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# Fuzzy Soft Connectedness on Fuzzy Soft Topological Spaces

Thangaraj Beaula and R. Raja
Department of Mathematics, T.B.M.L. College,Porayar 609 307, Tamil Nadu, India.

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**ABSTRACT**
In this paper fuzzy soft connectedness on fuzzy soft topological spaces are defined. Some related properties regarding the newly defined concepts are proved.



## 1. Introduction

Uncertainty is an attribute of information and to solve the complicated problems in economics, engineering and environment - classical methods cannot be successfully used. A wide range of theories such as probability theory, fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, vague set theory and the interval mathematical approaches for modeling uncertainties have emerged. Each of these theories has its inherent difficulties as pointed out by Molodtsov[6]. The reason for these difficulties is possibly, the inadequacy of the parameterization tool of the theories. Molodtsov [6] initiated the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the existing theoretical approaches. This theory has proven useful in many fields such as decision making [1,2,4,9,12], data analysis [18] forecasting [15] and simulation[5]. The concept and basic properties of soft set theory were presented in [6,7]. In the classical soft set theory, a situation may be complex in the real world because of the fuzzy nature of the parameters with this point of view, the classical soft sets have been extended to fuzzy soft sets [7,11], intuitionistic fuzzy soft sets [8] vague soft sets [16], interval-valued fuzzy soft sets [17] and interval-valued intutionistic fuzzy soft sets [3].

## 2. Preliminaries

In this section some basic definitions of fuzzy soft set are presented. Throughout our discussion, U refers to an intial universe, E the set of all parameters for U and $P(\tilde{U})$ the set of all fuzzy sets of U. (U,E) means the universal set U and the parameter set E.

**Definition 2.1 [6]**

A pair $(F, E)$ is called a soft set (over $U$) if and only if $F$ is a mapping of $E$ into the set of all subsets of the set $U$.

**Definition 2.2 [7]**

A pair (F, A) is called a fuzzy soft set over U where $F : A \to P(\tilde{U})$ is a mapping from A into $P(\tilde{U})$.

**Definition 2.3 [7]**

For two fuzzy soft sets $(F, A)$ and $(G, B)$ in a fuzzy soft class $(U, E)$, we say that $(F, A)$ is a fuzzy soft subset of $(G, B)$, if

(*i*) $A \subseteq B$

(*ii*) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \tilde{\subseteq} (G, B)$.

**Definition 2.4 [7]**

Union of two fuzzy soft sets $(F, A)$ and $(G, B)$ in a soft class $(U, E)$ is a fuzzy soft set $(H,C)$ where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

**Tele:**
**E-mail address: edwinbeaula@yahoo.co.in**

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

**Definition 2.5 [7]**

Intersection of two fuzzy soft sets $(F, A)$ and $(G, B)$ in a soft class $(U, E)$ is a fuzzysoft set $(H, C)$ where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ *or* $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A) \tilde{\cap} (G, B) = (H, C)$.

**Definition 2.6 [13]**

Let $A \subseteq E$ then the mapping $F_A : E \to \tilde{P}(U)$, defined by $F_A(e) = \mu^e F_A$ ( a fuzzy subset of U), is called soft set over (U,E), where $\mu^e F_A = \tilde{0}$ if $e \in E - A$ and $\mu^e F_A \neq \tilde{0}$ if $e \in A$. The set of all fuzzy soft set over (U,E) is denoted by FS (U,E).

**Definition 2.7 [13]**

The fuzzy soft set $F_\varphi \in FS(U, E)$ is called null fuzzy soft set and it is denoted by $\tilde{\Phi}$. $F_\phi(e) = \tilde{0}$ for every $e \in E$.

**Definition 2.8 [13]**

Let $F_E \in FS(U,E)$ and $F_E(e) = \tilde{1}$ for all $e \in E$. Then $F_E$ is called absolute fuzzy soft set. It is denoted by $\tilde{E}$.

**Definition 2.9 [13]**

Let $F_A, G_B \in FS(U,E)$. If $F_A(e) \subseteq G_B(e)$ for all $e \in E$, *i.e.*, if $\mu^e F_A \subseteq \mu^e G_B$ for all $e \in E$, *i.e.*, if $\mu^e F_A(x) \leq \mu^e G_B(x)$ for all $x \in U$ and for all $e \in E$, then $F_A$ is said to be fuzzy soft subset of $G_B$, denoted by $F_A \tilde{\subseteq} G_B$.

**Definition 2.10 [13]**

Let $F_A, G_B \in FS(U,E)$. Then the union of $F_A$ and $G_B$ is also fuzzy softset $H_C$, defined by $H_C(e) = \mu^e H_C = \mu^e F_A \cup \mu^e G_B$ for all $e \in E$ where $C = A \cup B$. Here we write $H_C = F_A \tilde{\cup} G_B$.

**Definition 2.11 [13]**

Let $F_A, G_B \in FS(U,E)$. Then the intersection of $F_A$ and $G_B$ is also a fuzzy soft set $H_C$, defined by $H_C(e) = \mu^e H_C = \mu^e F_A \cap \mu^e G_B$ for all $e \in E$ where $C = A \cap B$. Here we write $H_C = F_A \tilde{\cap} G_B$.

**Definition 2.12**

Let $F_A \in FS(U,E)$. The complement of $F_A$ is denoted by $F_A^{\ C}$ and is defined By $F_A^{\ C} : E \to \tilde{P}(U)$ is a mapping given by $F_A^{\ C}(\varepsilon) = [F(\varepsilon)]^C$, $\forall \varepsilon \in E$.

**3.Fuzzy Soft Connected Spaces**

**Definition 3.1**

Let $(U, E, \Im)$ be a fuzzy soft topological space. Let $F_A, G_A$ be nonempty disjoint fuzzy soft open subsets of $U$ such that $F_A \tilde{\cup} G_A = U$ then $F_A, G_A$ is said to constitute the separation for U.

**Definition 3.2**

If there is no separation for U then it is said to be fuzzy soft connected.

**Example 3.3**

Let U = { a,b,c }, E = $\{e_1,e_2,e_3\}$ and A=$\{e_1,e_2\}$, B=$\{e_1,e_3\}$ where $A \subseteq E$, $B \subseteq E$. Define $\Im = \{\phi, \tilde{E}\}$ then $\Im$ is the fuzzy soft indiscrete topology on (U,E). Then $(U, E, \Im)$ is the fuzzy soft connected space under indiscrete topology.

**Example 3.4**

Let U = $\{h_1,h_2,h_3\}$, E=$\{e_1,e_2,e_3\}$ $\Im = \{\phi, U, F_A, F_B\}$ where $F_A$ and $F_B$ are two fuzzy soft sets over U defined as, If A=$\{e_1,e_2\}$, B= $\{e_2,e_3\}$ then

$$F_A = \begin{cases} F(e_1) = \{(h_1, 0.5), (h_2, 0.1), (h_3, 0)\} \\ F(e_2) = \{(h_1, 0.6), (h_2, 0), (h_3, 0.1)\} \end{cases}$$

$$F_B = \begin{cases} F(e_2) = \{(h_1, 0.1), (h_2, 0.2), (h_3, 0.3)\} \\ F(e_3) = \{(h_1, 0.7), (h_2, 0.8), (h_3, 0)\} \end{cases}$$

Then $(U, E, \Im)$ is a fuzzy soft topological space there exists no $F_A, F_B \in \Im - \{\tilde{\varphi}\}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = \tilde{U}$. In this case $(U, E, \Im)$ is fuzzy soft connected.

**Proposition 3.5**

Let $(U,E,\Im)$ be a fuzzy soft topological space over U. Then the following are equivalent

(i) $(U,E,\Im)$ is fuzzy soft connected

(ii) There exists no $F_A, F_B \in \Im' - \tilde{\phi}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = \tilde{U}$ where $\Im' = \{F_{A'} / F_A \in \Im\}$

(iii) There exist no $F_A, F_B \in FS(U,E) - \tilde{\varphi}$ such that $(F_A \tilde{\cap} \tilde{F}_B) \tilde{\cup} (F_A \tilde{\cap} F_B) = \tilde{\varphi}$ and $F_A \tilde{\cup} F_B = \tilde{U}$ where $FS(U,E)$ is the set of all fuzzy soft subsets of U.

(iv) There exist no $F_A \in FS(U,E) - \{\tilde{\phi}, \tilde{U}\}$ such that $F_A \in \Im \cap \Im'$.

Proof

(i) $\Rightarrow$(ii) Assume there exist $F_A, F_B \in \Im' - \tilde{\varphi}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U$.

Then for all $e \in E$, $F_A(e) \tilde{\cap} F_B(e) = \tilde{\phi}$ and $F_A(e) \tilde{\cup} F_B(e) = U$ .Thus for all $e \in E$, $F_A{}'(e) = U - F_A(e) = F_B(e)$ and $F_B{}'(e) = U - F_B(e) = F_A(e)$ which implies that $F_A{}' = F_B \in \Im' - \tilde{\phi}$ and $F_B{}' = F_A \in \Im' - \tilde{\phi}$. Then there exist $F_A, F_B \in \Im - \tilde{\phi}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U$. However $(U,E,\Im)$ is fuzzy soft connected. This is a contradiction.

(ii) $\Rightarrow$(iii) Assume there exist $F_A, F_B \in FS(U,E) - \tilde{\phi}$ such that $F_A \tilde{\cap} \bar{F}_B \tilde{\cup} \bar{F}_A \tilde{\cap} F_B = \tilde{\varphi}$ and $F_A \tilde{\cup} F_B = U$. Obviously, $F_A \tilde{\cap} F_B = \tilde{\phi}$.

Also $\bar{F}_A = \bar{F}_B \tilde{\cap} U = \bar{F}_B \tilde{\cap} F_A \tilde{\cup} F_B$

$$= \bar{F}_B \tilde{\cap} F_A \tilde{\cup} \bar{F}_B \tilde{\cap} F_B = F_B$$

which implies that $F_B$ is a fuzzy soft closed set.By using the same methods, it can be shown that $F_A$ is also a fuzzy soft closed set. Hence there exist $F_A, F_B \in \Im' - \tilde{\phi}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U$. This is a contradiction. So (iii) holds.

(iii) $\Rightarrow$(iv) Assume there exists $F_A \in \Im \tilde{\cap} \Im' - \{\tilde{\phi}, U\}$. If take $F_B = F_A{}'$ then $F_A, F_B \in \Im \tilde{\cap} \Im' - \tilde{\varphi} (\tilde{\subset} FS(U,E) - \tilde{\varphi})$. Besides , we have $F_A \tilde{\cap} \bar{F}_B \tilde{\cup} (\bar{F}_A \tilde{\cap} F_B) = F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U$. This contradiction so (iv) holds.

(iv) $\Rightarrow$(i) Assume $(U,E,\Im)$ is not fuzzy soft connected. Then there exist $F_A, F_B \in \Im - \tilde{\phi}$, such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U$. It is easy to see that $F_A{}' = F_B$ and $F_B{}' = F_A$. Thus $F_A, F_B \in \Im \tilde{\cap} \Im' - \{\tilde{\phi}, U\}$. This is a contradiction .

**Definition 3.6**

The difference $F_C$ of two sets $F_A$ and $F_B$ over U, denoted by $F_A - F_B (or \ F_A / F_B)$ is defined as $F_C = F_A - F_B$ for all $e \in E$.

**Definition 3.7**

Let $F_A$ be a fuzzy soft set over U and U' be a non-empty subset of U. Then the fuzzy soft subset of $F_A$ over U' denoted by ${}^Y F_A$ is defined as ${}^Y F_A(e) = U' \tilde{\cap} F_A(e)$ for all $e \in E$. In other words ${}^Y F_A = U' \tilde{\cap} F_A$.

**Definition 3.8**

Let $(U,E,\Im)$ be a fuzzy soft topological space over U and U' be a non-empty subset of U. Then ${}^Y\Im = \{{}^Y F_A / F_A \in \Im\}$ is said to be the fuzzy soft relative topology on U' and $(U', E, {}^Y\Im)$ is called a fuzzy soft subspace of $(U,E,\Im)$. In fact ${}^Y\Im$ is a fuzzy soft topology on U'.

**Theorem 3.9**

Let $(U', E, {}^Y\Im)$ be a fuzzy soft subspace of a fuzzy soft topological space $(U,E,\Im)$. If $(U'', E, {}^Y\Im)$ is a fuzzy soft subspace of $(U', E, {}^Y\Im)$ then $(U'', E, {}^Y\Im)$ is also a fuzzy soft subspace of $(U,E,\Im)$.

Proof

$$
\begin{aligned}
{}^Y\Im &= \{U' \tilde{\cap} F_A / F_A \in \Im\} \\
&= \{U'' \tilde{\cap} U' \tilde{\cap} (F_A / F_A \in \Im\} \\
&= \{U'' \tilde{\cap} F_A / F_A \in \Im\} \\
&= \Im
\end{aligned}
$$

So $(U'', E, {}^{Y}\Im)$ is a fuzzy soft subspace of $(U, E, \Im)$.

**Definition 3.10**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U and U' be a non-empty subset of U. If $(U', E, {}^{Y}\Im)$ is fuzzy soft connected then U' is called a fuzzy soft connected subset of U.

**Definition 3.11**

Let $(U', E, {}^{Y}\Im)$ be a fuzzy soft subspace of a fuzzy soft topological space $(U, E, \Im)$. For a fuzzy soft set $F_A \in FS(U,E)$, $\overline{F}_A$ and ${}^{Y}\overline{F}_A$ will denoted the fuzzy soft closures of $F_A$ in $(U, E, \Im)$ and $(U', E, {}^{Y}\Im)$ respectively.

**Theorem 3.12**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U. If U' is a fuzzy soft connected sub set of U, then there exist no $F_A, F_B \in \Im - \tilde{\varphi}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U'$.

Proof

If there exist $F_A, F_B \in \Im - \tilde{\varphi}$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U'$ then $F_A = U' \tilde{\cap} F_A \in {}^{Y}\Im - \tilde{\phi}$ and $F_B = U' \tilde{\cap} F_B \in {}^{Y}\Im - \tilde{\phi}$. However , U' is a fuzzy soft connected subset of U. This is a contradiction.

**Corollary 3.13**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U and U' be a fuzzy soft connected subset of U. If there exist $F_A, F_B \in \Im$ such that $F_A \tilde{\cap} F_B = \tilde{\phi}$ and $U' \tilde{\subseteq} F_A \tilde{\cup} F_B$ then $U' \tilde{\subseteq} F_A$ or $U' \tilde{\subseteq} F_B$.

**Lemma 3.14**

Let $(U', E, {}^{Y}\Im)$ be a fuzzy soft subspace of a fuzzy soft topological space $(U, E, \Im)$ and $F_A$ be a fuzzy soft set over U, then $F_A$ is fuzzy soft closed in U' if and only if $F_A = U' \tilde{\cap} F_B$ for some fuzzy soft closed set $F_B$ in U.

**Definition 3.15**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U and $F_A$ be a fuzzy soft set over U. Then the fuzzy soft closure of $F_A$, denoted by $\overline{F}_A$ is the intersection of all fuzzy soft closed super sets of $F_A$.

$\overline{F}_A$ is the smallest fuzzy soft closed set over U which contains $F_A$.

**Proposition 3.16**

Let $F_A, F_B$ and $F_C$ be three fuzzy soft sets over a common universe U. Then

i) $F_A \tilde{\cup} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cap} (F_A \tilde{\cup} F_C)$

ii) $(F_A \tilde{\cap} F_B) \tilde{\cup} F_C = (F_A \tilde{\cup} F_C) \tilde{\cap} (F_B \tilde{\cup} F_C)$

iii) $F_A \tilde{\cap} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\cap} F_C)$

iv) $(F_A \tilde{\cup} F_B) \tilde{\cap} F_C = (F_A \tilde{\cap} F_C) \tilde{\cup} (F_B \tilde{\cap} F_C)$

v) $F_A \tilde{\cap} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cap} F_C$

$F_A \tilde{\cup} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cup} F_C$

**Proposition 3.17**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U and $F_A, F_B$ be two fuzzy soft sets over U. Then

i) $\overline{\phi} = \phi$, $\overline{\tilde{U}}$ = U

ii) $F_A \tilde{\subseteq} \overline{F}_A$

iii) $F_A$ is a fuzzy soft closed set if and only if $F_A = \overline{F}_A$

iv) $\overline{\overline{F}}_A = \overline{F}_A$

v) $F_A \tilde{\subseteq} F_B$ implies $\overline{F}_A \tilde{\subseteq} \overline{F}_B$

**Proposition 3.18**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U and U' be a non-empty subset of U. Then U' is a fuzzy soft connected subset of U if and only if there exist no $F_A, F_B \in FS(U,E) - \tilde{\phi}$ such that $F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U'$.

**Proof**

For all $F_A, F_B \in FS(U', E) - \tilde{\phi}$ by definition 3.14 , proposition 3.15 and lemma 3.13 we have

$F_A \tilde{\cap} {}^{Y}\overline{F}_B = F_A \tilde{\cap} (\tilde{\cap}\{F_C / F_C \in ({}^{Y}\Im)', F_C \tilde{\supseteq} F_B\})$

$= F_A \tilde{\cap} (\tilde{\cap}\{U' \tilde{\cap} R_E / R_E \in \Im', R_E \tilde{\supset} F_B\})$
$= (F_A \tilde{\cap} U') \tilde{\cap} (\tilde{\cap}\{R_E / R_E \in \Im', R_E \tilde{\supset} F_B\})$
$= (F_A \tilde{\cap} (\tilde{\cap}\{R_E / R_E \in \Im', R_E \tilde{\supset} F_B\})$
$= F_A \tilde{\cap} \overline{F}_B$

Similarly we can show that ${}^{Y}\overline{F}_A \tilde{\cap} F_B = \overline{F}_A \tilde{\cap} F_B$ by proposition 3.5, $(U', E, {}^{Y}\Im)$ is fuzzy soft connected if and only if there exists no $F_A, F_B \in FS(U', E) - \tilde{\phi}$ such that $F_A \tilde{\cap} {}^{Y}\overline{F}_B \tilde{\cup} {}^{Y}\overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U'$. Then $(U', E, {}^{Y}\Im)$ is fuzzy soft connected if and only if there exist no $F_A, F_B \in FS(U', E) - \tilde{\phi}$ such that $F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U'$ ie U' is a fuzzy soft connected subset of U if and only if there exist no $F_A, F_B \in FS(U, E) - \tilde{\phi}$ such that $F_A \tilde{\cap} \overline{F}_B \cup \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U'$.

**Proposition 3.19**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U and U' be fuzzy soft connected subset of U. If there exist $F_A, F_B \in FS(U', E)$ such that $F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $U' \tilde{\subset} F_A \tilde{\cup} F_B$ then $U' \tilde{\subset} F_A$ or $U' \tilde{\subset} F_B$.

**Proof**

If there exist $F_A, F_B \in FS(U', E)$ such that $F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $U' \tilde{\subset} F_A \cup F_B$ then by proposition 3.15 and 3.16 we have

$((U' \cap F_A) \cap (U' \cap F_A) \cup ((U' \cap F_A) \cap (U' \cap F_B))$
$\tilde{\subset} ((U' \tilde{\cap} F_A) \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} U' \tilde{\cap} F_B))$
$= U' \tilde{\cap} F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B$
$= U' \tilde{\cap} \tilde{\phi}$
$= \tilde{\phi}$

Besides, we have $U' \tilde{\cap} F_A, U' \tilde{\cap} F_B \in FS(U', E)$, and $(U' \tilde{\cap} F_A) \tilde{\cup} (U' \tilde{\cap} F_B) = U' \tilde{\cap} (F_A \tilde{\cup} F_B) = U'$. Since $U'$ is a fuzzy soft connected subset of U, $U' \tilde{\cap} F_A = \tilde{\phi}$ or $U' \tilde{\cap} F_B = \tilde{\phi}$ by proposition3.18. If $U' \tilde{\cap} F_A = \tilde{\phi}$ then by $U' \tilde{\cap} F_A \tilde{\cup} U' \tilde{\cap} F_B = U'$, we have $U' \tilde{\subset} F_B$. Similarly, if $U' \tilde{\cap} F_B = \tilde{\phi}$ then $U' \tilde{\subset} F_A$.

**Proposition 3.20**

Let $(U, E, \Im)$ be a fuzzy soft topological space over U , U' be a fuzzy soft connected subset of U and $U''$ be a non-empty subset of U. If $U' \tilde{\subset} U'' \tilde{\subset} U'$ then $U''$ is also a fuzzy soft connected subset of U.

**Proof**

Assume that $U''$ is not a fuzzy soft connected subset of U. By proposition 3.18, there exist $F_A, F_B \in FS(U'', E) - \tilde{\phi}$ $(\tilde{\subset} FS(U', E))$ such that $F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = U''$. Then $U' \tilde{\subset} F_A \tilde{\cup} F_B$. Since $U'$ is a fuzzy soft connected subset of U, we have $U' \tilde{\subset} F_A$ or $U' \tilde{\subset} F_B$ by proposition 3.19.If $U' \tilde{\subset} F_A$, then $U'' \tilde{\cap} \overline{F}_B \tilde{\subset} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ for $U''_B \tilde{\subset} U' \tilde{\subset} \overline{F}_A$. Thus $F_B = U'' \tilde{\cap} F_B = \phi$. This is a contradiction. Similarly if $U' \tilde{\subset} F_B$ then $F_A = \tilde{\phi}$ . This is also a contradiction.

**Proposition 3.21**

Let f be a fuzzy soft continuous mapping from fuzzy soft topological space $(U, E, \Im_1)$ to fuzzy soft topological space $(U', E, \Im_2)$. If $(U, E, \Im_1)$ is fuzzy soft connected and $f(U) \neq \tilde{\phi}$ then $f(U)$ is fuzzy soft connected subset of U'.

**Proof**

Assume $f(U)$ is not a fuzzy soft connected subset of U'. By proposition 3.18, there exist $F_A, F_B \in FS(U, E) - \tilde{\phi}$ $(\tilde{\subset} FS(U', E) - \tilde{\varphi})$ such that $F_A \tilde{\cap} \overline{F}_B \tilde{\cup} \overline{F}_A \tilde{\cap} F_B = \tilde{\phi}$ and $F_A \tilde{\cup} F_B = f(U)$. Then $f^{\leftarrow}(F_A)$, $f^{\leftarrow}(F_B) \in FS(U, E) - \tilde{\phi}$ and by proposition 3.26 and 3.24,

$f^{\leftarrow}(F_A) \tilde{\cap} \overline{f^{\leftarrow}(F_B)} \tilde{\cup} \overline{f^{\leftarrow}(F_A)} \tilde{\cap} f^{\leftarrow}(F_B)$
$\tilde{\subset} f^{\leftarrow}(F_A) \tilde{\cap} f^{\leftarrow}(\overline{F}_B) \tilde{\cup} f^{\leftarrow}(\overline{F}_A) \tilde{\cap} f^{\leftarrow}(F_B)$
$= f^{\leftarrow}(F_A) \tilde{\cap} (\overline{F}_B) \tilde{\cup} (\overline{F}_A) \tilde{\cap} (F_B)$
$= f^{\leftarrow}(\tilde{\phi})$

$=\tilde{\phi}$

Besides by proposition 3.24 and 3.25, we have

$$f^{\leftarrow}(F_A)\tilde{\cup} f^{\leftarrow}(F_B) = f^{\leftarrow}(F_A)\tilde{\cup}(F_B)$$
$$= f^{\leftarrow}\tilde{f}(U)$$
$$= f^{\leftarrow} f^{\rightarrow}(U)$$
$$=U$$

It follows that $(U,E,\Im_1)$ is not fuzzy soft connected. This is a contradiction. So $f(U)$ is a fuzzy soft connected subset of U'.

Definition 3.22

Let f be a mapping from U to U',

1) The fuzzy soft set mapping induced by f, denoted by the notation $f^{\rightarrow}$, is a mapping from FS(U,E) to FS(U',E') that maps $F_A$ to $f^{\rightarrow}(F_A)=(f^{\rightarrow}F_A)$, where $f^{\rightarrow}(F_A)$ is defined by $f^{\rightarrow}(F_A)(e)=\{f(x)\mid x\in F_A(e)\}$ for all $e\in E$.

2) The inverse fuzzy soft set mapping induced by f, denoted by the notation $f^{\leftarrow}$, is a mapping from FS(U',E') to FS(U,E) that maps $F_B$ to $f^{\leftarrow}(F_B)=(f^{\leftarrow}F_B)$, where $f^{\leftarrow}(F_B)$ is defined by $f^{\leftarrow}(F_B)(e)=\{x\mid f(x)\in F_B(e)\}$ for all $e\in E$.

**Example 3.23**

Let $U=\{h_1,h_2,h_3\}$ $U'=\{p_1,p_2\}$ and $E=\{e_1,e_2\}$. The mapping f is given by $f(h_1)=p_1$, $f(h_2)=p_1$, $f(h_3)=p_2$.

1) If $F_A\in FS(U,E)$ is defined by $\{F_A(e_1)=\{h_1,h_2\},F_A(e_2)=\{h_1,h_3\}\}$ then $f^{\rightarrow}(F_A)=(f^{\rightarrow}F_A)=\{f^{\rightarrow}(F_A)(e_1)=\{p_1\},f^{\rightarrow}(F_A)(e_2)=u'\}\in FS(U',E')$.

2) If $F_B\in FS(U',E')$ is defined by $\{F_B(e_1)=\{p_2\},F_B(e_2)=\{p_1\}\}$ then $f^{\leftarrow}(F_B)=(f^{\leftarrow}F_B)=\{f^{\leftarrow}(F_B)(e_1)=\{h_3\},f^{\leftarrow}(F_B)(e_2)=\{h_1,h_2\}\}\in FS(U,E)$.

**Proposition 3.24**

Let f be a mapping from U to U' and $F_{A_1},F_{A2},\in FS(U',E')$. Then

1) $f^{\leftarrow}(\phi)=\phi$, $f^{\leftarrow}(U')=\tilde{U}$

2) $F_{A_1}\tilde{\subset}F_{A_2}\Rightarrow f^{\leftarrow}(F_{A_1})\tilde{\subset}(f^{\leftarrow}F_{A_2})$

3) $f^{\leftarrow}(F_{A_1})\tilde{\cup}(F_{A_2})=f^{\leftarrow}(F_{A_1})\tilde{\cup}(f^{\leftarrow}F_{A_2})$

4) $f^{\leftarrow}(F_{A_1})\tilde{\cap}(F_{A_2})=f^{\leftarrow}(F_{A_1})\tilde{\cap}(f^{\leftarrow}F_{A_2})$

5) $f^{\leftarrow}(F_{A_1})'=(f^{\leftarrow}(F_{A_2}))'$

**Proposition 3.25**

Let f be a mapping from U to U' and $F_A\in FS(U,E)$, $F_B\in FS(U',E')$. Then

1) $f^{\leftarrow}(f^{\rightarrow}(F_A)\tilde{\supset}F_A$. If f is one-one, then $f^{\leftarrow}(f^{\rightarrow}(F_A)=F_A$

2) $f^{\rightarrow}(f^{\leftarrow}(F_B)\tilde{\subset}F_B$. If f is surjective, then $f^{\rightarrow}(f^{\leftarrow}(F_B)=F_B$

**Proof**

1)Let $f^{\rightarrow}(F_A)=F_B$. Then for all $e\in E$, $f^{\leftarrow}(F_B(e))=\{x\mid f(x)\in F_B(e)\}$ $=\{x\mid f(x)\in f(t)\mid t\in F_A(e)\}\}\tilde{\supset}F_A(e)$ which implies that $f^{\leftarrow}(f^{\rightarrow}(F_A)\tilde{\supset}F_A$ If f is one-one, notice that $\{x\mid f(x)\in\{f(t)\mid t\in F_A(e)\}\}=F_A(e)$, thus $f^{\leftarrow}(f^{\rightarrow}(F_A)=F_A$.

2)Let $f^{\rightarrow}(F_B)=F_A$. Then for all $e\in E$, $f^{\rightarrow}(F_A(e))=\{f(x)\mid x\in F_A(e)\}$ $=\{f(x)\mid x\in t\mid f(t)\in F_B(e)\}\}\tilde{\subset}F_B(e)$ which implies that $f^{\rightarrow}(f^{\leftarrow}(F_B)\tilde{\subset}F_B$ If f is surjective, notice that $\{f(x)\mid x\in\{t\mid f(t)\in F_B(e)\}\}=F_B(e)$, thus $f^{\rightarrow}(f^{\leftarrow}(F_B)=F_B$.

**Proposition 3.26**

Let $(U,E,\Im_1)$ (resp., $(U',E,\Im_2)$ ) be a fuzzy soft topological space over U(resp.,(U') and f be a mapping from U to U'. The following condition are equivalent :

1)F is a fuzzy soft continuous mapping from $(U,E,\Im_1)$ to $(U',E,\Im_2)$

2)For each fuzzy soft closed set $F_B$ in U', $f^{\leftarrow}(F_B)$ is a fuzzy soft closed set in U.

3)For every fuzzy soft set $F_A$ over U $f^{\rightarrow}(\overline{F}_A) \tilde{\subseteq} \overline{f^{\rightarrow}(F_A)}$

4)For every fuzzy soft set $F_B$ over U' $f^{\rightarrow}(\overline{F}_B) \tilde{\supseteq} \overline{f^{\leftarrow}(F_B)}$

**Proof**

(1) $\Rightarrow$ (2) Let $F_B$ be a fuzzy soft closed set in U'. Then ( $F_B$ )' be a fuzzy soft open set in U'. By (1) and proposition 3.24, $f^{\leftarrow}(F_{B_1})' = (f^{\leftarrow}(F_{B_1}))'$ is a fuzzy soft open set in U. Hence $f^{\leftarrow}(F_B)$ is a fuzzy soft closed set in U.

(2) $\Rightarrow$ (3) Let $F_A$ be fuzzy soft set over U. By proposition 2.17 $f^{\rightarrow}(F_A) \tilde{\subseteq} \overline{f^{\rightarrow}(F_A)}$. Then by proposition 3.24 and 3.25 , $F_A \tilde{\subseteq} f^{\leftarrow}(f^{\rightarrow}(F_A)) \tilde{\subseteq} f^{\leftarrow}(\overline{f^{\rightarrow}(F_A)})$ since $\overline{f^{\rightarrow}(F_A)}$ is a fuzzy soft closed set in U', then by (2) $f^{\leftarrow}(\overline{f^{\rightarrow}(F_A)})$ is a fuzzy soft closed set in U. Thus $F_A \tilde{\subseteq} f^{\leftarrow}(\overline{f^{\rightarrow}(F_A)})$ also by proposition 3.24 and 3.25 $f^{\rightarrow}(\overline{F}_A) \tilde{\subseteq} f^{\rightarrow}(\overline{f^{\leftarrow}(F_A)}) \tilde{\subseteq} \overline{f^{\rightarrow}(F_A)})$. So $f^{\rightarrow}(\overline{F}_A) \tilde{\subseteq} \overline{f^{\rightarrow}(F_A)}$.

(3) $\Rightarrow$ (4) Let $F_B$ be a fuzzy soft closed set in U'. By (3) proposition 3.25 and 3.17 $f^{\rightarrow}(\overline{f^{\leftarrow}(F_B)} \tilde{\subseteq} \overline{f^{\rightarrow}(f^{\leftarrow}(F_B))} \tilde{\subseteq} \overline{F}_B$. Then by proposition 3.24 and 3.25 $f^{\leftarrow}(F_B) \tilde{\supseteq} f^{\leftarrow}(f^{\rightarrow}(\overline{f^{\rightarrow}(F_B)})) \tilde{\supseteq} \overline{(f^{\leftarrow}(F_B))}$.

(4) $\Rightarrow$(1) If $F_B$ is a fuzzy soft open set in U', then ( $F_B$ )' is a fuzzy soft closed set in U'. By (4) and proposition 3.17 $\overline{f^{\leftarrow}(F_B)'} \tilde{\subseteq} f^{\leftarrow}(F_B)'$ obviously, $\overline{f^{\leftarrow}(F_B)'} \tilde{\supseteq} f^{\leftarrow}(F_B)'$. Thus $\overline{f^{\leftarrow}(F_B)'} = f^{\leftarrow}(F_B)'$ which implies that $f^{\leftarrow}(F_B)' = (f^{\leftarrow}(F_B))'$ by proposition 3.24 is a fuzzy soft closed set in U. Therefore , $f^{\leftarrow}(F_B)$ is a fuzzy soft open set in U. So f is a fuzzy soft continuous mapping from $(U, E, \Im_1)$ to $(U', E, \Im_2)$.

**Proposition 3.27**

Let f be a mapping from U to U', $F_{A_1}, F_{A_2} \in FS(U, E)$. Then

1) $f^{\rightarrow}(\phi) = \phi$

2) $F_{A_1} \tilde{\subseteq} F_{A_2} \Rightarrow f^{\rightarrow}(F_{A_1}) \tilde{\subseteq} f^{\rightarrow}(F_{A_2})$

3) $f^{\rightarrow}(F_{A_1}) \tilde{\cup} (F_{A_2})) = f^{\rightarrow}(F_{A_1}) \tilde{\cup} f^{\rightarrow}(F_{A_2})$

4) $f^{\rightarrow}(F_{A_1}) \tilde{\cap} (F_{A_2})) = f^{\rightarrow}(F_{A_1}) \tilde{\cap} f^{\rightarrow}(F_{A_2})$.

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