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Expectation Identities of Left Truncated Logistic Distribution Based on Generalized Order Statistics

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ABSTRACT

In this paper, we establish some expectation identities satisfied by single and product moments of Generalized Order Statistics from Left Truncated Logistic Distribution. These identities are independent of left truncation point and therefore also applicable to Logistic as well as for half Logistic distributions studied in Balakrishnan (1985) and Saran and Pandey (2012) . A particular case of these results verify the corresponding results of Saran and Pandey (2004).

Introduction

Generalized order statistics (GOS) have been introduced and extensively studied in Kamps (1995 a,b) as a unified theoretical set-up which contains a variety of models of ordered random variables with different interpretations. Examples of such models are: Ordinary order statistics, Sequential order statistics, Progressive type II censored order statistics, Record values, k^{th} record value and Pfeifer's records. There is no natural interpretation of generalized order statistics in terms of observed random samples but these models can be effectively applied in life testing and reliability analysis, medical and life time data, and models related to software reliability analysis, etc. The structural similarities and common approach of these models makes it possible to define several distributional properties at once.

Left Truncated Logistic Distribution

A random variable X is said to have Left Truncated Logistic Distribution (LTLD) if its probability density function is of the form

$$f(x)=\frac{\left(e^{\beta}+1\right)e^{-x}}{\left(e^{-x}+1\right)^{2}},\quad \beta<x<\infty,\ \beta>0 \tag{2.1}$$

and its cumulative distribution function is given by

$$F(x)=\frac{\left(e^{\beta}+1\right)\left(e^{-\beta}-e^{-x}\right)}{e^{-x}+1},\quad \beta<x<\infty,\ \beta>0. \tag{2.2}$$

The characterization differential equation for LTLD is given by

$$\frac{1-F(x)}{f(x)}=1+e^{-x}=\sum_{j=0}^{\infty}\alpha_j x^j,\text{where } \alpha_j=\begin{cases}2, & j=0,\\ \dfrac{(-1)^j}{j!}, & j\ge 1.\end{cases}$$

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The mathematical form of the cdf, as given in (2.3), plays an important role for deriving the expectation identities for single and product moments of GOS from left truncated logistic distribution (3.1).

Generalized Order Statistics

Let be a sequence of absolutely continuous, independent and identically distributed random variables with cdf and pdf f(x). Assume k > 0, n { 2,3,…}, , , such that for all . Then , r = 1, 2,…,n , are called GOS if their joint pdf is given by

$$f^{X(1,n,\tilde{m},k),\,X(2,n,\tilde{m},k),\dots,X(n,n,\tilde{m},k)}(x_1,x_2,\dots,x_n)=k\left(\prod_{j=1}^{n-1}\gamma_j\right)\left(\prod_{i=1}^{n-1}\left(1-F(x_i)\right)^{m_i} f(x_i)\right)\times\left(1-F(x_n)\right)^{k-1} f(x_n), \quad (3.1)$$

where $F^{-1}(0+) < x_1 \le x_2 \le \dots \le x_n < F^{-1}(1)$.

By choosing appropriate values of parameters, we get the distribution of a few very common statistics as shown in the Table – 1

Table 1

S.No.	Choice of parameters for i = 1, 2,…, n	GOS becomes
1	$\gamma_i = n-i+1,\ m_1 = m_2 = \dots = m_{n-1} = 0$ and k = 1	Joint distribution of n order statistics
2	$\gamma_i = k,\ m_1 = m_2 = \dots = m_{n-1} = -1,\ k \in N$	k^{th} record value
3	$\gamma_i = (n-i+1)\alpha_i,\ \alpha_i > 0$	Sequential order statistics
4	$\gamma_i = \alpha - i + 1,\ \alpha > 0$	Order statistics with non integer sample size
5	$\gamma_i = \beta_i,\ \beta_i > 0$	Pfeifer's record values
6	$m_i \in N_o,\ k \in N$	Progressively type-II right censored order statistics

The joint pdf of first r, GOS is given by :

$$f^{X(1,n,\tilde{m},k),\,X(2,n,\tilde{m},k),\dots,X(r,n,\tilde{m},k)}(x_1,x_2,\dots,x_r)=c_{r-1}\left(\prod_{i=1}^{r-1}\left(1-F(x_i)\right)^{m_i} f(x_i)\right)\times\left(1-F(x_r)\right)^{k+n-r+M_r-1} f(x_r), \quad (3.2)$$

where, $F^{-1}(0+) < x_1 \le x_2 \le \dots \le x_r < F^{-1}(1)$.

We now consider two cases:

Case I : $m_1 = m_2 = \dots = m_{n-1} = m$

Case II: $\gamma_i \ne \gamma_j\ ; i \ne j,\quad i, j = 1, 2, \dots, n-1.$

For case I, the GOS will be denoted by X(r, n, m, k). The pdf of X(r, n, m, k) is given by

$$f^{X(r,n,m,k)}(x)=\frac{c_{r-1}}{(r-1)!}\left(1-F(x)\right)^{\gamma_r-1} f(x)\, g_m^{r-1}\left(F(x)\right),\quad x\in R \quad (3.3)$$

and the joint pdf of X(r, n, m, k) and X(s, n, m, k), 1 ≤ r < s ≤ n is given by :

$$f^{X(r,n,m,k),\,X(s,n,m,k)}(x,y)=\frac{c_{s-1}}{(r-1)!\,(s-r-1)!}\left(\left(1-F(x)\right)^{m} f(x)\right) g_m^{r-1}\left(F(x)\right)\left[h_m\left(F(y)\right)-h_m\left(F(x)\right)\right]^{s-r-1}\left(1-F(y)\right)^{\gamma_s-1} f(y),\quad x<y \quad (3.4)$$

where,

$$c_{r-1}=\prod_{j=1}^{r}\gamma_j,\qquad \gamma_j=k+(n-j)(m+1),\ r=1,2,\dots,n,$$

$g_m(x) = h_m(x) - h_m(0),\ x \in (0,1)$ and

$$h_m(x)=\begin{cases}-\dfrac{(1-x)^{m+1}}{m+1}, & m\ne -1,\\ -\log(1-x), & m=-1.\end{cases} \quad (3.5)$$

For case II, the pdf of $X(r,n,\tilde{m},k)$ is given by

$$f^{X(r,n,\tilde{m},k)}(x)=c_{r-1}\, f(x)\sum_{i=1}^{r} a_i(r)\left(1-F(x)\right)^{\gamma_i-1} f(x),\quad x\in R \quad (3.6)$$

Also, the joint pdf of $X(r,n,\tilde{m},k)$ and $X(s,n,\tilde{m},k)$ $1 \le r < s \le n$ is given by

$$f^{X(r,n,\tilde{m},k),X(s,n,\tilde{m},k)}(x,y)=c_{s-1}\left\{\sum_{i=r+1}^{s}a_i^r(s)\left(\frac{1-F(y)}{1-F(x)}\right)^{\gamma_i}\right\}$$

$$\times\left\{\sum_{i=1}^{r}a_i(r)\left(1-F(x)\right)^{\gamma_i}\right\}\frac{f(x)}{1-F(x)}\frac{f(y)}{1-F(y)}, \tag{3.7}$$

Wher

$$c_{s-1}=\prod_{j=1}^{s}\gamma_j,\quad \gamma_j=k+n-j+M_j,\ s=1,2,\dots,n.$$

Further it can be proved that

(i) $a_i(r)=\prod_{j(\neq i)=1}^{r}\left(\gamma_j-\gamma_i\right)^{-1}$, $1\le i\le r\le n$

(ii) $a_i^r(s)=\prod_{j(\neq i)=r+1}^{s}\left(\gamma_j-\gamma_i\right)^{-1}$, $r+1\le i\le s\le n\cdot$

(iii) $a_i(r)=\left(\gamma_{r+1}-\gamma_i\right)a_i(r+1)$

(iv) $c_r=c_{r-1}\gamma_{r+1}$

(v) $\sum_{i=1}^{r+1}a_i(r+1)=0$

(vi) $\sum_{i=1}^{r}a_i(r)\left(1-F(x)\right)^{\gamma_i}=\frac{\left(1-F(x)\right)^{\gamma_r}}{(r-1)!}g_m^{r-1}\left(F(x)\right)$

(vii) $\sum_{i=r+1}^{s}a_i^r(s)\left(\frac{1-F(y)}{1-F(x)}\right)^{\gamma_i}=\frac{\left(1-F(x)\right)^{-(m+1)(s-r-1)}}{(s-r-1)!}\left(\frac{1-F(y)}{1-F(x)}\right)^{\gamma_s}$

$\times\left(h_m\left(F(y)\right)-h_m\left(F(x)\right)\right)^{s-r-1}$

The moments of order statistics have generated considerable interest in the recent years. The expressions for several recurrence relations and identities satisfied by single as well as product moments of order statistics have been obtained by several authors in the past. These relations help in reducing the quantum of computations involved. Joshi (1978, 1982) established recurrence relations for exponential distribution with unit mean and were further extended by Balakrishnan and Joshi (1984) for doubly truncated exponential distribution. For linear-exponential distribution, Balakrishnan and Malik(1986) derived the similar type of relations which were extended to doubly truncated linear exponential distribution by Mohie El-Din et al. (1997) and Saran and Pushkarna (1999). Nain (2010 a, b) obtained recurrence relations for ordinary order statistics and k^{th} record values from p^{th} order exponential and generalized Weibull distributions, respectively. Saran and Nain (2012 a, b) obtained recurrence relatio ns for single and product moments arising from double truncated p^{th} ordinary order statistics and also for k^{th} record values and generalized Weibull distribution.

The recurrence relations for the moments of generalized order statistics based on non identically distributed random variable s were developed by Kamps (1995 a, b). Pawlas and Szynal (2001) obtained recurrence relations for single and product moments of generalized order statistics from Pareto, generalized Pareto and Burr distributions. Saran and Pandey (2004, 2009) obtained recurrence relations for single and product moments of generalized order statistics from linear-exponential and Burr distributions. Saran and Pandey (2011) obtained recurrence relations for marginal and joint moment generating functions of dual (lower) generalized order statistics from inverse Weibull distribution.

In this paper, we establish expectation identities for single and product moments of GOS from LTLD. The left truncated logistic distribution has many application in statistical analysis viz. modeling of categorical dependent variables, regression analysis of continuous random variables like income and population models, fitting of rain fall data in heavy rainy areas, describing energy levels (or electron properties) in semiconductors and metals, calculating chess rating, measuring the risk incurred by financial assets returns and to stable growth by applying diversification and hydrology.

Notations

For n = 1, 2, 3, ... $1\le r<s\le n,\ k\ge 1$ and $u,v\in\{0,1,2,\dots\}$, we denote by

(i) $\mu_{r:m,n,k}^{u}=E\left(X^u(r,n,m,k)\right)$ (3.8)

(ii) $\mu_{r,s:m,n,k}^{u,v}=E\left(X^u(r,n,m,k)X^v(s,n,m,k)\right)$ (3.9)

(iii) $\mu_{r:\tilde{m},n,k}^{u}=E\left(X^u(r,n,\tilde{m},k)\right)$ (3.10)

(iv) $\mu_{r,s:\tilde{m},n,k}^{u,v}=E\left(X^u(r,n,\tilde{m},k)X^v(s,n,\tilde{m},k)\right)$ (3.11)

Review of a Few Identities of Athar and Islam (2004)

Here we briefly look at few identities, stated in the form of lemmas 2.1, 2.3, 3.1 and 3.2, respectively, in Athar and Islam (2004) for Borel measurable functions $\omega(x)$ and $\omega(x,y)$ with support (α,∞).

(i) $$E\left(\omega(X(r,\ n,\ m,\ k))\right)-E\left(\omega(X(r\text{-}1,\ n,\ m,\ k))\right)=\frac{c_{r-2}}{(r-1)!}\int_{\alpha}^{\infty}\frac{\partial\omega(x)}{\partial x}\left(\bar{F}(x)\right)^{\gamma_r} f(x)\, g_m^{r-1}\left(F(x)\right)dx \qquad (4.1)$$

(ii) $$E\left(\omega(X(r,\ n,\ \tilde{m},\ k))\right)-E\left(\omega(X(r\text{-}1,\ n,\ \tilde{m},\ k))\right)=\frac{c_{r-1}}{\gamma_r}\int_{\alpha}^{\infty}\frac{\partial\omega(x)}{\partial x}\left[\sum_{i=1}^{r} a_i(r)\left(1-F(x)\right)^{\gamma_i}\right]dx \qquad (4.2)$$

(iii) $$E\left(\omega(X(r,\ n,\ m,\ k),\ X(s,\ n,\ m,\ k))\right)-E\left(\omega(X(r,\ n,\ m,\ k),\ X(s\text{-}1,\ n,\ m,\ k))\right)=$$

$$\frac{c_{s-2}}{(r-1)!(s-r-1)!}\int_{\alpha}^{\infty}\int_{x}^{\infty}\frac{\partial\omega(x,y)}{\partial y}\left(\bar{F}(x)\right)^{m} f(x)\, g_m^{r-1}\left(F(x)\right)$$

$$\times\left[h_m\left(F(y)\right)-h_m\left(F(x)\right)\right]^{s-r-1}\left(\bar{F}(y)\right)^{\gamma_s} dy\,dx \qquad (4.3)$$

(iv) $$E\left(\omega(X(r,\ n,\ \tilde{m},\ k),\ X(s,\ n,\ \tilde{m},\ k))\right)-E\left(\omega(X(r,\ n,\ \tilde{m},\ k),\ X(s\text{-}1,\ n,\ \tilde{m},\ k))\right)$$

$$=c_{s-2}\int_{\alpha}^{\infty}\int_{x}^{\infty}\frac{\partial\omega(x,y)}{\partial y}\left[\sum_{i=1}^{r} a_i(r)\left(\bar{F}(x)\right)^{\gamma_i}\right]\frac{f(x)}{\bar{F}(x)},$$

$$\times\left[\sum_{i=r+1}^{s} a_i^{r}(s)\left(\frac{\bar{F}(y)}{\bar{F}(x)}\right)^{\gamma_i}\right]dy\,dx \qquad (4.4)$$

where $\bar{F}(x)=1-F(x)$.

Expectation Identities for Single and Product Moments of Gos From LTLD

In this section, we shall derive expectation identities for single and product moments of generalized order statistics from LTLD given in (2.1).

Case I : $m_1 = m_2 = \ldots = m_{n-1} = m$.

Theorem 1. Fix a positive integer k. For $n\in N,\ m\in Z$ and $1\le r\le n$,

$$\mu_{r:n,m,k}^{u}=\mu_{r-1:n,m,k}^{u}+\frac{u+1}{\gamma_r}\sum_{j=0}^{\infty}\alpha_j\mu_{r:n,m,k}^{u+j} \qquad (5.1)$$

Proof.

Using (3.3), the u^{th} moment of X(r, n, m, k) from the LTLD (2.1) is given by

$$\mu_{r:n,m,k}^{u}=\frac{c_{r-1}}{(r-1)!}\int_{\beta}^{\infty}x^{u}\left(\bar{F}(x)\right)^{\gamma_r-1} f(x)\, g_m^{r-1}\left(F(x)\right)dx. \qquad (5.2)$$

Substituting $\omega(x)=x^{u+1}$ in (4.1) and using (2.3), we have

$$\mu_{r:n,m,k}^{u}-\mu_{r-1:n,m,k}^{u}=\frac{(u+1)c_{r-1}}{\gamma_r(r-1)!}\int_{\beta}^{\infty}x^{u}\left(\bar{F}(x)\right)^{\gamma_r-1}$$

$$\times\left(\sum_{j=0}^{\infty}\alpha_j x^{j}\right)f(x)\, g_m^{r-1}\left(F(x)\right)dx, \qquad (5.3)$$

which on using (5.2) leads to (5.1).

Theorem 2. Fix a positive integer k. For $n\in N,\ m\in Z$ and $1\le r<s\le n$,

$$\mu_{r,s:n,m,k}^{u,v}=\mu_{r,s-1:n,m,k}^{u,v}+\frac{v+1}{\gamma_s}\sum_{j=0}^{\infty}\alpha_j\mu_{r,s:n,m,k}^{u,v+j} \qquad (5.4)$$

Proof

Using (3.4), the expression for product moments of X(r, n, m, k) and X(s, n, m, k) is:

$$\mu^{u,v}_{r,\, s:\, n,\, m,\, k} = \frac{c_{s-1}}{(r-1)!\,(s-r-1)!}\int_{\beta}^{\infty}\int_{x}^{\infty} x^u\, y^v \left(\bar{F}(x)\right)^m f(x)\, g_m^{r-1}\left(F(x)\right)$$

$$\times\left[h_m\left(F(y)\right)-h_m\left(F(x)\right)\right]^{s-r-1}\left(\bar{F}(y)\right)^{\gamma_s-1} f(y)\,dy\,dx\,. \tag{5.5}$$

Substituting $\omega\left(x, y\right) = x^u y^{v+1}$ in (4.3) and using (2.3) , we have

$$\mu^{u,v}_{r,\, s:\, n,\, m,\, k} - \mu^{u,v}_{r,\, s-1:\, n,\, m,\, k} = \frac{(v+1)\,c_{s-2}}{(r-1)!(s-r-1)!}\int_{\beta}^{\infty}\int_{x}^{\infty} x^u y^v \left(\bar{F}(x)\right)^m$$

$$\times f(x)\, g_m^{r-1}\left(F(x)\right)\left[h_m\left(F(y)\right)-h_m\left(F(x)\right)\right]^{s-r-1}\left(\bar{F}(y)\right)^{\gamma_s} dy\,dx \tag{5.6}$$

$$= \frac{(v+1)\,c_{s-1}}{\gamma_s(r-1)!(s-r-1)!}\int_{\beta}^{\infty}\int_{x}^{\infty} x^u\, y^v\left(\sum_{j=0}^{\infty}\alpha_j\, y^j\right)\left(\bar{F}(x)\right)^m f(x)\, g_m^{r-1}\left(F(x)\right)$$

$$\times\left[h_m\left(F(y)\right)-h_m\left(F(x)\right)\right]^{s-r-1}\left(\bar{F}(y)\right)^{\gamma_s-1} f(y)\,dy\,dx, \tag{5.7}$$

which on further simplification takes the form as stated in (6.69).

Case II: $\gamma_i \neq \gamma_j$; $i \neq j,\quad i, j = 1, 2, \ldots, n-1.$

Theorem 3. Fix a positive integer k. For $n \in N,\ m \in Z$ and $1 \le r \le n,$

$$\mu^{u}_{r:\, n, \tilde{m},\, k} = \mu^{u}_{r-1:\, n, \tilde{m},\, k} + \frac{u+1}{\gamma_r}\sum_{j=0}^{\infty}\alpha_j\, \mu^{u+j}_{r:\, n, \tilde{m},\, k} \tag{5.8}$$

Proof:

Using (3.5), the moment generating function of $X\left(r, n, \tilde{m}, k\right)$ is given by

$$\mu^{u}_{r:\, n,\, \tilde{m},\, k} = c_{r-1}\int_{\beta}^{\infty} x^u \sum_{i=1}^{r} a_i(r)\left(1-F(x)\right)^{\gamma_i-1} f(x)dx\,. \tag{5.9}$$

Substituting $\omega\left(x\right) = x^{u+1}$ in (4.2) and from (2.3), we have

$$\mu^{u}_{r:\, n, \tilde{m},\, k} - \mu^{u}_{r-1:\, n, \tilde{m},\, k} = \frac{(u+1)\,c_{r-1}}{\gamma_r}\int_{\beta}^{\infty} x^u\left(\sum_{i=1}^{r} a_i(r)\left(1-F(x)\right)^{\gamma_i-1}\right)$$

$$\times\left(\sum_{j=0}^{\infty}\alpha_j\, x^j\right) f(x)dx, \tag{5.10}$$

which on further simplification gives (5.8).

Theorem 4. For n = 1, 2, 3,... , $1 \le r < s \le n$ and $k \ge 1$

$$\mu^{u,v}_{r,s\,:\, n,\, \tilde{m},\, k} = \mu^{u,v}_{r,s-1\,:\, n,\, \tilde{m},\, k} + \frac{v+1}{\gamma_s}\sum_{j=0}^{\infty}\alpha_j\, \mu^{u,v+j}_{r,s\,:\, n,\, \tilde{m},\, k} \tag{5.11}$$

Proof: From (3.6), the product moment function of $X\left(r, n, \tilde{m}, k\right)$ and $X\left(s, n, \tilde{m}, k\right)$ is given by

$$\mu_{r,s:n,\tilde{m},k}^{u,v} = c_{s-1}\int_{\beta}^{\infty}\int_{x}^{\infty} x^u\, y^v \left[\sum_{i=1}^{r} a_i(r)\left(\bar{F}(x)\right)^{\gamma_i}\right]\frac{f(x)}{\bar{F}(x)}$$

$$\times\left[\sum_{i=r+1}^{s} a_i^r(s)\left(\frac{\bar{F}(y)}{\bar{F}(x)}\right)^{\gamma_i}\right]\frac{f(y)}{\bar{F}(y)}dy\,dx\,. \tag{5.12}$$

Substituting $\omega(x,y) = x^u y^{v+1}$ in (4.4) and using (2.3), we have

$$\mu_{r,s:n,\tilde{m},k}^{u,v} - \mu_{r,s-1:n,\tilde{m},k}^{u,v} = \frac{(v+1)c_{s-1}}{\gamma_s}\int_{\beta}^{\infty}\int_{x}^{\infty} x^u y^v \left[\sum_{i=1}^{r} a_i(r)\left(\bar{F}(x)\right)^{\gamma_i}\right]\frac{f(x)}{\bar{F}(x)}$$

$$\times\left(\sum_{i=r+1}^{s} a_i^r(s)\left(\frac{\bar{F}(y)}{\bar{F}(x)}\right)^{\gamma_i}\right)\left(\sum_{j=0}^{\infty}\alpha_j\, y^j\right)\frac{f(y)}{\bar{F}(y)}dy\,dx\,, \tag{5.13}$$

which on using (5.12) leads to the expectation identities as stated in (5.11).

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