abc}(G)$ for some families of graphs, and some basic properties of $ED_mG^{abc}(G)$ are studied. Also, the characterization of $ED_mG^{abc}(G)$ are established.

In section 3, we have defined and studied the properties of $EDG^{abc}(G)$.

The following results are needed to study $ED_mG^{abc}(G)$.

Theorem: 1.1[4] A graph G is Eulerian if and only if every vertex of G is of even degree.

Theorem: 1.2[1]

(i) $\gamma_{ED}(P_{3k}) = k$.

(ii) $\gamma_{ED}(P_{3k+1}) = 1$.

(iii) $\gamma_{ED}(P_{3k+2}) = \dfrac{k^2+3k+2}{2}$.

Theorem: 1.3[13]

(i) $\gamma_D(C_{3k}) = 3$.

(ii) $\gamma_D(C_{3k+1}) = (3k+1)(k+2)/2$.

(iii) $\gamma_D(C_{3k+2}) = 3k+2$.

Theorem: 1.4[5]

$\gamma_{ed}(K_n) = 1$.

Theorem: 1.5[5]

$\gamma_{ed}(K_{m,\ n}) = 2$.

Theorem: 1.6[5]

$\gamma_{ed}(K_{1,\ n}) = 2$, $n \ge 2$.

Theorem: 1.7[5]

$\gamma_{ed}(W_3) = 1$, $\gamma_{ed}(W_4) = 2$, $\gamma_{ed}(W_5) = 3$, $\gamma_{ed}(W_6) = 2$ and $\gamma_{ed}(W_n) = 3$ for $n \ge 7$.

2. The eccentric dominating graph $ED_mG^{abc}(G)$ of a Graph G

We define a new class of intersection graphs in the field of domination theory as follows.

Definition: 2.1

Let G be a graph with vertex set V(G) and let S be the set of all γ_{ed}-sets of G. Then two elements in V′ are said to satisfy property ‘a’ if u, v ∈ V and are adjacent in G. Two elements in V′ are said to satisfy property ‘b’ if $u = D_1$, $v = D_2 \in S$ and have a common vertex. Two elements in V′ are said to satisfy property ‘c’ if u ∈ V(G), v = D ∈ S such that u ∈ D. A graph having vertex set $V' = V \cup S$, where V = V(G) and S is the set of all γ_{ed}-sets of G and any two elements in V′ are adjacent if and only if they satisfy any one of the property a, b, c is denoted by $ED_mG^{abc}(G)$. Here, the elements of V(G) are called as point vertices and the elements of S are known as set vertices.

Remark: 2.1

(i) Total number of vertices in $ED_mG^{abc}(G)$ is $p + \gamma_{ED}(G)$.

(ii) Total number of edges in $ED_mG^{abc}(G)$ is $\le q + \gamma_{ED}(G)\gamma_{ed}(G) + \dfrac{\gamma_{ED}(G)(\gamma_{ED}(G)-1)}{2}$.

(iii) G is an induced sub graph of $ED_mG^{abc}(G)$.

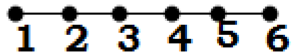
(iv) Number of edges in $ED_mG^{abc}(G) > q$.

(v) $\deg_{ED_mG^{abc}(G)} v_i \le \deg_G v_i + \gamma_{ED}(G)$, $1 \le i \le p$. The equality holds when v ∈ V(G) lie on all γ_{ed}-sets of G.

(vi) $\text{Deg } D_j \le \gamma_{ed}(G) + \gamma_{ED}(G) - 1$, $1 \le j \le n$. The equality holds when $D_i \cap D_j \ne \phi$.

Example

G:



ED_mG^{abc} (G):

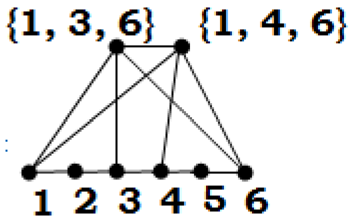


Fig. 2.1

{1, 3, 6} and {1, 4, 6} are γ_{ed}-sets of G.

Theorem: 2.1

Let G = K_p. Then ED_mG^{abc} (G) is $K_p \circ K_1$ and ED_mG^{abc} (G) is bi-eccentric with radius 2.

Proof

When G = K_p, each vertex form a γ_{ed}-set. By the definition, $ED_mG^{abc}(G)$ is $K_p \circ K_1$. The eccentricity of pendant vertices is 3 and the eccentricity of other vertices is 2. Hence, ED_mG^{abc} (G) is bi-eccentric with radius 2.

Theorem: 2.2

Let G = $\overline{K_p}$. Then ED_mG^{abc} (G) is $K_{1,\ p}$.

Proof

When G = $\overline{K_p}$, the whole vertex set is a γ_{ed}-set of G. By the definition, ED_mG^{abc} (G) is $K_{1,\ p}$.

Lemma: 2.1

(i) If G = W_3, then $\gamma_{ED}(G) = 4$.

(ii) If G = W_4, then $\gamma_{ED}(G) = 4$.

(iii) If G = W_p, then a) $\gamma_{ED}(G) = 15$ if p = 5.

b) $\gamma_{ED}(G) = 3$ if p = 6.

c) $\gamma_{ED}(G) = 28$ if p = 7.

d) $\gamma_{ED}(G) = 28$ if p = 8.

e) $\gamma_{ED}(G) = 12$ if p = 9.

f) $\gamma_{ED}(G) = p(p-3)/2$ if $p \ge 10$.

Proof:

Let G = $W_p = C_n + K_1$.

(i) G = $W_3 = K_4$. Hence $\gamma_{ed}(G) = 1$. Then it follows that, $\gamma_{ED}(G) = 4$.

(ii) **When G = W_4**, any two adjacent non-central vertices form γ_{ed} -set of G. Thus, we get four such γ_{ed}-sets. Hence, $\gamma_{ED}(G) = 4$.

(iii) **a) When p = 5.** Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where x and y are adjacent vertices in C_n and D_2 = {x, y, z} where x, y and z are three consecutive vertices in C_n and D_3 = {x, y, z} where x and y are adjacent vertices and d(x, z) = d(y, z) = 2 in C_n form γ_{ed}-sets of G. Therefore, we get p+p+p = 3p such γ_{ed}-sets of G. Hence, $\gamma_{ED}(G)$ = 3p = 15.

b) When p = 6. Let V = {v_1, v_2, v_3, v_4, v_5, v_6} be the vertices of C_6. D_1 = {v_1, v_4}, D_2 = {v_2, v_5} and D_3 = {v_3, v_6} are γ_{ed}-sets of W_6. Hence, $\gamma_{ED}(G)$ = 3.

c) When p = 7 = 3k+1. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n are γ_{ed}-sets of G. Therefore, we get p(p–3)/2 γ_{ed}-sets which contains u_1 and (3k+1)(k+2)/2 γ_{ed}-sets which contain vertices of C_7, since $\gamma_D(C_n)$ = (3k+1)(k+2)/2, by Theorem 1.3. Therefore, $\gamma_{ED}(G)$ = $\dfrac{(3k+1)(3k+1-3)+(3k+1)(k+2)}{2}$ = 2k(3k+1) = 28.

d) When p = 8 = 3k+2. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n are γ_{ed}-sets of G. Therefore, we get p(p–3)/2 γ_{ed}-sets which contains u_1 and (3k+2) γ_{ed}-sets which contain vertices of C_8, since $\gamma_D(C_p)$ = 3k+2, by Theorem 1.3.

Therefore, $\gamma_{ED}(G)$ = $\dfrac{(3k+2)(3k+2-3)}{2}+(3k+2)=\dfrac{(3k+1)(3k+2)}{2}=28.$

e) When p = 9 = 3k. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n. Therefore, we get p(p–3)/2 γ_{ed}-sets which contains u_1 and 3 γ_{ed}-sets which contains vertices of C_9, since $\gamma_D(C_n)$ = 3, by Theorem 1.3. Therefore, $\gamma_{ED}(G)$ = $\dfrac{(3k)(3k-3)}{2}+3=\dfrac{3(k(k-1)+2)}{2}=\dfrac{3(k^2-k+2)}{2}$ = 12.

f) When p ≥ 10. Let u_1 be the central vertex of G. D_1 = {x, y, u_1}, where d(x, y) ≠ 2 in C_n. Therefore, we get p(p–3)/2 γ_{ed}-sets such that each γ_{ed}-set contains central vertex u_1. Hence, $\gamma_{ED}(G)$ = p(p–3)/2.

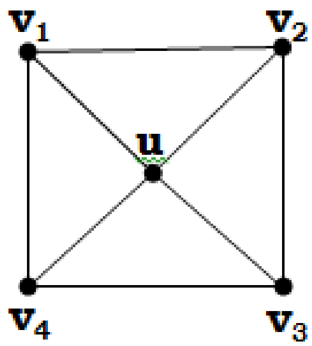
Theorem: 2.3

(i) If G = W_3, then $ED_mG^{abc}(G)$ is $K_4 \circ K_1$.
(ii) If G = W_4, then $ED_mG^{abc}(G)$ is a 2-self-centered graph.
(iii) If G = W_p, p = 5, 7, 8 and 9, then $ED_mG^{abc}(G)$ is a 2-self-centered graph. If p = 6, then $ED_mG^{abc}(G)$ is bi-eccentric with radius 2.
(iv) If G = W_p, p ≥ 10, then $ED_mG^{abc}(G)$ is of radius 1 and diameter 2.

Proof

(i) By Lemma 2.1, $\gamma_{ed}(G)$ = 1 and $\gamma_{ED}(G)$ = 4. Hence, by definition, $ED_mG^{abc}(G)$ is $K_4 \circ K_1$.
(ii) By Lemma 2.1, $\gamma_{ED}(G)$ = 4.

G:



$ED_mG^{abc}(G)$

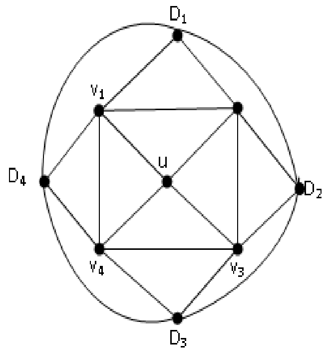


Fig 2.2

D_1 = {v_1, v_2}, D_2 = {v_2, v_3}, D_3 = {v_3, v_4} and D_4 = {v_4, v_1} are γ_{ed}-sets of G.

Thus, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

(iii) **a) When G = W_5.** By Lemma 2.1, $\gamma_{ED}(G)$ = 15. Consider the following cases.

Case (i) Suppose u, v ∈ V and d_G(u, v) ≤ 2

Since G is an induced sub graph of G, in $ED_mG^{abc}(G)$, d(u, v) = 1 or 2.

Case (ii) Suppose u ∈ V and v = D ∈ S

If u ∈ D, then in $ED_mG^{abc}(G)$, d(u, v) =1.

If u ∉ D, then there exists a vertex u′ ∈ V such that u′ dominates u and u′ ∈ D, then it follows that in $ED_mG^{abc}(G)$, d(u, v) = d(u, u′)+d(u′, D) = 2.

Case (iii) Suppose u, v ∈ S

If u and v have a vertex in common, then in $ED_mG^{abc}(G)$, d(u, v) = 1, otherwise d(u, v) = 2. Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

b) When G = W_6. By Lemma 2.1, $\gamma_{ED}(G)$ = 3. Consider the following cases.

Case (i) Suppose u, v ∈ V and d_G(u, v) ≤ 2

As in case (i) of (iii) a), d(u, v) ≤ 2 in $ED_mG^{abc}(G)$.

Case (ii) Suppose u ∈ V and v = D ∈ S

As in case (ii) of (iii) a), d(u, v) ≤ 2 in $ED_mG^{abc}(G)$.

Case (iii) Suppose u, v ∈ S

Set vertices are disjoint. Let u = D_1 and v = D_2 be two γ_{ed}-sets of G. There exists some vertices of D_1 is adjacent to some vertices of D_2. Then in $ED_mG^{abc}(G)$, uv_1v_2v is a path. Therefore, d(u, v) = 3.

Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is 3. Hence, $ED_mG^{abc}(G)$ is bi-eccentric with radius 2.

c) When p = 7, 8, 9, G has atleast two disjoint γ_{ed}-sets.

Let u = D_1 and v = D_2 be two γ_{ed}-sets of G. If D_1 and D_2 are disjoint, then there exists a γ_{ed}-set D_3 such that D_3 is adjacent to both D_1 and D_2. In $ED_mG^{abc}(G)$, d(D_1, D_2) = d(D_1, D_3)+d(D_3, D_2) = 2. Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

(iv) **When G = W_p, p ≥ 10.** By Lemma 2.1, $\gamma_{ED}(G)$ = p(p–3)/2. Let u_1 be the central vertex of G, every γ_{ed}-set contains the central vertex. Thus, eccentricity of central vertex is 1 in $ED_mG^{abc}(G)$.

Eccentricity of point vertices except central vertex is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is of radius 1 and diameter 2.

Theorem: 2.4

If $G = K_{m,n}$, then $ED_mG^{abc}(G)$ is a 2- self-centered graph.

Proof

$G = K_{m,n}$. $V(G) = V_1 \cup V_2$. $|V_1| = m$ and $|V_2| = n$. $D = \{u, v\}$, $u \in V_1$ and $v \in V_2$ is a γ_{ed}-sets of G. Thus, we get mn such γ_{ed}-sets. Since G is an induced sub graph of $ED_mG^{abc}(G)$, $e(u) = e(v) = 2$. Suppose $x, y \notin V$. Then $x = D_1$ and $y = D_2$ are two γ_{ed}-sets of G. If D_1 and D_2 have a common vertex, then, in $ED_mG^{abc}(G)$, $d(D_1, D_2) = 1$. Suppose D_1 and D_2 are disjoint. Then there exists a γ_{ed}-set D_3 such that D_3 is adjacent to both D_1 and D_2. Then, in $ED_mG^{abc}(G)$, $d(D_1, D_2) = d(D_1, D_3)+d(D_3, D_2) = 2$. Thus, the eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence $ED_mG^{abc}(G)$ is a 2-self-centered graph.

Theorem: 2.5

If $G = K_{1,n}$, $n \geq 3$, then $K_{1,2n}$ and K_n^+ are edge disjoint sub graphs of $ED_mG^{abc}(G)$.

Proof

$G = K_{1,n}$, $n \geq 3$. Let $D = \{u, v\}$, where v is the central vertex. The central vertex dominates all vertices in V–D and u is an eccentric vertex of V–D. Hence, D is a γ_{ed}-set of G. We get n such γ_{ed} -sets. In $ED_mG^{abc}(G)$, set vertices form a clique. Central vertex v is adjacent to n point vertices and n set vertices. These edges form $K_{1, 2n}$ and the remaining edges form K_n^+. Hence, $K_{1,2n}$ and K_n^+ are edge disjoint sub graphs of $ED_mG^{abc}(G)$.

Theorem: 2.6

If $G = P_p$, then

(i) $ED_mG^{abc}(G)$ is of radius 2 and diameter 4, when $p = 3k$, $3k+1$, $k \geq 3$.

(ii) $ED_mG^{abc}(G)$ is bi-eccentric with radius 2, when $p = 3k+2$, $k \geq 3$.

(iii) $ED_mG^{abc}(G)$ is 2-self-centered, when p = 3, 4 and 5.

Proof

(i) $G = P_p$, $p = 3k$, $3k+1$.

Two end vertices of G are eccentric vertices. Every γ_{ed}-set contains two end vertices. Hence, set vertices are adjacent to each other in $ED_mG^{abc}(G)$.

From Theorem 1.2, $\gamma_{ED}(G) = \begin{cases} k & if\ p = 3k \\ 1 & if\ p = 3k+1 \end{cases}$

Consider the following sub cases.

case (i) Suppose $u, v \in V$.

If there exists a γ_{ed}-set D_1 such that D_1 contains u and v, then in $ED_mG^{abc}(G)$, $d(u, v) = 2$.

If $u \in D_1$ and $v \in D_2$, then there exists a vertex u_1 such that u_1 dominates u and $u_1 \in D_2$. Thus in $ED_mG^{abc}(G)$, uu_1D_2v is a path. Therefore, $d(u, v) = 3$.

If there is no γ_{ed}-set which contains u and v, then there exists γ_{ed}-set D_3 such that D_3 contains u' and v', u' dominates u and v' dominates v. Then in, $ED_mG^{abc}(G)$, $uu'D_3v'v$ is a path. Therefore, $d(u, v) = 4$.

case (ii) Suppose $x \in V$, $y \in S$, $y = D_4$ is the γ_{ed}-set of G.

If $x \in D_4$, then in $ED_mG^{abc}(G)$, $d(x, y) = 1$.

If $x \notin D_4$, then there exists a vertex $x' \in V(G)$ such that x' dominates x and $x' \in D_4$. Then in $ED_mG^{abc}(G)$, $d(x, y) = d(x, x')+d(x', y) = 2$.

Hence, $rad(ED_mG^{abc}(G)) = 2$, $diam(ED_mG^{abc}(G)) = 4$.

(ii) When p = 3k+2. From Theorem 1.2, $\gamma_{ED}(G) = \dfrac{k^2+3k+2}{2}$. Here, all vertices are ed-good.

case (i) Suppose $u, v \in V$.

If there exists a γ_{ed}-set D_1 such that D_1 contains u and v, then in $ED_mG^{abc}(G)$, $d(u, v) = 2$ in $ED_mG^{abc}(G)$.

If $u \in D_1$ and $v \in D_2$, then there exists a vertex u_1 such that u_1 dominates u and $u_1 \in D_2$. In $ED_mG^{abc}(G)$, uu_1D_2v is a path. Therefore, $d(u, v) = 3$.

case (ii) Suppose $x \in V$, $y \in S$, $y = D_3$ is the γ_{ed}-set of G.

If $x \in D_3$, then in $ED_mG^{abc}(G)$, $d(x, y) = 1$.

If $x \notin D_3$, then there exists a vertex $x' \in V(G)$ such that x' dominates x and $x' \in D_3$. Then in $ED_mG^{abc}(G)$, $d(x, y) = d(x, x')+d(x', y) = 2$.

Hence, $rad(ED_mG^{abc}(G)) = 2$, $diam(ED_mG^{abc}(G)) = 3$.

(iii) When p = 3, 4 and 5. Eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

Theorem: 2.7

If $G = K_{2n}–F$, where F is a 1-factor, then $ED_mG^{abc}(G)$ is a 2-self-centered graph.

Proof

Let G be a graph obtained from the complete graph K_{2n} by deleting edges of a linear factor. That is, $G = K_{2n}–F$, where F is a 1-factor. Let u_i and u'_i, i = 1, 2, 3,..., n be a pair of non-adjacent vertices in G. Then u_i and u'_i are eccentric to each other. $D = \{v_1, v_2, v_3, ..., v_n\}$, where, $v_i = u_i$ or u'_i is a γ_{ed}-set of G. Therefore, there are 2^n such γ_{ed}-sets of G, u_i, u'_i cannot be in a single γ_{ed}-set of G, i = 1, 2, 3, ..., n. Let $u, v \in V'$. Consider the following cases.

Case (i) $u, v \in V$.

In this case, G is an induced sub graph of $ED_mG^{abc}(G)$. Then in, $ED_mG^{abc}(G)$, $d(u, v) = 2$.

Case (ii) $u \in V$ and $v \in S$, $v = D$ is the γ_{ed}-set of G.

If $u \in D$, then in $ED_mG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D$, then there exists a vertex $u' \in V(G)$ such that u' dominates u and $u' \in D$. Then it follows that, in $ED_mG^{abc}(G)$, $d(u, v) = d(u, u')+d(u', v) = 2$.

Case (iii) $u, v \in S$, $u = D_1$ and $v = D_2$ are two γ_{ed}-sets of G.

If D_1 and D_2 have a vertex in common, then in $ED_mG^{abc}(G)$, $d(u, v) = 1$.

If D_1 and D_2 are disjoint, then there exists a γ_{ed}-set D_3 such that D_3 is adjacent to both D_1 and D_2. Then it follows that, in $ED_mG^{abc}(G)$, $d(D_1, D_2) = d(D_1, D_3,)+d(D_3, D_2) = 2$.

Hence, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

Corollary: 2.7 Let $G = K_n–F$, where F is a 1- factor, n is even. Then $ED_mG^{abc}(G)$ is Eulerian if $n \equiv 0 (mod\ 4)$.

Proof: Number of vertices in $ED_mG^{abc}(G)$ is $n+2^{n/2.}$

Degree of point vertex in $ED_mG^{abc}(G)$ is $n–2+2^{n/2–1}$.

Degree of set vertex in $ED_mG^{abc}(G)$ is $n/2+2^{n/2–1}$. All vertices have even degree in $ED_mG^{abc}(G)$, since n is even. Then by Theorem 1.1, $ED_mG^{abc}(G)$ is Eulerian.

Theorem: 2.8 For any graph G, $ED_mG^{abc}(G)$ is connected.

Proof: Case (i) G is connected.

G is an induced sub graph of $ED_mG^{abc}(G)$. If D is any γ_{ed}-set, it is adjacent to some vertices of G. Therefore, $ED_mG^{abc}(G)$ is connected.

Case (ii) G is disconnected.

Let D be a γ_{ed}-set of G. Then D contains vertices from each component of G. In ED_mG^{abc} (G), D is adjacent to those vertices. Therefore, in $ED_mG^{abc}(G)$, any two vertices are connected by a path. Hence, $ED_mG^{abc}(G)$ is connected.

Theorem: 2.9

$ED_mG^{abc}(G)$is complete if and only if $G = K_1$.

Proof

Suppose $ED_mG^{abc}(G)$ is complete. Then G is complete and each γ_{ed} -set contains all the vertices of G. That is, G has exactly one γ_{ed}-set. Hence, $G = K_1$.

Conversely, $G = K_1$, by the definition, we get $ED_mG^{abc}(G) = K_2$. This implies that $ED_mG^{abc}(G)$ is complete.

Theorem: 2.10

For any graph G, $ED_mG^{abc}(G)$ is a tree if and only if G is K_2 or $\overline{K_p}$.

Proof

Suppose $ED_mG^{abc}(G)$ is a tree. Then G has no cycle. To prove that G is K_2 or $\overline{K_p}$. On the contrary, suppose $G \neq \overline{K_p}$ or K_2. Consider the following two cases.

Case (i) If $\Delta(G) = p-1$, $p \geq 3$, then G is a star. By Theorem 2.5, $ED_mG^{abc}(G)$ has a cycle, a contradiction.

Case (ii) If $\Delta(G) \leq p-2$, Since G is a tree, then there exists three vertices u, v and $w \in V$ such that u and v are adjacent and w is not adjacent to both u and v and is an eccentric vertex. This implies that, in $ED_mG^{abc}(G)$, u and v are connected by at least two paths, a contradiction. Hence from the above cases, $G = K_2$ or $\overline{K_p}$.

Conversely, Suppose, $G = K_2$ or $\overline{K_p}$. $ED_mG^{abc}(G)$ is $K_{1,p-1}$ or P_4. Hence, $ED_mG^{abc}(G)$ is a tree.

Theorem: 2.11

(i) $\beta_o(ED_mG^{abc}(G)) \geq \max\{\beta_o(G), d_{ed}(G)\}$.

(ii) For any graph G, $\kappa(ED_mG^{abc}(G)) \leq \min\{\min \deg_{ED_mG^{abc}(G)} v_i, 1 \leq i \leq p, \gamma_{ed}(G)\}$.

(iii) For any graph G, $\lambda(ED_mG^{abc}(G)) \leq \min\{\min \deg_{ED_mG^{abc}(G)} v_i, 1 \leq i \leq p, \gamma_{ed}(G)\}$.

(iv) $\chi(G) \leq \chi(ED_mG^{abc}(G)) \leq \chi(G)+\gamma_{ED}(G)$. Furthermore, both bounds are sharp.

Proof

(i) Proof is obvious.

(ii) **Case (i)** Let $v \in V$ and is of minimum degree among the all vertices of $ED_mG^{abc}(G)$. Then by deleting the vertices adjacent to v, the resulting graph is disconnected. Thus, $\kappa(ED_mG^{abc}(G)) \leq \min\{ \deg_{ED_mG^{abc}} v_i, 1 \leq i \leq p\}$.

Case (ii) Let S be the set of all γ_{ed}-sets of G. Cardinality of each set is $\gamma_{ed}(G)$. Suppose $\gamma_{ed}(G) \leq \delta(G)$. Then by deleting the vertices adjacent to any one γ_{ed}-set, the resulting graph is disconnected. Hence, $\kappa(ED_mG^{abc}(G)) \leq \min\{\min \deg_{ED_mG^{abc}(G)} v_i, 1 \leq i \leq p, \gamma_{ed}(G)\}$.

(iii) As in ii), $\lambda(ED_mG^{abc}(G)) \leq \min\{\min \deg_{ED_mG^{abc}(G)} v_i, 1 \leq i \leq p, \gamma_{ed}(G)\}$.

(iv) Proof is obvious.

Theorem: 2.12

For any graph G, distance between any two vertices in $ED_mG^{abc}(G)$ is at most four.

Proof

Suppose G has at least two vertices then $ED_mG^{abc}(G)$ has at least three vertices. Let u, $v \in V'$. We consider the following cases.

Case (i) Suppose u, $v \in V$.

If u and v are adjacent in G, then in $ED_mG^{abc}(G)$, d(u, v) = 1.

Suppose u and v are not adjacent in G.

Sub Case (i) In this case, there exists a γ_{ed}-set containing u and v. This implies that, in $ED_mG^{abc}(G)$, d(u, v) = 2.

Sub Case (ii) In this case, there exists a vertex w such that w is adjacent to both u and v. Then, in $ED_mG^{abc}(G)$, d(u, v) = d(u, w)+d(w, v) = 2.

Sub Case (iii) y = D is a γ_{ed}-set of G. Suppose the vertices w, $x \in D$ are adjacent to u and v respectively, then in $ED_mG^{abc}(G)$, $d(u, v) \leq d(u, w) + d(w, y) + d(y, x) + d(x, v) = 4$.

Case (ii) Suppose u$\in$ V and v $\in$ S, v = D is the γ_{ed}-set of G.

If $u \in D$, then in $ED_mG^{abc}(G)$, d(u, v) = 1. If $u \notin D$, then there exists a vertex $w \in D$ adjacent to u and hence in $ED_mG^{abc}(G)$, d(u, v) = d(u, w) + d(w, v) = 2.

Case (iii) Suppose u, v $\in$ S, u = D and v = D′ are two γ_{ed}-sets of G.

If D and D′ have a vertex in common, then in $ED_mG^{abc}(G)$, d(u, v) = 1.

If D and D′ are disjoint. Consider the following sub cases.

Sub case (i) If there exists a γ_{ed}-set D″ such that D″ is adjacent to both D and D′. Thus, in $ED_mG^{abc}(G)$, d(D, D′) = d(D, D″)+d(D″, D′) = 2.

Sub case (ii) every vertex of $w \in D$ is adjacent to some vertex $x \in D'$ and vice versa. Thus, it follows that in $ED_mG^{abc}(G)$, uwxv is a path. Therefore, $d(u, v) \leq 3$.

Hence, from the above cases, distance between any two vertices in $ED_mG^{abc}(G)$ is at most four.

Theorem: 2.13

Let G be a connected graph with rad(G) = 1 and diam(G) = 2. Any central vertex lies on all the γ_{ed}-set if and only if radius of $ED_mG^{abc}(G)$ is one.

Proof

Let G be a connected graph with rad(G) = 1, diam(G) = 2 and let u be any central vertex. Suppose u lies on all the γ_{ed}-sets of G. Then, in $ED_mG^{abc}(G)$, all the γ_{ed}-sets are adjacent to each other and deg u = (p–1)+ $\gamma_{ED}(G)$. Therefore, eccentricity of u in ED_mG^{abc} (G) is one. Since G is connected $\gamma_{ed}(G) \leq p-1$, implies eccentricity of set vertices is not equal to one. Suppose there exists a vertex $u \in V$ such that u is not in any γ_{ed}-set, then also eccentricity of u in $ED_mG^{abc}(G)$ is not equal to one. Therefore, $rad(ED_mG^{abc}(G)) = 1$ if and only if there exists $u \in V$ such that u belongs to every γ_{ed}-set of G.

Theorem: 2. 14

Let G be a 2-self-centered graph. If $D_i \cap D_j \neq \phi$ for $i \neq j$, then $ED_mG^{abc}(G)$ is a 2-self-centered graph. Otherwise, $ED_mG^{ac}(G)$ is bi-eccentric with diameter three.

Proof

Let G be a 2-self-centered graph. Let u, $v \in V'$. Consider the following cases.

Case (i) Suppose u, v $\in$ V. Since G is an induced sub graph of $ED_mG^{abc}(G)$. Then, it follows that, in $ED_mG^{abc}(G)$, d(u, v) = 2.

Case (ii) Suppose u $\in$ V and v $\notin$ V. Then v = D is a γ_{ed}-set of G.

If $u \in D$, then in $ED_mG^{abc}(G)$, d(u, v) = 1. If $u \notin D$, then there exists a vertex $w \in V$ such that w dominates u and $w \in D$.

Thus, it follows that, in $ED_mG^{abc}(G)$, u-w-v is a path, $d(u, v) = d(u, w)+d(w, v) \leq 2$.

Case (iii) Suppose u, v $\notin$ V. Then $u = D_1$ and $v = D_2$ are two γ_{ed}-sets of G. If $D_i \cap D_j \neq \phi$ for $i \neq j$, then any two set vertices have a common vertex. There exists a vertex $y \in V$ such that $y \in D_1$ and D_2. Thus, it follows that, in $ED_mG^{abc}(G)$, u-y-v is a path, $d(u, v) = d(u,y)+d(y, v) \leq 2$.

Therefore, eccentricity of point vertices is 2 and eccentricity of set vertices is also 2. Hence, $ED_mG^{abc}(G)$ is a 2-self-centered graph.

Suppose $D_i \cap D_j = \phi$, for $i \neq j$. If D_1 and D_2 are disjoint. Then each vertex $x \in D_1$ is adjacent to some vertex $z \in D_2$ and vice versa. Thus, it follows that, in $ED_mG^{abc}(G)$, u-x-z-v is a path, $d(u, v) = d(u, x)+d(x, z)+d(z, v) \leq 3$.

Hence, $ED_mG^{abc}(G)$ is bi-eccentric with diameter three.

Theorem: 2.15

Let G be a graph with diameter 3. Then $diam(ED_mG^{abc}(G))$ is 2 or 3.

Proof

Let G be a graph with diameter 3 and let $u, v \in V'$. We consider the following cases.

Case (i) Suppose u, v $\in$ V.

If u and v are adjacent in G, then in $ED_mG^{abc}(G)$, $d(u, v) = 1$, since $ED_mG^{abc}(G)$ contains G.

Suppose u and v are not adjacent in G.

Sub case (i) d(u, v) = 2.

a) There exists a γ_{ed} -set D such that D contains u and v, then in $ED_mG^{abc}(G)$, $d(u, v) = 2$.

b) There exists no γ_{ed} -set containing both u and v, and u, v belongs to some γ_{ed} -sets, then in $ED_mG^{abc}(G)$, $d(u, v) = 2$.

c) u and v are not ed-good vertices, $d(u, v) = 2$ in $ED_mG^{abc}(G)$.

Sub case (ii) d(u, v) = 3.

a) There exists a γ_{ed} -set D_1 such that D_1 contains u and v, then in $ED_mG^{abc}(G)$, $d(u, v) = d(u, D_1)+d(D_1, v) = 2$.

b) $u \in D_1$ and $v \in D_2$. If there exists a vertex w such that w is adjacent to both u and v, then in $ED_mG^{abc}(G)$, $d(u, v) = d(u, w)+d(w, v) = 2$.

c) u and v are not ed-good vertices, in $ED_mG^{abc}(G)$, $d(u, v) = 3$.

Case (ii) Suppose u $\in$ V and v $\in$ S, v = D_3 is the γ_{ed}-set of G.

If $u \in D_3$, then in $ED_mG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D_3$, then there exists a vertex u′ such that u′ dominates u and $u' \in D_3$. It follows that, in $ED_mG^{abc}(G)$, $d(u, v) = d(u, u')+d(u', v) = 2$.

Case (iii) Suppose u, v $\in$ S, u = D_4 and v = D_5 are two γ_{ed}-sets of G.

If D_4 and D_5 have a common vertex, then in $ED_mG^{abc}(G)$, $d(D_4, D_5) = 1$.

Suppose D_4 and D_5 are disjoint. Then there exists a γ_{ed}-set D_6 such that D_6 is adjacent to both D_4 and D_5. Then, it follows that, in $ED_mG^{abc}(G)$, $d(D_4, D_5) = d(D_4, D_6)+d(D_6, D_5) = 2$. If there does not exist, then every vertex $w \in D_4$ is adjacent to some vertex $w' \in D_5$ and vice versa. This implies that, in $ED_mG^{abc}(G)$, uww′v is a path. Therefore, $d(u, v) = 3$.

Hence, diameter of $ED_mG^{abc}(G)$ is 2 or 3.

Corollary: 2.15

(i) If $G = P_4$, then $ED_mG^{abc}(G)$ is C_5 which is 2-self-centered.

(ii) If $G = C_6$, then $ED_mG^{abc}(G)$ is 3-self-centered.

Theorem: 2.16

Let G be a graph with diameter greater than or equal to 4. Then $diam(ED_mG^{abc}(G)) \leq 4$.

Proof

Let G be a graph with $diam(G) \geq 4$ and let $u, v \in V'$. Consider the following cases.

Case (i) Suppose u, v $\in$ V.

If u and v are adjacent in G, then it follows that, in $ED_mG^{abc}(G)$, $d(u, v) = 1$.

Suppose u and v are not adjacent in G.

Sub case (i) If there exists a vertex $x \in V$ such that x is adjacent to both u and v. Then, in $ED_mG^{abc}(G)$, $d(u, v) = d(u, x)+d(x, v) = 2$.

Sub case (ii) supposes u and v are eccentric vertices of G.

If there exists a γ_{ed}-set D_1 such that D_1 contains u and v. Then it follows that, in $ED_mG^{abc}(G)$, $d(u, v) = d(u, D_1)+d(D_1, v) = 2$.

Sub case (iii) u $\in$ D_2, v $\in$ D_3.

In this case, there exists a vertex $y \in V$ such that y dominates v and $y \in D_2$. It follows that, in $ED_mG^{abc}(G)$, uD_2yv is a path. Therefore, $d(u, v) \leq 3$.

Sub case (iv) u and v are not ed-good vertices.

In this case, there exists vertices $u', v' \in V$ such that u′ dominates u, v′ dominates v and D_3 contains u′ and v′. Then, it follows that, in $ED_mG^{abc}(G)$, $uu'D_3v'v$ is a path. Therefore, $d(u, v) \leq 4$.

Case (ii) Suppose u $\in$ V and v $\in$ S, v = D_4 is the γ_{ed}-set of G.

If $u \in D_4$, then in $ED_mG^{abc}(G)$, $d(u, v) = 1$. If $u \notin D_4$, then there exists a vertex $z \in V$ such that z dominates u and $z \in D_4$. It follows that, in $ED_mG^{abc}(G)$, $d(u, v) = d(u, z)+d(z, v) = 2$.

Case (iii) Suppose u, v $\in$ S, u = D_5 and v = D_6 are two γ_{ed}-sets of G.

If D_5 and D_6 have a common vertex, then in $ED_mG^{abc}(G)$, $d(D_5, D_6) = 1$.

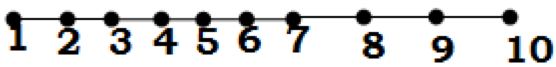
Suppose D_5 and D_6 are disjoint. If there exists a γ_{ed}-set D_7 such that D_7 is adjacent to both D_5 and D_6, it follows that, in $ED_mG^{abc}(G)$, $d(D_5, D_6) = d(D_5, D_7)+d(D_7, D_6) = 2$. If D_5 and D_6 are disjoint, then every vertex $w \in D_5$ is adjacent to some vertex $w' \in D_6$ and vice versa. This implies that, in $ED_mG^{abc}(G)$, uww′v is a path. Therefore, $d(u, v) \leq 3$.

So, $diam(ED_mG^{abc}(G)) \leq 4$.

Example:

Consider $G = P_{10}$. $diam(ED_mG^{abc}(G))$ is 4.

G:



D = {1, 4, 7, 10} is a γ_{ed} -set of G.

$ED_mG^{abc}(G)$:

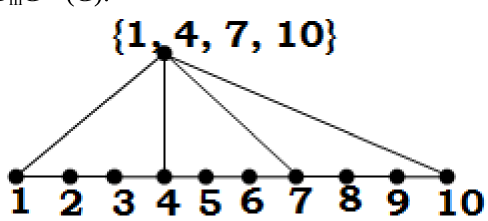


Fig. 2.3

Theorem: 2.17

$ED_mG^{abc}(G)$ is self-centered with diameter 2 if G is any one of the following:

(i) $rad(G) = 1$, $diam(G) = 2$ and any central vertex does not lie on any γ_{ed} -set of G.

(ii) G is self-centered with diameter 2 and $D_i \cap D_j \neq \phi$ for $i \neq j$.

(iii) G = $\overline{K_m} + K_1 + K_1 + \overline{K_n}$, m, n ≥ 2.

(iv) G is a wounded spider with k legs, k > 2 and one non-wounded leg.

Proof

(i) When rad(G) = 1, diam(G) = 2. Consider the following two cases. Let u be the central vertex.

Case (i) a) Suppose u ∉ D, where D is a γ_{ed}-set of G.

Let v = D be the γ_{ed}-set of G. If u ∉ D, then there exists a vertex w ∈ V such that w dominates u and w ∈ D. This implies that, in $ED_mG^{abc}(G)$, d(u, v) = d(u, w)+d(w, v) = 2.

Let D_1 and D_2 be two disjoint γ_{ed}-sets of G. Suppose vertices x ∈ D_1 and x′ ∈ D_2, and there exists a γ_{ed}-set D_3 containing x and x′. Thus, it follows that, d(D_1, D_2) = d(D_1, D_3)+d(D_3, D_2) = 2. If D_1, D_2 are not disjoint, d(D_1, D_2) = 1. Also, rad(G) = 1, diam(G) = 2 implies $\gamma_{ed}(G) < p - 1$. Hence, there exists u ∈ V such that d(u, D) = 2 for any D. Hence, $ED_mG^{abc}(G)$ is self-centered with diameter 2.

b) Suppose the central vertex lies on some γ_{ed}-sets of G.

Let D_1 be the γ_{ed}-set of G. Suppose u ∉ D_1, then there exists a γ_{ed}-set D_2 contains u such that D_1 and D_2 are adjacent. This follows that, in $ED_mG^{abc}(G)$, d(u, D_1) = d(u, D_2)+d(D_2, D_1) = 2.

Let w ∈ V and D_3 be the γ_{ed}-set of G. Suppose w ∉ D_3, then there exists a vertex w′ adjacent to w such that D_3 contains w′. This, it follows that d(w, D_3) = d(w, w′)+d(w′,D_3) = 2. Hence, $ED_mG^{abc}(G)$ is self-centered with diameter 2.

(ii) G is self-centered with diameter 2 and $D_i \cap D_j \neq \phi$ for i ≠ j.

By Theorem 2.14, $ED_mG^{abc}(G)$ is self-centered with diameter 2.

(iii) When G = $\overline{K_m} + K_1 + K_1 + \overline{K_n}$, m, n ≥ 2.

Let u, v ∈ V. d_G(u, v) = 3. Suppose the γ_{ed}-set D contains u and v. Then in $ED_mG^{abc}(G)$, d(u, v) = d(u, D)+d(D, v) = 2. The two central vertices lie on all γ_{ed}-sets of G. Let w be the pendant vertex and D_1 be the γ_{ed}-set of G. Suppose w ∉ D_1. Then there exists a γ_{ed}-set D_2 such that D_2 contains w. This follows that, in $ED_mG^{abc}(G)$, d(w, D_1) = d(D_1, D_2)+d(D_2, w) = 2. Hence, ED_mG^{abc} (G) is self-centered with diameter 2.

(iv) Let G be a wounded spider with k legs, k > 2 and one non-wounded leg.

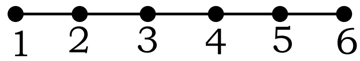
Let usv represent the non wounded leg, where v is pendant, u support vertex of the wounded legs. D_1 = {u, s, v}, D_2 = {u, w, v} where w is a pendant vertex of wounded leg are γ_{ed}-sets of G. Let x, y ∈ V, d_G(x, y) = 3. Then there exists a γ_{ed}-set D such that D contains x, y. Thus, in $ED_mG^{abc}(G)$, d(x, y) = d(x, D)+d(D, y) = 2. Every γ_{ed}-set contains u and v. Thus, γ_{ed}-sets are adjacent to each other. Hence, in $ED_mG^{abc}(G)$, d(D_i, D_j) = 1. Suppose x ∉ D_1. In $ED_mG^{abc}(G)$, d(x, D_1) = d(x, u)+d(u, D_1) = 2. Hence, $ED_mG^{abc}(G)$ is self-centered with diameter 2.

3. The Eccentric dominating graph $EDG^{abc}(G)$ of a graph G

Definition: 3.1

The eccentric dominating graph $EDG^{abc}(G)$ of a graph G is obtained from G with vertex set V′ = V ∪ S, where V = V(G) and S is the set of all minimal eccentric dominating sets of G. Then two elements in V′ are said to satisfy property 'a' if u, v ∈ V and are adjacent in G. Two elements in V′ are said to satisfy property 'b' if u = D_1, v = D_2 ∈ S and have a common vertex. Two elements in V′ are said to satisfy property 'c' if u ∈ V, v = D ∈ S such that u ∈ D. Two elements in V′ are said to satisfy property'd' if u, v ∈ V and there exists D ∈ S such that u, v ∈ D. A graph having vertex set V′ and any two elements in V′ are adjacent if and only if they satisfy any one of the property a, b, c is denoted by $EDG^{abc}(G)$.

Example: 3.1

G: 

{1, 3, 6}, {1, 4, 6} and {1, 2, 5, 6} are minimal eccentric dominating sets of G.

EDG^{abc}(G)

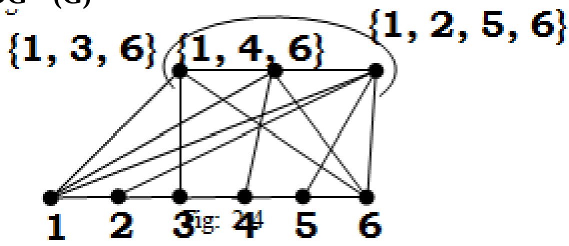


Fig. 2.4

Remarks: 3.1

(i) G is an induced sub graph of $EDG^{abc}(G)$.

(ii) $ED_mG^{abc}(G)$ is a sub graph of $EDG^{abc}(G)$.

(iii) Number of vertices in $EDG^{abc}(G)$ is p+number of minimal eccentric dominating sets of G.

(iv) Number of edges in $EDG^{abc}(G)$ is greater than q.

(v) $\deg_{EDG^{abc}(G)} v_j = \deg_G v_j + S_j$, 1 ≤ j ≤ p, where S_j is the number of minimal eccentric dominating set containing v_j.

(vi) deg D_i ≤ $|D_i|+|S|(|S|-1)/2$, 1 ≤ i ≤ n, where S is the set of all minimal eccentric dominating sets of G.

Observation: 3.1

If all γ_{ed}-sets of G are minimal, then $EDG^{abc}(G) \cong ED_mG^{abc}(G)$.

Theorem: 3.1

If G = K_p, then $EDG^{abc}(G) = ED_mG^{abc}(G)$. That is, $EDG^{abc}(G)$ is $K_p \circ K_1$ and $EDG^{abc}(G)$ is of radius 2 and diameter 3.

Proof

Proof is similar to the proof of Theorem 2.1.

Theorem: 3.2

If G = $\overline{K_p}$, then $EDG^{abc}(G) = ED_mG^{abc}(G)$. That is, $EDG^{abc}(G)$ is $K_{1,p}$.

Proof

Proof is similar to the proof of Theorem 2.2.

Theorem: 3.3

If G = $K_{1,p-1}$, then $EDG^{abc}(G)$ is a 2-self-centered graph.

Proof

Let G = $K_{1,p-1}$. D = {u, v}, where u is the central vertex and v is the non-central vertex of G and all pendant vertices form minimal eccentric dominating sets of G. Thus, we get p such minimal eccentric dominating sets of G. Let x, y ∈ V′. Consider the following cases.

Case (i) Suppose x, y ∈ V.

If d_G(x, y) ≤ 2, then G is an induced sub graph of $EDG^{abc}(G)$. Then in $EDG^{abc}(G)$, d(x, y) ≤ 2.

Case (ii) x ∈ V and y ∈ S, y = D_1 is the minimal eccentric dominating set of G.

If x ∈ D_1, then in $EDG^{abc}(G)$, d(x, y) = 1.

If x ∉ D_1, then there exists a vertex x′ ∈ V such that x′ dominates x and x′ ∈ D_1. Thus, in $EDG^{abc}(G)$, d(x, y) = d(x, x′)+d(x′, y) = 2.

Case (iii) Suppose x, y ∈ S, x = D_2, y = D_3 are two minimal eccentric dominating sets of G.

In this case, set vertices are adjacent to each other. Then, it follows that, in $EDG^{abc}(G)$, d(x, y) = 1.

Hence, $EDG^{abc}(G)$ is a 2-self-centered graph.

Theorem: 3.4

If G = $K_{m,n}$, then $EDG^{abc}(G)$ is a 2-self-centered graph. Here, $EDG^{abc}(G) = ED_mG^{abc}(G)$.

Proof:

Proof is similar to the proof of Theorem 2.4.

Theorem: 3.5

For any graph G, $EDG^{abc}(G)$ is connected.

Proof:

Proof is similar to the proof of Theorem 2.8.

Theorem: 3.6

$EDG^{abc}(G)$ is complete if and only if G = K_1.

Proof:

Proof is similar to the proof of Theorem 2.9.

Theorem: 3.7

$EDG^{abc}(G)$ is a tree if and only if G is $\overline{K_p}$ or K_2.

Proof:

Proof is similar to the proof of Theorem 2.10.

Theorem: 3.8

For any graph G, distance between any two vertices in $EDG^{abc}(G)$ is at most four.

Proof:

Let u, v ∈ V′. Consider the following cases:

Case (i) u, v ∈ V.

If u and v are adjacent in G, then in $EDG^{abc}(G)$, d(u, v) = 1.

Suppose u and v are not adjacent in G.

a) There exists a minimal eccentric dominating set containing u and v. In $EDG^{abc}(G)$, d(u, v) = 2.

b) y = D is a γ_{ed}-set of G. Suppose the vertices w, x ∈ D are adjacent to u and v respectively, then in ED_mG^{abc} (G), d(u,v) ≤ d(u, w) + d(w,y) + d(y,x) + d(x,v) = 4.

Case (ii) u ∈ V and v ∈ S.

In this case, v ∈ S, thus v = D is a minimal eccentric dominating set of G. If u ∈D, then in $EDG^{abc}(G)$, d(u, v) = 1. If u ∉ D, then there exists a vertex w ∈ D dominates u and hence in $EDG^{abc}(G)$, d(u, v) = d(u, w)+d(w, v) = 2.

Case (iii) u, v ∈ S.

In this case, u = D_1 and v = D_2 are two minimal eccentric dominating sets of G. If D_1 and D_2 are disjoint, then every vertex z ∈ D_1 is adjacent to some vertex in x ∈ D_2 and vice versa. Then it follows that, in $EDG^{abc}(G)$, uzxv is a path. Therefore, d(u, v) ≤ 3. If there exists a minimal eccentric dominating set D_3 such that D_3 is adjacent to both D_1 and D_3. Then it follows that, in $EDG^{abc}(G)$, $d(D_1, D_2) = d(D_1, D_3)+d(D_3, D_2) = 2$.

If D_1 and D_2 have a vertex in common, then in $EDG^{abc}(G)$ $d(D_1, D_2) = 1$.

Thus, from all the three cases, distance between any two vertices in $EDG^{abc}(G)$ is at most four.

Conclusion

In this paper, we have defined and studied the new eccentric dominating graphs $ED_mG^{abc}(G)$ and $EDG^{abc}(G)$.

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