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Effect of Rotation and Hall Current on Unsteady MHD Flow Past an Impulsively Started Vertical Plate with Heat and Mass Transfer in Porous Medium

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ARTICLE INFO Article history: Received: 11 April 2016; Received in revised form: 21 May 2016; Accepted: 26 May 2016; Keywords Rotation_effects.	ABSTRACT The present study is carried out to examine the effects of rotation and Hall current on unsteady natural convection flow of a viscous, incompressible and electrically conducting fluid past an impulsively started vertical plate in a porous medium under the influence of transversely applied uniform magnetic field. Exact solution of energy equation is obtained in closed form by Laplace transform technique, and solution of momentum equation is obtained by defining a complex variable. The expression for the shear stress at the plate due to the primary and secondary flows is obtained. The results obtained are shown by graphs and table.
MHD, Porous medium, Hall current, Heat Transfer, Mass Transfer.	© 2016 Elixir All rights reserved.

Introduction

The impulsively started flow past a semi infinite flat plate is one of the classical problems in the fluid dynamics. Significant study to understand the behaviour of the fluids in unsteady boundary layer was done by Steworten [1, 2]. His research was completely based within the context of boundary layer equations. And influence of magnetic field on such flow within porous and nonporous media is of much significance in geothermal energy extraction, thermal insulation of buildings, sensible heat storage bed, enhanced recovery of petroleum products, and plasma studies. Due to the importance of the study, many researchers worked on impulsively started vertical plate with heat and mass transfer using different analytical and numerical methods. For instance, Soundalkar [3] analysed the effects of mass transfer and free-convection currents on the flow past an impulsively started vertical plate, and his research shows that there is a rise in the velocity due to the presence of a foreign mass. But an increase in Schmidt number leads to a fall in the velocity. Further Soundhangar et al. [4] studied the heat and mass transfer effect on flow past impulsively started vertical plate. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate was studied by Prasad et al. [5], and they solved the governing equations using an implicit finitedifference method of Crank-Nicolson type. Recently many other researchers [6, 7, 8, 9, 10, 11, 12, 13] analysed the effect of heat and mass transfer on impulsively started vertical flat plate in porous or non porous medium under the influence of external transverse magnetic field. However when the strength of magnetic field is very strong, one cannot neglect the effect of Hall current.

Also, the rotating flows of viscous, incompressible and electrically conducting fluid have attracted attention of investigators due to their abundant geophysical and astrophysical applications. It is well known that a number of astronomical bodies possess fluid interiors and magnetic fields. Many scholars have studied such models, for instance, Mazumdar et al. [14] studied the hydrodynamic study flow with the effect of Hall current. Agarwal et al. [15] analysed the combined influence of dissipation and Hall effect on free convective flow in a rotating fluid, and they analysed that the primary shear-stress increases and secondary shear-stress decreases with increase in magnetic and Hall parameters. Also the influence of Hall effect on heat transfer was studied by Seth et al. [16].

This paper deals with an analysis of effects of Hall current on unsteady free convective flow past an impulsively started vertical plate in the presence of transversely applied uniform magnetic field with heat and mass transfer in rotating system. The problem is solved analytically using the Laplace Transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction have been tabulated.

Mathematical Analysis-

Consider an unsteady MHD flow of a viscous, incompressible, electrically conducting fluid past an impulsively started vertical infinite flat plate in porous medium. Let \overline{x} - axis be chosen along the plate in the direction of flow, the \overline{z} - axis normal to the plate, the \overline{y} - axis normal to the $\overline{x} - \overline{z}$ plane and the plate is assumed to coincide with \overline{z} = 0 plane. The fluid and the plate rotate as a rigid body with a uniform angular velocity $\overline{\Omega}$ about \overline{z} - axis. A uniform magnetic field B_o is applied along \overline{z} - axis and the plate is taken to be electrically non-conducting. The fluid motion is induced due to the impulsive movement of the plate as well as the free convection due to heating of the plate. As the plate occupying the plane $\frac{1}{z} = 0$ is of infinite extent, all the physical quantities depend only on \overline{z} and \overline{t} . Initially, at time $\overline{t} \leq 0$, the fluid and the plate are at rest and at a uniform concentration \overline{C}_{∞} and temperature \overline{T}_{∞} . At time $\overline{t} > 0$, the plate starts moving with a velocity u_o in its own plane and the concentration and temperature of the plate is raised to \overline{C}_{w} and \overline{T}_{w} respectively. Since the fluid is electrically conducting whose magnetic Reynolds number is very small, therefore the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Also, due to the conservation of electric charge, current density along $\frac{1}{7}$ direction J_{z} is constant. Since the plate is assumed to be nonconducting therefore J_{z} can be assumed to be zero. So, under the above assumptions, the governing equations with Boussinesq's approximation are as follows:

$$\frac{\partial \overline{u}}{\partial \overline{t}} - 2\overline{\Omega}\overline{v} = v \frac{\partial^2 \overline{u}}{\partial \overline{z}^2} + g\beta(\overline{C} - \overline{C}_{\infty}) + g\beta^*(\overline{T} - \overline{T}_{\infty}) + \frac{B_o}{\rho}\overline{J}_y - \frac{\mu}{\overline{K}}\overline{u},$$

$$\frac{\partial \overline{v}}{\partial t} + 2\overline{\Omega u} = v \frac{\partial^2 \overline{v}}{\partial \overline{z}^2} - \frac{B_o}{\rho} \overline{J}_x - \frac{\mu}{\overline{K}} \overline{v}, \qquad 2$$

$$\frac{\partial \overline{C}}{\partial \overline{t}} = D \frac{\partial^2 \overline{C}}{2^{-2}}, \qquad 3$$

$$\frac{\partial \overline{T}}{\partial \overline{t}} = \alpha \frac{\partial^2 \overline{T}}{\partial \overline{z}^2}.$$

The boundary conditions taken are as under

$$\vec{t} \le 0: \vec{u} = 0, \vec{v} = 0, \vec{C} = \vec{C}_{\infty}, \vec{T} = \vec{T}_{\infty} \forall \vec{z},$$

$$\vec{t} \succ 0: \vec{u} = u_o, \vec{v} = 0, \vec{C} = \vec{C}_w, \vec{T} = \vec{T}_w at \vec{z} = 0,$$

$$\vec{u} \to 0, \vec{C} \to \vec{C}_{\infty}, \vec{T} \to \vec{T}_{\infty} as \vec{z} \to 0,$$

where the symbols are: \overline{C} -concentration of the fluid, \overline{T} -temperature of the fluid, \overline{C}_{∞} - concentration of the fluid far away from the plate, \overline{T}_{∞} – temperature of the fluid far away from the plate, \overline{C}_{w} - concentration at the wall, \overline{T}_{w} - temperature at the wall, B_{a} - external magnetic field, \bar{u} - primary velocity of the fluid, \bar{v} - secondary velocity of the fluid, u_o – velocity of the Plate, \overline{K} – permeability parameter, \bar{z} spatial coordinate normal to the plate, \bar{t} time, β -volumetric coefficient of concentration expansion, β^* – volumetric coefficient of thermal expansion, α – thermal diffusivity, g-acceleration due to gravity, ρ -density, v-kinematic viscosity, σ -Stefan-Boltzmann constant, J_x – current density along x-axis, J_y – current density along y - axis.

Further, on computing current density components, we $J_x = \frac{\sigma B_o(v+mu)}{1+m^2}$ and $J_y = \frac{\sigma B_o(mv-u)}{1+m^2}$. Thus obtained

equations (1) and (2) become:

$$\frac{\partial \overline{u}}{\partial t} - 2\overline{\Omega v} = v \frac{\partial^2 \overline{u}}{\partial \overline{z}^2} + g \beta (\overline{C} - \overline{C}_{\infty}) + g \beta^* (\overline{T} - \overline{T}_{\infty}) + \frac{\sigma B_o^2}{\rho(1 + m^2)} (m\overline{v} - \overline{u}) - \frac{\mu}{\overline{K}} \overline{u},$$
6

$$\frac{\partial \overline{v}}{\partial \overline{t}} + 2\overline{\Omega}\overline{u} = v \frac{\partial^2 \overline{v}}{\partial \overline{z}^2} - \frac{\sigma B_o^2}{\rho(1+m^2)} (\overline{v} + \overline{u}m) - \frac{\mu}{\overline{K}}\overline{v},$$
⁷

To obtain the equations in dimensionless form, the following non-dimensional quantities are introduced:

$$u = \frac{\overline{u}}{u_{o}}, v = \frac{\overline{v}}{u_{o}}, t = \frac{u_{o}^{2}}{v}\overline{t}, K = \frac{u_{o}^{2}}{v^{2}}\overline{K}, z = \frac{u_{o}}{v}\overline{z}, \theta = \frac{(\overline{T} - \overline{T}_{\infty})}{(\overline{T}_{w} - \overline{T}_{\infty})}$$

$$G_{m} = \frac{g\beta^{*}v(\overline{C}_{w} - \overline{C}_{\infty})}{u_{o}^{3}}, \Omega = \frac{v}{u_{o}^{2}}\overline{\Omega}, S_{c} = \frac{v}{D}, P_{r} = \frac{v}{\alpha}$$

$$c = \frac{(\overline{C} - \overline{C}_{\infty})}{(\overline{C}_{w} - \overline{C}_{\infty})}, M = \frac{\sigma B_{o}^{2}v}{\rho u_{o}^{2}}, G_{r} = \frac{g\beta v(\overline{T}_{w} - \overline{T}_{\infty})}{u_{o}^{3}}$$

where u-dimensionless primary velocity of the fluid, v-dimensionless secondary velocity of the fluid. z-dimensionless spatial coordinate normal to the plate, *c* – dimensionless concentration, θ – dimensionless temperature, S_c – Schmidt number, P_r – Prandlt number, G_m – Mass Grashof number, G_r – Thermal Grashof number, t-dimensionless time, Ω -dimensionless rotation parameter and M – magnetic field parameter, m – Hall parameter. The equations (3) to (7) become:

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{\partial^2 u}{\partial z^2} + \frac{M}{(1+m^2)}(mv-u) + G_m c + G_r \theta - \frac{u}{K},$$
⁹

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} - \frac{M}{(1+m^2)}(v+mu) - \frac{v}{K},$$
¹⁰

$$\frac{\partial c}{\partial t} = \frac{1}{S_c} \frac{\partial^2 c}{\partial z^2},$$
¹¹

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2}.$$
 12

$$t \le 0: u = 0, v = 0, c = 0, \theta = 0(\forall z),$$

$$t \succ 0: u = 1, v = 0, c = 1, \theta = 1(atz = 0),$$

$$u \to 0, c \to 0, \theta \to 0(asz \to \infty).$$

$$13$$

To solve above system, take q = u + iv. Then using equations 6 and 7, we get,

$$\frac{\partial q}{\partial t} = G_m c + \frac{\partial^2 q}{\partial z^2} - bq,$$
¹⁴

The boundary conditions given in eq. 9 are reduced to

$$t \le 0: q = 0, c = 0(\forall z),$$

$$t \succ 0: q = 1, c = t(atz = 0),$$

$$q \rightarrow 0, c \rightarrow 0(asz \rightarrow \infty).$$

$$15$$

The governing non-dimensional partial differential equations (11), (12) and (14) subject to the above boundary conditions prescribed in equation (15) are solved using the Laplace Transform technique. The solution is as under:

$$q(\mathbf{z}, \mathbf{t}) = A \left\{ e^{z\sqrt{d}} \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{bt}\right) + e^{-z\sqrt{d}} \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{bt}\right) \right\} + 16$$

$$\left\{ e^{z\sqrt{b+B_j}} \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{(b+B_j)t}\right) + e^{-z\sqrt{b+B_j}} \operatorname{Erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{(b+B_j)t}\right) + 2e^{-B_j t} \operatorname{Erfc}\left(\frac{z}{2}\sqrt{\frac{P_j}{t}}\right) - e^{z\sqrt{P_jB_j}} \operatorname{Erfc}\left(\frac{z}{2}\sqrt{\frac{P_j}{t}} + \sqrt{B_j t}\right) - e^{-z\sqrt{P_jB_j}} \operatorname{Erfc}\left(\frac{z}{2}\sqrt{\frac{P_j}{t}} - \sqrt{B_j t}\right) \right\}$$

$$c(\mathbf{z}, \mathbf{t}) = \operatorname{Erfc}\left(\frac{z}{2}\sqrt{\frac{S_c}{t}}\right), \qquad 17$$

$$\theta(\mathbf{z}, \mathbf{t}) = Erfc\left(\frac{z}{2}\sqrt{\frac{P_r}{r}}\right).$$
18

Skin Friction

The skin-friction components τ_x and τ_y are obtained as

Where

$$b = \frac{Mi}{m+i} + 2i\Omega + \frac{1}{K}, A_1 = \frac{G_r}{P_r - 1}, A_2 = \frac{G_m}{S_c - 1},$$

$$B_1 = \frac{b}{P_r - 1}, B_2 = \frac{b}{S_c - 1}, N_1 = \frac{A_1}{2B_1}, N_2 = \frac{A_2}{2B_2},$$

$$P_1 = P_r, P_2 = S_c, A = \frac{1}{2} - \frac{A_1}{2B_1} - \frac{A_2}{2B_2}.$$

Result and Discussion

In order to get a physical insight of the problem, a representative set of numerical results is shown graphically in figures 1-22 and table -1. It is noticed from figures 1 to 18 that primary velocity u and secondary velocity v attain a distinctive maximum value near the surface of plate, and then decrease on increasing boundary layer coordinate z to approach free stream value. From figures 5 and 6, it is observed that increasing the permeability parameter K of the porous medium, the primary and secondary velocities increase. This is because an increase in K implies that there is a decrease in the resistance of the porous medium, which tends to accelerate primary velocity as well as secondary velocity in the boundary layer region. Figures 9 and 10 show the influence of Hall current on primary and secondary velocities. Primary velocity u increases rapidly near the surface of the plate whereas secondary velocity v increases throughout the boundary layer region on increasing Hall current parameter m. This shows that Hall current tends to accelerate primary velocity in the region near the surface of the plate, whereas it tends to accelerate secondary velocity throughout the boundary layer region. Effect of rotation on flow behaviour is shown by figures 15 and 16, and it is observed that when rotation parameter Ω increases primary velocity *u* decreases

throughout the boundary layer region, whereas secondary velocity v increases near the surface of the plate. This implies that rotation tends to accelerate secondary velocity, whereas it retards primary velocity in the boundary layer region. As time t increases both the primary and secondary velocities increase (figures 17,18). Effect of magnetic parameter M on velocity profile is shown by figures 7, 8 and it is observed that effect is almost similar to that of rotation parameter. Figures 1 to 4 shows the buoyancy effect, and it is observed that both the primary and secondary velocities increase on increasing thermal Grashof number G_r and mass Grashof number G_m . Therefore, it concludes that buoyancy force tends to accelerate primary and secondary velocities. Figures 11 to 14 show the effect of Prandlt number and Schmidt number on velocity profile and it is observed that primary and secondary velocities go on decreasing with increase in P_r and S_c . This is due to the fact that with large values of P_r and S_c the viscous diffusivity dominates the behaviour.

Concentration and temperature profiles are illustrated in figure – 19 and 22 for different values of S_c , P_r and time. In figure 21, it can be seen that the concentration of the fluid is inversely proportional to the value of Schmidt number S_c . Thus, the increase in S_c reduces the concentration in the system. This is due to the fact that there would be a decrease of concentration boundary layer thickness with the increase of Schmidt number S_c . Similar effect can be seen for temperature profile with Prandlt number P_r (figure-19). Also concentration and temperature boundary layer increases with time (figure – 20, 22).

The effects of various parameters on the skin-friction are shown in table -1. It is found from table -1, that the value of τ_x increases when the values of m, G_m , G_r , K and t are increased (keeping other parameters fixed), but if values of M, S_c , P_r and Ω are increased, it gets decreased. Also, it is observed that τ_y decreases with $P_r \& S_c$, and it is increased when m, G_m , G_r , K, M, Ω and t are increased.













								par ameters).		
G_m	G_r	K	М	m	P_r	S _c	Ω	t	$ au_x$	$ au_y$
5	5	0.5	2	1.5	0.71	2.01	0.5	0.2	0.327	0.533
10	5	0.5	2	1.5	0.71	2.01	0.5	0.2	1.299	0.579
5	7	0.5	2	1.5	0.71	2.01	0.5	0.2	0.825	0.564
5	5	0.7	2	1.5	0.71	2.01	0.5	0.2	0.487	0.553
5	5	0.9	2	1.5	0.71	2.01	0.5	0.2	0.578	0.564
5	5	0.5	2	1.5	0.71	2.01	0.5	0.2	0.327	0.533
5	5	0.5	4	1.5	0.71	2.01	0.5	0.2	0.126	0.757
5	5	0.5	2	0.5	0.71	2.01	0.5	0.2	0.067	0.471
5	5	0.5	2	1.0	0.71	2.01	0.5	0.2	0.220	0.542
5	5	0.5	2	1.5	0.71	2.01	0.5	0.2	0.327	0.533
5	5	0.5	2	1.5	1.51	2.01	0.5	0.2	0.128	0.510
5	5	0.5	2	1.5	0.71	4.01	0.5	0.2	0.148	0.518
5	5	0.5	2	1.5	0.71	7.01	0.5	0.2	0.015	0.508
5	5	0.5	2	1.5	0.71	2.01	1.5	0.2	0.229	1.074
5	5	0.5	2	1.5	0.71	2.01	2.5	0.2	0.070	1.587
5	5	0.5	2	1.5	0.71	2.01	0.5	0.2	0.606	0.605
5	5	0.5	2	1.5	0.71	2.01	0.5	0.3	0.826	0.670

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Conclusion-

It is found that Hall current has tendency to accelerate the fluid flow in both the primary and secondary flow directions. Rotation retards primary flow whereas it accelerates secondary flow. Also the flow in both the directions is accelerated by the porosity of the medium. As the value of Schmidt number and Prandlt number increase the fluid velocity in both the directions gets decreased, but the velocity in both the directions increases with time. In addition,

> Temperature increases with time but decreases with Prandlt number.

 \triangleright Concentration increases with time but decreases with Schmidt number.

Skin friction

• τ_x increases when m, G_m , G_r , K and t are increased, but it decreases with M, S_c , P_r and Ω .

• τ_y increases when m, G_m , G_r , K, M, Ω and t are

increased, but it decreases if $P_r \& S_c$ are increased.

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