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An Interval Parametric Technique for Solving Fuzzy Matrix Games

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ABSTRACT
The aim of this paper is to develop a methodology for solving matrix games with fuzzy payoffs. Here, a fuzzy matrix game has been considered and its solution method has been proposed using interval parametric technique. This technique is based on parametric representation of interval number. In this technique, the fuzzy number has been converted into interval number using interval approximation of fuzzy number and the interval number has been presented to its parametric interval functional form. Then the corresponding matrix game has been converted into crisp game using the said technique. The value of the matrix game for each player is obtained by solving corresponding crisp game problems using the existing method. Finally, a numerical example has been considered and solved in support of the solution method.

1. Introduction

Many real world decision making problems with competitive situation, it is often required to take the decision where there are two or more opposite parties with conflicting interests and the action of one depends upon the action which is taken by the opponent. A variety of competitive situation is seen in real life viz., in political campaign, elections, advertisement, etc. Game theory is a mathematical way out for finding of conflicting interests with competitive situations, which includes players or decision makers (DM) who select several strategies from the set of admissible strategies. During the past, several researchers formulated and solved matrix game considering crisp/precise payoff. This means that every probable situation to select the payoff involved in the matrix game is perfectly determinable. In this case, it is usually assumed that there exists some complete information about the payoff matrix. But in some realistic situations, it is observed that the players of games may be not able to evaluate exactly the outcomes of games due to uncertain and lack of information between games' players. As a result, payoffs of some games are not measured by precise/fixed valued numbers. In this situation, the problem can be formulated considering vague sense and, payoffs are treated as imprecise/uncertain in nature. In such cases fuzzy mathematics [8] is a vital tool to handle such situation. In this paper, we have treated imprecise parameters considering fuzzy sets/fuzzy numbers. Therefore, the fuzzy game theory provides an efficient framework which solves real-life conflict problems with fuzzy valued/imprecise valued information. In the last few years, several attempts have been made in the existing literature for solving game problem with fuzzy payoff [1-10, 15, 19]. Fuzziness in game problem has been well discussed by Campos [10]. Compos introduced fuzzy linear programming model to solve fuzzy matrix game. Sakawa and Nishizaki [13] solved multi-objective fuzzy games by introducing Max-min solution procedure. Based on fuzzy duality theory [3], Bector et al. [2,3,4], and Vijay et al. [20] proved that a two person zero-sum matrix game with fuzzy goals and fuzzy payoffs is equivalent to a pair of linear programming problems. Nayak and Pal [16,17] well studied the interval and fuzzy matrix games. Çevikel and Ahlatçıoglu [1] described new concepts of solutions for multi-objective two person zero-sum games with fuzzy goals and fuzzy payoffs using linear membership functions. Li and Hong [6] gave an approach for solving constrained matrix games with payoffs of triangular fuzzy numbers. Bandyopadhyay et al. [18] well studied a matrix game with payoff as triangular intuitionistic fuzzy number. Mijanur et al. [12] introduced an alternative approach for solving fuzzy matrix games. Very recently, Sahoo [11] discussed effect of defuzzification methods in solving fuzzy matrix games. In this paper, two person matrix games have taken into consideration. The element of payoff matrix is considered to be fuzzy number [8]. Then the corresponding problem has been converted into crisp equivalent two person matrix game using parametric interval functional form. The value of the matrix game for each player is obtained by solving corresponding crisp game problems using the existing method. Finally, to illustrate the methodology, a numerical example has been solved and the computed results have been presented. The rest of the paper is organized as follows. Section 2, deals with preliminary definitions about fuzzy numbers. In Sec. 2 presents the background about of Fuzzy set and fuzzy Numbers. Parametric form of interval number is defined in Sec. 3. Mathematical model of matrix game is described in Sec. 4. Solution of matrix game is discussed in Sec. 5. Numerical example and Computational results are reported in Sec. 6 and a conclusion has been drawn in last section.

2. Background

2.1. Preliminary definitions about fuzzy number

Let X be a non-empty set. A fuzzy set $\tilde{A}$ is defined as the set of pairs $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$, where $\mu_{\tilde{A}}:X\rightarrow[0,1]$ is a

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mapping and $\mu_{\tilde{A}}(x)$ is called the membership function of $\tilde{A}$ or grade of membership of x in $\tilde{A}$.

Definition 1 (*α -level set or α -cut)*

The α -cut of a fuzzy set $\tilde{A}$ is a crisp subset of X and is denoted by $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$, where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$ and $\alpha \in [0,1]$. The lower and upper points of $\tilde{A}_\alpha$, are represented by $A_L(\alpha) = \inf \tilde{A}_\alpha$ and $A_U(\alpha) = \sup \tilde{A}_\alpha$.

Definition 2 ***(Strong α -level set or strong α -cut)***

If $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}$, it is called strong α -level set or strong α -cut.

Definition 3 ***(Normal fuzzy set)***

A fuzzy set $\tilde{A}$ is called a normal fuzzy set if there exists at least one $x \in X$ such that $\sup_x \mu_{\tilde{A}}(x) = 1$.

Definition 4 ***(Convex fuzzy set)***

A fuzzy set $\tilde{A}$ is called convex iff for every pair of $x_1, x_2 \in X$, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\lambda \in [0,1]$. Alternatively, a fuzzy set is convex if all α -level sets are convex.

Definition 5 ***(Fuzzy number)***

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) There exists at least one $x_0 \in R$ for which $\mu_{\tilde{A}}(x_0) = 1$.

(ii) $\mu_{\tilde{A}}(x)$ is pair wise continuous.

(iii) $\tilde{A}$ Must be convex and normal.

Definition 6 ***(Triangular fuzzy number)***

The triangular fuzzy number (TFN) is a normal fuzzy number denoted as $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \leq a_2 \leq a_3$ and its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 7 ***(Parabolic fuzzy number)***

The parabolic fuzzy number (PFN) is a normal fuzzy number denoted as $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \leq a_2 \leq a_3$ and its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \left(\dfrac{a_2 - x}{a_2 - a_1}\right)^2 & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ 1 - \left(\dfrac{x - a_2}{a_3 - a_2}\right)^2 & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 8 ***(The Nearest interval Approximation of a fuzzy Number)***

If $\tilde{A}$ be a fuzzy number with interval of confidence at the level α is $[A_L(\alpha), A_U(\alpha)]$ then according to Grzegorzewski [14], the interval approximation of fuzzy number $\tilde{A}$ is denoted as $[a_L, a_R]$ and is defined by

$$[a_L, a_R] = \left[\int_0^1 A_L(\alpha)d\alpha, \int_0^1 A_U(\alpha)d\alpha\right].$$

Definition 9 ***(The nearest interval approximation of triangular fuzzy number)***

Let $\tilde{A} = (a_1, a_2, a_3)$ is a triangular fuzzy number. The α -level interval of $\tilde{A}$ is $[A_L(\alpha), A_R(\alpha)]$ where $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $A_R(\alpha) = a_3 - (a_3 - a_2)\alpha$ .By the nearest interval approximation method, the interval number considering $\tilde{A} = (a_1, a_2, a_3)$ as a TFN is

$$[a_L, a_R] = \left[\int_0^1 [a_1 + (a_2 - a_1)\alpha]d\alpha, \int_0^1 [a_3 - (a_3 - a_2)\alpha]d\alpha\right]_0^1 = \left[\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_2 + a_3)\right]$$

Definition 10 ***(The nearest interval approximation of parabolic fuzzy number)***

Let $\tilde{A} = (a_1, a_2, a_3)$ be a parabolic fuzzy number. The α -level interval of $\tilde{A}$ is $[A_L(\alpha), A_R(\alpha)]$ where

$A_L(\alpha)=a_2-(a_2-a_1)\sqrt{1-\alpha}$ and $A_R(\alpha)=a_2+(a_3-a_2)\sqrt{1-\alpha}$. By the nearest interval approximation method, the interval number considering $\tilde{A}=(a_1,a_2,a_3)$ as a PFN is

$$[a_L,a_R]=\left[\int_0^1[a_2-(a_2-a_1)\sqrt{1-\alpha}]d\alpha,\int_0^1[a_2+(a_3-a_2)\sqrt{1-\alpha}]d\alpha\right]=\left[\frac{1}{3}(2a_1+a_2),\frac{1}{3}(a_2+2a_3)\right]$$

3. Parametric representation of interval number

Let $[a_L,a_R]$ be an interval number then the parametric representation of interval number is denoted by $h(p)$ and is defined as follows:

$$h(p)=\begin{cases}(a_L)^{1-p}(a_R)^p & \text{if } a_L>0\\ -(|a_L|)^{1-p}(|a_R|)^p & \text{if } a_R<0\end{cases},\ p\in[0,1]$$

If $0\in[a_L,a_R]$ i.e. if $a_L<0$ and $a_R>0$

Then

$$h(p)=\begin{cases}-(|a_L|)^{1-p}(|\varepsilon|)^p,\varepsilon\to 0^- & \text{if } (a_R-|a_L|)<0\\ (\varepsilon)^{1-p}(a_R)^p,\varepsilon\to 0^+ & \text{if } (a_R-|a_L|)>0\end{cases},\ p\in[0,1]$$

If $a_L=0$ and $a_R>0$ then $h(p)=(\varepsilon)^{1-p}(a_R)^p,\varepsilon\to 0^+$

If $a_R=0$ and $a_L<0$ then $h(p)=-(|a_L|)^{1-p}(|\varepsilon|)^p,\varepsilon\to 0^-$

The choice of ε depends on decision maker's (DM). Generally ε is very small positive quantity with value 0.001 or less and ε depends on the choice of the decision maker's (DM) as per his/her desire of accuracy.

3.1. Algorithm for finding crisp number from fuzzy number

Step-1: Evaluate an interval number $[a_L,a_R]$ using nearest interval approximation of fuzzy number.

Step-2: Represent $[a_L,a_R]$ to its parametric functional form $h(p)$.

Step-3: For different values of $p\in[0,1]$, $h(p)$ is a crisp number and $h(p)\in[a_L,a_R]$

Step-4: End

4. Mathematical Model of a Matrix Game

Let $A_i\in\{A_1,A_2,...,A_m\}$ be a pure strategy available for player A and $B_j\in\{B_1,B_2,...,B_n\}$ be a pure strategy available for player B. When player A chooses a pure strategy A_i and the player B chooses a pure strategy B_j, then g_{ij} is the payoff for player A and $-g_{ij}$ be a payoff for player B. The two-person zero-sum matrix game G can be represented as a pay-off matrix $G=(g_{ij})_{m\times n}$.

4.1. Fuzzy Payoff matrix

Let $\tilde{g}_{ij}$ be the fuzzy payoff which is the gain of player A from player B if player A chooses strategy A_i whereas player B chooses B_j. Then the fuzzy payoff matrix of player A and B is $\tilde{G}=(\tilde{g}_{ij})_{m\times n}$.

Where $A_i\in\{A_1,A_2,...,A_m\}$, $B_j\in\{B_1,B_2,...,B_n\}$ and it is assumed that each player has his/her choices from amongst the pure strategies.

4.2. Interval Payoff matrix

Let $[g_{ijL},g_{ijR}]$ be the fuzzy payoff which is the gain of player A from player B if player A chooses strategy A_i whereas player B chooses B_j. Then the interval payoff matrix of player A and B is $[G_L,G_R]=([g_{ijL},g_{ijR}])_{m\times n}$.

Where $A_i\in\{A_1,A_2,...,A_m\}$, $B_j\in\{B_1,B_2,...,B_n\}$ and it is assumed that each player has his/her choices from amongst the pure strategies.

4.3. Interval parametric form of Payoff matrix

Let $[g_{ijL},g_{ijR}]$ be the fuzzy payoff which is the gain of player A from player B if player A chooses strategy A_i where as player B chooses B_j. Then the interval parametric form of payoff matrix of player A and B is

$$\left((g_{ijL})^{1-p}(g_{ijR})^p\right)_{m\times n}.$$

Where $A_i\in\{A_1,A_2,...,A_m\}$, $B_j\in\{B_1,B_2,...,B_n\}$ and it is assumed that each player has his/her choices from amongst the pure strategies.

4.4 Mixed strategy

Let us consider the fuzzy matrix game whose payoff matrix is $\left(\left(g_{ijL}\right)^{1-p}\left(g_{ijR}\right)^{p}\right)_{m\times n}$. The mixed strategy for the player-A, is denoted by $x=(x_1,\cdots,x_m)'$, where $x_i \geq 0$, $i=1,2,...,m$ and $\sum_{i=1}^{m} x_i = 1$.

Similarly, a mixed strategy for the player-B is denoted by $y=(y_1,y_2,\cdots,y_n)'$ where $y_j \geq 0,\ j=1,2,\cdots,n$ and $\sum_{j=1}^{n} y_j = 1$.

Definition 4.1

A pair (x,y) of mixed strategies for the players in a matrix game is called a situation in mixed strategies. In a situation (x,y) of mixed strategies each usual situation (*i*, *j*) in pure strategies becomes a random event occurring with probabilities x_i and y_j. Since in the situation (*i*, *j*), player-A receives a payoff $(\tilde{g}_{ij})$, the mathematical expectation of his payoff under (x,y) is equal to

$$E(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left(g_{ijL}\right)^{1-p}\left(g_{ijR}\right)^{p}\right)x_i\,y_j$$

Theorem 4.2

Let $E(x,y)$ be such that both $\min_y \max_x E(x,y)$ and $\max_x \min_y E(x,y)$ exist,

Then

$$\min_y \max_x E(x,y) \geq \max_x \min_y E(x,y)$$

4.5. Saddle point of a function

Let $E(x,y)$ be a function of two variables (vectors) χ and y. The point $(x_\circ, y_\circ)$ is said to be the saddle point of the function $E(x,y)$ if $E(x,y_\circ) \leq E(x_\circ,y_\circ) \leq E(x_\circ,y)$

Theorem 4.3

Let $E(x,y)$ be a function of two variables such that $\max_x \min_y E(x,y)$ and $\min_y \max_x E(x,y)$ exist. Then the necessary and sufficient condition for the existence of a saddle point (x_0,y_0) of $E(x,y)$ is that

$$E(x_\circ,y_\circ)=\max_y \min_x E(x,y)=\min_y \max_x E(x,y)$$

4.6. Value of a Matrix Game

The common value of $\max_x\left\{\min_y E(x,y)\right\}$ and $\min_y\left\{\max_x E(x,y)\right\}$ is called the value of the matrix game with payoff matrix $G=\left(\left(g_{ijL}\right)^{1-p}\left(g_{ijR}\right)^{p}\right)$ and denoted by $v(G)$ or simply v.

5 Solution of Matrix Game

Let us consider a 2 $\times$ 2 Matrix game whose payoff matrix is given by

$$G=\begin{bmatrix}\left(g_{11L}\right)^{1-p}\left(g_{11R}\right)^{p} & \left(g_{12L}\right)^{1-p}\left(g_{12R}\right)^{p}\\ \left(g_{21L}\right)^{1-p}\left(g_{21R}\right)^{p} & \left(g_{22L}\right)^{1-p}\left(g_{22R}\right)^{p}\end{bmatrix}$$

If G has a saddle point, solution is obvious.

Let G have no saddle point. Let the player-A has the strategy $X=(x_1,x_2)'\equiv(x,1-x)(0\leq x\leq 1)$ and the player-B has the strategy $Y=(y,1-y)'(0\leq y\leq 1)$.

Then

$$E(x,y)=\sum_{i=1}^{2}\sum_{j=1}^{2}\left(\left(g_{ijL}\right)^{1-p}\left(g_{ijR}\right)^{p}\right)x_i\,y_j$$

If $X^*=\left(x^*,1-x^*\right)',\ Y^*=\left(y^*,1-y^*\right)$ be optimal strategies, then from

$$E\left(X,Y^*\right)\leq E\left(X^*,Y^*\right)\leq E\left(X^*,Y\right)$$

we have

$$E\left(x,y^*\right)\leq E\left(x^*,y^*\right)\leq E\left(x^*,y\right)\ \ \forall x\in(0,1),\ \ y\in(0,1).$$

From the first part of the inequality, we set that $E\left(X,Y^*\right)$ regarded as a function of x has a maximum at χ^* thus,

$$\left.\frac{\partial E}{\partial x}\right|_{\left(x^*,y^*\right)}=0\Rightarrow y^*=\frac{\left(g_{22L}\right)^{1-p}\left(g_{22R}\right)^{p}-\left(g_{12L}\right)^{1-p}\left(g_{12R}\right)^{p}}{\left(\left(g_{11L}\right)^{1-p}\left(g_{11R}\right)^{p}+\left(g_{22L}\right)^{1-p}\left(g_{22R}\right)^{p}\right)-\left(\left(g_{12L}\right)^{1-p}\left(g_{12R}\right)^{p}+\left(g_{21L}\right)^{1-p}\left(g_{21R}\right)^{p}\right)}.$$

Provided that $\left(\left(g_{11L}\right)^{1-p}\left(g_{11R}\right)^{p}+\left(g_{22L}\right)^{1-p}\left(g_{22R}\right)^{p}\right)-\left(\left(g_{12L}\right)^{1-p}\left(g_{12R}\right)^{p}+\left(g_{21L}\right)^{1-p}\left(g_{21R}\right)^{p}\right)\neq 0$

Similarly, from the second part of the inequality, it is seen that $E(x^*,y)$ regard as a function of y has a minimum at y^* i.e.,

$$\left.\frac{\partial E}{\partial y}\right|_{(x^*,y^*)}=0\Rightarrow x^*=\frac{(g_{22L})^{1-p}(g_{22R})^p-(g_{21L})^{1-p}(g_{21R})^p}{\left((g_{11L})^{1-p}(g_{11R})^p+(g_{22L})^{1-p}(g_{22R})^p\right)-\left((g_{12L})^{1-p}(g_{12R})^p+(g_{21L})^{1-p}(g_{21R})^p\right)}.$$

Provided that

$$\left((g_{11L})^{1-p}(g_{11R})^p+(g_{22L})^{1-p}(g_{22R})^p\right)-\left((g_{12L})^{1-p}(g_{12R})^p+(g_{21L})^{1-p}(g_{21R})^p\right)\neq 0$$

And

$$v^*=E\left(X^*,Y^*\right)=\frac{(g_{11L})^{1-p}(g_{11R})^p(g_{22L})^{1-p}(g_{22R})^p-(g_{12L})^{1-p}(g_{12R})^p(g_{21L})^{1-p}(g_{21R})^p}{\left((g_{11L})^{1-p}(g_{11R})^p+(g_{22L})^{1-p}(g_{22R})^p\right)-\left((g_{12L})^{1-p}(g_{12R})^p+(g_{21L})^{1-p}(g_{21R})^p\right)}$$

It can be proved that

$\left((g_{11L})^{1-p}(g_{11R})^p+(g_{22L})^{1-p}(g_{22R})^p\right)-\left((g_{12L})^{1-p}(g_{12R})^p+(g_{21L})^{1-p}(g_{21R})^p\right)=0$ implies that $\tilde{G}$ has a saddle point.

6. Numerical Example

To illustrate the proposed method, we have solved a numerical example. In this example, the elements of payoff matrix are fuzzy valued (taken from Mijanur et al. [12]). Using parametric form of interval number, the matrix game has been converted into crisp matrix game. Finally, we have solved the matrix game and computed results have been presented in Table 1.

Example 1

Suppose that there are two companies A and B to enhance the market share of a new product by competing in advertising. The two companies are considering two different strategies to increase market share: strategy I (adv. by TV), II (adv. by Newspaper). Here it is assumed that the targeted market is fixed, i.e. the market share of the one company increases while the market share of the other company decreases and also each company puts all its advertisements in one. The above problem may be regarded as matrix game. Namely, the company A and B are considered as players A and B respectively. The marketing research department of company A establishes the following pay-off matrix.

$$\tilde{G}=\begin{array}{c c} & \begin{array}{cc}\text{Adv. by TV} & \text{Adv. by Newspaper}\end{array}\\ \begin{array}{c}\text{Adv. by TV}\\ \text{Adv by Newspaper}\end{array} & \begin{pmatrix}(175,180,190) & (150,156,158)\\ (80,90,100) & (175,180,190)\end{pmatrix}\end{array}$$

Where the element (175, 180, 190) in the matrix $\tilde{G}$ indicates that the sales amount of the company A increase by "about 180" units when the company A and B use the strategy I (adv. by TV) simultaneously. The other elements in the matrix $\tilde{G}$ can be explained similarly. The crisp equivalent game of Example-1 is as follows:

$$G(p)=\begin{array}{c c} & \begin{array}{cc}\text{Adv. by TV} & \text{Adv. by Newspaper}\end{array}\\ \begin{array}{c}\text{Adv. by TV}\\ \\ \end{array} & \begin{pmatrix}(177.5)^{1-p}*(185)^p & (153)^{1-p}*(157)^p\\ (85)^{1-p}*(95)^p & (177.5)^{1-p}*(185)^p\end{pmatrix}\end{array}, p\in[0,1]$$

Table 1. Solution of matrix game

p	Player-A (for TFN)			Player-A (for PFN)		
	x	$1-x$	V^*	x	$1-x$	V^*
0.00	0.790598	0.209402	158.1303	0.790960	0.209040	157.1564
0.05	0.789259	0.210741	158.3966	0.789219	0.210781	157.5090
0.10	0.787915	0.212085	158.6637	0.787468	0.212532	157.8633
0.15	0.786565	0.213435	158.9319	0.785707	0.214293	158.2193
0.20	0.785209	0.214791	159.2009	0.783938	0.216062	158.5769
0.25	0.783848	0.216152	159.4710	0.782158	0.217841	158.9363
0.30	0.782482	0.217518	159.7421	0.780369	0.219631	159.2974
0.35	0.781109	0.218891	160.0141	0.778570	0.221430	159.6602
0.40	0.779731	0.220269	160.2871	0.776761	0.223239	160.0248
0.45	0.778347	0.221653	160.5612	0.774941	0.225059	160.3911
0.50	0.776957	0.223043	160.8362	0.773111	0.226889	160.7593
0.55	0.775561	0.224439	161.1123	0.771271	0.228729	161.1292
0.60	0.774159	0.225841	161.3893	0.769419	0.230581	161.5010
0.65	0.772751	0.227249	161.6675	0.767556	0.232444	161.8747
0.70	0.771336	0.228664	161.9466	0.765682	0.234318	162.2502
0.75	0.769915	0.230085	162.2269	0.763797	0.236203	162.6276
0.80	0.768488	0.231512	162.5082	0.761899	0.238101	163.0069
0.85	0.767054	0.232946	162.7905	0.759990	0.240010	163.3881
0.90	0.765613	0.234387	163.0739	0.758069	0.241931	163.7714
0.95	0.764166	0.235834	163.3585	0.756136	0.243864	164.1565
1.00	0.762712	0.237288	163.6441	0.754190	0.245810	164.5437

From Table-1 it is observed that for trapezoidal fuzzy number the value of the game $V^* \in [158.1303, 163.6441]$ and $x \in [0.762712, 0.790598]$. Whereas in case of parabolic fuzzy number the value of the game $V^* \in [157.1564, 164.5437]$ and $x \in [0.754190, 0.790960]$

7. Conclusions

In this paper, first time a method of solving fuzzy game problem using parametric form of interval numbers has been considered. A Numerical example in existing literature is solved to illustrate the proposed method. Due to the choices of decision makers', the payoff value in a zero sum game might be imprecise rather than precise value. This impreciseness may be represented by various ways. In this paper, we have represented this by fuzzy number. Then the fuzzy game problem has been converted into crisp game problem after parametric representation in which all the payoff values are crisp valued. Here, a new technique has been proposed to solve the fuzzy game. Finally, the games with their strategies and value of the game have been presented and compared and it is claimed that, this method is more effective method for solving fuzzy nonlinear programming problems in the near future.

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