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A Study on Multi Fuzzy Rw-Continuous Maps and Multi Fuzzy Rw-Irresolute Maps in Multi Fuzzy Topological Spaces

V.Murugan[1], M.Marudai[2] and U.Karuppiah[3]

[1]Head of the Department of Mathematics, H.H.The Rajahs College, Pudukkottai – 622001, Tamilnadu, India.
[2]Head of the Department of Mathematics, Bharathidasan University, Thiruchirappalli, Tamilnadu, India.
[3]Department of Mathematics, St. Joseph's College, Tamilnadu, India.

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ABSTRACT
In this paper, we have studied some of the properties of multi fuzzy rw-continuous mappings and multi fuzzy rw-irresolute mappings in multi fuzzy topological spaces and have proved some results on these.

Introduction

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [16] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C.L.Chang [5] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren [15], K.K.Azad [2], G.Balasubramanian and P.Sundaram [3, 4], S.R.Malghan and S.S.Benchalli [11, 12] and many others have contributed to the development of fuzzy topological spaces. We have introduced the concept of multi fuzzy rw-continuous mappings and multi fuzzy rw-irresolute mappings in multi fuzzy topological spaces and established some results.

1.Preliminaries

1.1 Definition[16]

Let X be a non-empty set. A **fuzzy subset A** of X is a function $A: X \to [0, 1]$.

1.2 Definition: A **multi fuzzy subset**

A of a set X is defined as an object of the form $A = \{ \langle x, A_1(x), A_2(x), A_3(x), \dots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to [0, 1]$ for all i. It is denoted as $A = \langle A_1, A_2, A_3, \dots, A_n \rangle$.

1.3 Definition

Let A and B be any two multi fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $A_i(x) \le B_i(x)$ for all i and for all x in X.

(ii) $A = B$ if and only if $A_i(x) = B_i(x)$ for all i and for all x in X.

(iii) $A^c = 1 - A = \langle 1-A_1, 1-A_2, 1-A_3, \dots, 1-A_n \rangle$.

(iv) $A \cap B = \{ \langle x, \min\{A_1(x), B_1(x)\}, \min\{A_2(x), B_2(x)\}, \dots, \min\{A_n(x), B_n(x)\} \rangle / x \in X \}$.

(v) $A \cup B = \{ \langle x, \max\{A_1(x), B_1(x)\}, \max\{A_2(x), B_2(x)\}, \dots, \max\{A_n(x), B_n(x)\} \rangle / x \in X \}$.

1.4 Definition

Let X be a set and $\Im$ be a family of multi fuzzy subsets of X. The family $\Im$ is called a multi fuzzy topology on X if and only if $\Im$ satisfies the following axioms

(i) $\bar{0}, \bar{1} \in \Im$,

(ii) If $\{ A_i ; i \in I \} \subseteq \Im$, then $\bigcup_{i \in I} A_i \in \Im$,

(iii) If $A_1, A_2, A_3, \dots, A_n \in \Im$, then $\bigcap_{i=1}^{i=n} A_i \in \Im$.

The pair $(X, \Im)$ is called a multi fuzzy topological space. The members of $\Im$ are called multi fuzzy open sets in X. A multi fuzzy set A in X is said to be multi fuzzy closed set in X if and only if A^c is a multi fuzzy open set in X.

1.5 Definition

Let $(X, \Im)$ be a multi fuzzy topological space and A be a multi fuzzy set in X. Then $\cap\{ B : B^c \in \Im \text{ and } B \supseteq A \}$ is called multi fuzzy closure of A and is denoted by mfcl(A).

1.6 Definition

Let $(X, \Im)$ be a multi fuzzy topological space and A be a multi fuzzy set in X. Then $\cup\{B : B \in \Im \text{ and } B \subseteq A \}$ is called multi fuzzy interior of A and is denoted by mfint(A).

1.7 Definition

Let $(X, \Im)$ be a multi fuzzy topological space and A be multi fuzzy set in X. Then A is said to be

Tele:
E-mail address: murugan10463@gmail.com

(i) multi fuzzy semiopen if and only if there exists a multi fuzzy open set V in X such that $V \subseteq A \subseteq mfcl(V)$.

(ii) multi fuzzy semiclosed if and only if there exists a multi fuzzy closed set V in X such that $mfint(V) \subseteq A \subseteq V$.

(iii) multi fuzzy regular open set of X if mfint(mfcl(A)) = A.

(iv) multi fuzzy regular closed set of X if mfcl(mfint(A)) = A.

(v) multi fuzzy regular semiopen set of X if there exists a multi fuzzy regular open set V in X such that $V \subseteq A \subseteq mfcl(V)$. We denote the class of multi fuzzy regular semiopen sets in multi fuzzy topological space X by MFRSO(X).

(vi) multi fuzzy generalized closed (mfg-closed) if $mfcl(A) \subseteq V$ whenever $A \subseteq V$ and V is multi fuzzy open set and A is multi fuzzy generalized open if $\overline{1} - A$ is multi fuzzy generalized closed.

1.8 Definition

A multi fuzzy set A of a multi fuzzy topological space (X, $\Im$) is called:

(i) multi fuzzy g-closed if $mfcl(A) \subseteq V$ whenever $A \subseteq V$ and V is multi fuzzy open set in X. (ii) multi fuzzy g-open if its complement A^c is multi fuzzy g-closed set in X. (iii) multi fuzzy rg-closed if $mfcl(A) \subseteq V$ whenever $A \subseteq V$ and V is multi fuzzy regular open set in X. (iv) multi fuzzy rg-open if its complement A^c is multi fuzzy rg-closed set in X. (v) multi fuzzy w-closed if $mfcl(A) \subseteq V$ whenever $A \subseteq V$ and V is multi fuzzy semi open set in X. (vi) multi fuzzy w-open if its complement A^c is multi fuzzy w-closed set in X. (vii) multi fuzzy gpr-closed if $pcl(A) \subseteq V$ whenever $A \subseteq V$ and V is multi fuzzy regular open set in X. (viii) multi fuzzy gpr-open if its complement A^c is multi fuzzy gpr-closed set in X.

1.9 Definition

Let (X, $\Im$) be a multi fuzzy topological space. A multi fuzzy set A of X is called multi fuzzy regular w-clsoed(briefly, multi fuzzy rw-closed) if $mfcl(A) \subseteq U$ whenever $A \subseteq U$ and U is multi fuzzy regular semiopen in multi fuzzy topological space X.

NOTE

We denote the family of all multi fuzzy regular w-closed sets in multi fuzzy topological space X by MFRWC(X).

1.10 Definition

A multi fuzzy set A of a multi fuzzy topological space X is called a multi fuzzy regular w-open (briefly, multi fuzzy rw-open) set if its complement A^C is a multi fuzzy rw-closed set in multi fuzzy topological space X.

NOTE

We denote the family of all multi fuzzy rw-open sets in multi fuzzy topological space X by MFRWO(X).

1.11 Definition

A mapping f : $X \longrightarrow Y$ from a multi fuzzy topological space X to a multi fuzzy topological space Y is called

(i) multi fuzzy continuous if $f^{-1}(A)$ is multi fuzzy open in X for each multi fuzzy open set A in Y.

(ii)multi fuzzy generalized continuous (mfg-continuous) if $f^{-1}(A)$ is multi fuzzy generalized closed in X for each multi fuzzy closed set A in Y.

(iii)multi fuzzy semi continuous if $f^{-1}(A)$ is multi fuzzy semiopen in X for each multi fuzzy open set A in Y.

(iv)multi fuzzy almost continuous if $f^{-1}(A)$ is multi fuzzy open in X for each multi fuzzy regular open set A in Y.

(v)multi fuzzy irresolute if $f^{-1}(A)$ is multi fuzzy semiopen in X for each multi fuzzy semiopen set A in Y.

(vi)multi fuzzy gc-irresolute if $f^{-1}(A)$ is multi fuzzy generalized closed in X for each multi fuzzy generalized closed set A in Y.

(vii) multi fuzzy completely semi continuous if and only if $f^{-1}(A)$ is an multi fuzzy regular semiopen set of X for every multi fuzzy open set A in Y.

(viii)multi fuzzy w-continuous if and only if $f^{-1}(A)$ is an multi fuzzy w-closed set of X for every multi fuzzy closed A in Y.

(ix)multi fuzzy rg-continuous if $f^{-1}(A)$ is multi fuzzy rg-closed in X for each multi fuzzy closed set A in Y.

(x) multi fuzzy gpr-continuous if $f^{-1}(A)$ is multi fuzzy gpr-closed in X for each multi fuzzy closed set A in Y.

(xi) multi fuzzy almost-irresolute if $f^{-1}(A)$ is multi fuzzy semi open in X for each multi fuzzy regular semi open set A in Y.

1.12 Definition

A mapping f : $X \longrightarrow Y$ from a multi fuzzy topological space X to a multi fuzzy topological space Y is called

(i) multi fuzzy open mapping if f(A) is multi fuzzy open in Y for every multi fuzzy open set A in X.

(ii) multi fuzzy semiopen mapping if f(A) is multi fuzzy semiopen in Y for every multi fuzzy open set A in X.

1.13 Definition

Let X and Y be multi fuzzy topological spaces. A map f : $X \longrightarrow Y$ is said to be multi fuzzy rw-continuous if the inverse image of every multi fuzzy open set in Y is multi fuzzy rw-open in X.

1.14 Definition

Let X and Y be multi fuzzy topological spaces. A map f : $X \longrightarrow Y$ is said to be a multi fuzzy rw-irresolute map if the inverse image of every multi fuzzy rw-open set in Y is a multi fuzzy rw-open set in X.

1.15 Definition

Let (X, $\Im$) be a multi fuzzy topological space and A be a multi fuzzy set of X. Then multi fuzzy rw-interior and multi fuzzy rw-closure of A are defined as follows.

$mfrwcl(A) = \cap\{ K : K$ is a multi fuzzy rw-closed set in X and $A \subseteq K \}$.

$mfrwint(A) = \cup \{ G : G$ is a multi fuzzy rw-open set in X and $G \subseteq A \}$.

Remark

It is clear that $A \subseteq mfrwcl(A) \subseteq mfcl(A)$ for any multi fuzzy set A.

1.16 Theorem

If A is a multi fuzzy regular open and multi fuzzy rg-closed in multi fuzzy topological space (X, $\Im$), then A is multi fuzzy rw-closed in X.

2. Some Properties

2.1 Theorem

If a map f : $(X, \Im) \longrightarrow (Y, \sigma)$ is multi fuzzy continuous, then f is multi fuzzy rw-continuous.

Proof

Let A be a multi fuzzy open set in a multi fuzzy topological space Y. Since f is multi fuzzy continuous, $f^{-1}(A)$ is a multi fuzzy open set in multi fuzzy topological space X. As every multi fuzzy open set is multi fuzzy rw-open, we have $f^{-1}(A)$ as multi fuzzy rw-open set in multi fuzzy topological space X. Therefore f is multi fuzzy rw-continuous.

2.2 Remark

The converse of the above Theorem need not be true in general.

2.3 Example

Let X = Y = { 1, 2, 3 } and the multi fuzzy sets A, B, C be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }, C = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 > }. Consider $\Im = \{ 0_X, 1_X, A \}$, $\sigma = \{ 0_Y, 1_Y, B, C \}$. Now (X, $\Im$) and (Y, σ) are the multi fuzzy topological spaces. Define a map f : (X, $\Im$) $\longrightarrow$ (Y, σ) by f(1) = 2, f(2) = 3 and f(3) = 1. Then f is multi fuzzy rw-continuous but not multi fuzzy continuous as the inverse image of the multi fuzzy set C in (Y, σ) is D defined as D = {< 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }. This is not an multi fuzzy open set in (X, $\Im$).

2.4 Theorem

A map f : (X, $\Im$) $\longrightarrow$ (Y, σ) is multi fuzzy rw-continuous if and only if the inverse image of every multi fuzzy closed set in a multi fuzzy topological space Y is a multi fuzzy rw-closed set in multi fuzzy topological space X.

Proof: Let D be a multi fuzzy closed set in a multi fuzzy topological space Y. Then D^C is multi fuzzy open in multi fuzzy topological space Y. Since f is multi fuzzy rw-continuous, $f^{-1}(D^C)$ is multi fuzzy rw-open in multi fuzzy topological space X. But $f^{-1}(D^C) = 1_X - f^{-1}(D)$ and so $f^{-1}(D)$ is a multi fuzzy rw-closed set in multi fuzzy topological space X.

Conversely, assume that the inverse image of every multi fuzzy closed set in Y is multi fuzzy rw-closed in multi fuzzy topological space X. Let A be a multi fuzzy open set in multi fuzzy topological space Y. Then A^C is multi fuzzy closed in Y. By hypothesis $f^{-1}(A^C) = 1_X - f^{-1}(A)$ is multi fuzzy rw-closed in X and so $f^{-1}(A)$ is a multi fuzzy rw-open set in multi fuzzy topological space X. Thus f is multi fuzzy rw-continuous.

2.5 Theorem

If a function f : (X, $\Im$) $\longrightarrow$ (Y, σ) is multi fuzzy almost continuous, then it is multi fuzzy rw-continuous.

Proof

Let a function f : (X, $\Im$) $\longrightarrow$ (Y, σ) be a multi fuzzy almost continuous and A be a multi fuzzy open set in multi fuzzy topological space Y. Then $f^{-1}(A)$ is a multi fuzzy regular open set in multi fuzzy topological space X. Now $f^{-1}(A)$ is multi fuzzy rw-open in X, as every multi fuzzy regular open set is multi fuzzy rw-open. Therefore f is multi fuzzy rw-continluous.

2.6 Remark

The converse of the above theorem need not be true in general.

2.7 Example: Consider the multi fuzzy topological spaces (X, $\Im$) and (Y, σ) as defined in Example 2.3. Define a map f : (X, $\Im$) $\longrightarrow$ (Y, σ) by f(1) = 2, f(2) = 3 and f(3) = 1. Then f is multi fuzzy rw-continuous but it is not multi fuzzy almost continuous.

2.8 Theorem

Multi fuzzy semi continuous maps and multi fuzzy rw-continuous maps are independent.

Proof

Consider the following examples. Let X = Y = { 1, 2, 3 } and the multi fuzzy sets A, B be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = {< 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 > }. Consider $\Im = \{ 0_X, 1_X, A \}$, $\sigma = \{ 0_Y, 1_Y, B \}$. Now (X, $\Im$) and (Y, σ) are the multi fuzzy topological spaces. Define a map f : (X, $\Im$) $\longrightarrow$ (Y, σ) by f(1) = 1, f(2) = 2 and f(3) = 3. Then f is multi fuzzy rw-continuous but it is not multi fuzzy semi continuous, as the inverse image of multi fuzzy set B in (Y, σ) is D defined as D = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 > }. This is not a multi fuzzy semiopen set in multi fuzzy topological space X. And, let X = Y = { 1, 2, 3 } and the multi fuzzy sets A, B, C, D be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }, C = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > } and D = {< 1, 0, 0, 0 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 > }. Consider $\Im = \{ 0_X, 1_X, A, B, C \}$ and $\sigma = \{ 0_Y, 1_Y, D \}$. Now (X, $\Im$) and (Y, σ) are the multi fuzzy topological spaces. Define a map f : (X, $\Im$) $\longrightarrow$ (Y, σ) by f(1) = f(3) = 3 and f(2) = 2. Then f is multi fuzzy semi continuous but it is not multi fuzzy rw-continuous, as the inverse image of multi fuzzy set D in (Y, σ) is E defined as E = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 > }. This is not a multi fuzzy rw-open set in multi fuzzy topological space X.

2.9 Theorem

Multi fuzzy generalized continuous maps and multi fuzzy rw-continuous maps are independent.

Proof

Consider the multi fuzzy topological spaces (X, $\Im$) and (Y, σ) as defined in example in Theorem 2.8. Define a map f : (X, $\Im$) $\longrightarrow$ (Y, σ) by f(1) = 1, f(2) = 2 and f(3) = 3. Then f is multi fuzzy rw-continuous but it is not multi fuzzy g-continuous as the inverse image of multi fuzzy set D in (Y, σ) is E defined as E = { < 1, 0, 0, 0 >, < 2, 0, 0, 0 >, < 3, 1, 1, 1 > }. This is not a multi fuzzy g-open set in multi fuzzy topological space X. And, let X = Y = { 1, 2, 3, 4 } and the multi fuzzy sets A, B, C, D be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 > }, B = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 > }, C = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 >, < 4, 0, 0, 0 > }. Let Y = { 1, 2, 3 } and the multi fuzzy set D be defined as D = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 > }. Consider $\Im = \{ 0_X, 1_X, A, B, C \}$ and $\sigma = \{ 0_X, 1_X, D \}$. Now (X, $\Im$) and (Y, σ) are the multi fuzzy topological spaces. Define a map f : (X, $\Im$) $\longrightarrow$ (Y, σ) by f(1) = f(4) = 3, f(2) = 2 and f(3) = 3. Then f is multi fuzzy g-continuous but it is not multi fuzzy rw-continuous, as the inverse image of multi fuzzy set D in (Y, σ) is E defined as E = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 >, < 4, 0, 0, 0 > }. This is not a multi fuzzy rw-open set in multi fuzzy topological space X.

2.10 Theorem

If a function f : (X, $\Im$) $\longrightarrow$ (Y, σ) is multi fuzzy rw-continuous and multi fuzzy completely semi continuous then it is multi fuzzy continuous.

Proof

Let a function f : (X, $\Im$) $\longrightarrow$ (Y, σ) be a multi fuzzy rw-continuous and multi fuzzy completely semi continuous. Let E be a multi fuzzy closed set in multi fuzzy topological space Y. Then $f^{-1}(E)$ is both multi fuzzy regular semiopen and multi fuzzy rw-closed set in multi fuzzy topological space X. By Theorem 1.16, $f^{-1}(E)$ is a multi fuzzy closed set in multi fuzzy topological space X. Therefore f is multi fuzzy continuous.

2.11 Theorem

If f : (X, $\Im$) $\longrightarrow$ (Y, σ) is multi fuzzy rw-continuous and g: (Y, σ) $\longrightarrow$ (Z, η) is multi fuzzy continuous, then their composition g$\bullet$f : (X, $\Im$) $\longrightarrow$ (Z, η) is multi fuzzy rw-continuous.

Proof

Let A be a multi fuzzy open set in multi fuzzy topological space Z. Since g is multi fuzzy continuous, $g^{-1}(A)$ is a multi fuzzy open set in multi fuzzy topological space Y. Since f is multi fuzzy rw-continuous, $f^{-1}(g^{-1}(A))$ is a multi fuzzy rw-open set in multi fuzzy topological space X. But $(g\bullet f)^{-1}(A) = f^{-1}(g^{-1}(A))$. Thus g$\bullet$f is multi fuzzy rw-continuous.

2.12 Theorem

If a map f : X $\longrightarrow$ Y is multi fuzzy rw-irresolute, then it is multi fuzzy rw-continuous.

Proof: Let A be a multi fuzzy open set in Y. Since every multi fuzzy open set is multi fuzzy rw-open, A is a multi fuzzy rw-open set in Y. Since f is multi fuzzy rw-irresolute, $f^{-1}(A)$ is multi fuzzy rw-open in X. Thus f is multi fuzzy rw-continuous.

2.13 Remark

The converse of the above theorem need not be true in general.

2.14 Example

Let X = Y = { 1, 2, 3 } and the multi fuzzy sets A, B, C be defined as A = { < 1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > }, B = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }, C = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }. Consider $\Im_1 = \{0_X, 1_X$, A, B, C } and $\Im_2 = \{ 0_Y, 1_Y$, A }. Now (X, $\Im_1$) and (Y, $\Im_2$) are multi fuzzy topological spaces. Define a map f : (X, $\Im_1$) $\longrightarrow$ (Y, $\Im_2$) be the identity map. Then f is multi fuzzy rw-continuous but it is not multi fuzzy rw-irresolute. Since for the multi fuzzy rw-open set E defined by E = { < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 1, 1, 1 > } in Y, f^{-1}(E) = E is not multi fuzzy rw-open in (X, $\Im_1$).

2.15 Theorem

Let X, Y and Z be multi fuzzy topological spaces. If f : X $\longrightarrow$ Y is multi fuzzy rw-irresolute and g : Y $\longrightarrow$ Z is multi fuzzy rw-continuous, then their composition g$\bullet$f : X $\longrightarrow$ Z is multi fuzzy rw-continuous.

Proof

Let A be any multi fuzzy open set in multi fuzzy topological space Z. Since g is multi fuzzy rw-continuous, $g^{-1}(A)$ is a multi fuzzy rw-open set in multi fuzzy topological space Y. Since f is multi fuzzy rw-irresolute $f^{-1}(g^{-1}(A))$ is a multi fuzzy rw-open set in multi fuzzy topological space X. But $(g\bullet f)^{-1}(A) = f^{-1}(g^{-1}(A))$. Thus g$\bullet$f is multi fuzzy rw-continuous.

2.16 Theorem

Let X, Y and Z be multi fuzzy topological spaces and f : X $\longrightarrow$ Y and g : Y $\longrightarrow$ Z be multi fuzzy rw-irresolute maps, then their composition g$\bullet$f : X $\longrightarrow$ Z is multi fuzzy rw-irresolute map.

Proof

Let A be any multi fuzzy rw-open set in multi fuzzy topological space Z. Since g is multi fuzzy rw-irresolute, $g^{-1}(A)$ is a multi fuzzy rw-open set in multi fuzzy topological space Y. Since f is multi fuzzy rw-irresolute $f^{-1}(g^{-1}(A))$ is a multi fuzzy rw-open set in multi fuzzy topological space X. But $(g\bullet f)^{-1}(A) = f^{-1}(g^{-1}(A))$. Thus g$\bullet$f is multi fuzzy rw-continuous.

2.17 Theorem

Let A be a multi fuzzy w-closed set in a multi fuzzy topological space (X, $\Im$) and f : (X, $\Im$) $\to$ (Y, σ) is a multi fuzzy almost irresolute and multi fuzzy closed mapping then f (A) is a multi fuzzy rw-closed set in Y.

Proof: Let A be a multi fuzzy w-closed set in X and f : (X, $\Im$) $\to$ (Y, σ) is a multi fuzzy almost irresolute and multi fuzzy closed mapping. Let f(A) $\subseteq$ O where O is multi fuzzy regular semi open in Y then A $\subseteq$ $f^{-1}(O)$ and $f^{-1}(O)$ is multi fuzzy semi open in X because f is multi fuzzy almost irresolute. Now A be a multi fuzzy w-closed set in X, mfcl(A) $\subseteq$ $f^{-1}(O)$. Thus, f(mfcl(A)) $\subseteq$ O and f(mfcl(A)) is a multi fuzzy closed set in Y (since mfcl(A) is multi fuzzy closed in X and f is multi fuzzy closed mapping). It follows that mfcl(f(A)) $\subseteq$ mfcl(f(mfcl(A))) = f(mfcl(A)) $\subseteq$ O. Hence mfcl(f(A)) $\subseteq$ O whenever f(A) $\subseteq$ O and O is multi fuzzy regular semi open in Y. Hence f(A) is multi fuzzy rw-closed set in Y.

2.18 Theorem

Let (X, $\Im$) be a multi fuzzy topological space and MFRSO(X) (resp. MFC(X)) be the family of all multi fuzzy regular semi open (resp. multi fuzzy closed) sets of X. Then MFRSO(X) $\subseteq$ MFC(X) if and only if every multi fuzzy set of X is multi fuzzy rw-closed.

Proof

Suppose that MFRSO(X) $\subseteq$ MFC(X) and let A be any multi fuzzy set of X such that A $\subseteq$ U$\in$MFRSO(X). U is multi fuzzy regular semi open. Then, mfcl(A) $\subseteq$ mfcl(U) = U because U$\in$MFRSO(X) $\subseteq$ MFC(X). Hence mfcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is multi fuzzy regular semi open. Hence A is multi fuzzy rw-closed set.

Suppose that every multi fuzzy set of X is multi fuzzy rw-closed. Let U$\in$MFRSO(X), then since U $\subseteq$ U and U is multi fuzzy rw-closed, mfcl(U) $\subseteq$ U, then U$\in$MFC(X). Thus MFRSO(X) $\subseteq$ MFC(X).

2.19 Remark

Every multi fuzzy w-continuous mapping is multi fuzzy rw-continuous, but converse may not be true.

Proof

Consider the example, let X = { a, b }, Y = { x, y } and multi fuzzy sets U and V are defined as follows: U = { < a, 0.7, 0.7, 0.7 >, < b, 0.6, 0.6, 0.6 > }, V = { < x, 0.7, 0.7, 0.7 >, < y, 0.8, 0.8, 0.8 > }. Let $\Im = \{ 1_X, 0_X$, U } and $\sigma = \{ 1_Y, 0_Y$, V } be multi fuzzy topologies on X and Y respectively. Then the mapping f : (X, $\Im$) $\to$ (Y, σ) defined by f(a) = x and f (b) = y is multi fuzzy rw-continuous but not multi fuzzy continuous.

2.20 Remark

Every multi fuzzy rw-continuous mapping is multi fuzzy rg-continuous, but converse may not be true.

Proof

Consider the example, let X = { a, b, c, d } Y = { p, q r, s } and multi fuzzy sets O, U, V, W, T are defined as follows: O = { < a, 0.9, 0.9, 0.9 >, < b, 0, 0, 0 >, < c, 0, 0, 0 >,< d, 0, 0, 0 > }, U = { < a, 0, 0, 0 >, < b, 0.8, 0.8, 0.8 >, < c, 0, 0, 0 >, < d, 0, 0, 0 > }, V = { < a, 0.9, 0.9, 0.9 > ,< b, 0.8, 0.8, 0.8 >, < c, 0, 0, 0 >, < d, 0, 0, 0 > }, W = { < a, 0.9, 0.9, 0.9 >, < b, 0.8, 0.8, 0.8 >, < c, 0.7, 0.7, 0.7 >, < d, 0, 0, 0 > }, T = {< p, 0, 0, 0 >, < q, 0, 0, 0 > < r, 0.7, 0.7, 0.7 >, < s, 0, 0, 0 > }. Let $\Im = \{1_X, 0_X$, O, U, V, W } and σ

$= \{1_Y, 0_Y, T\}$ be multi fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by f (a) = p, f (b) = q, f (c) = r, f (d) = s is multi fuzzy rg-continuous but not multi fuzzy rw-continuous.

2.21 Remark

Every multi fuzzy rw-continuous mapping is multi fuzzy gpr-continuous, but converse may not be true.

Proof

Consider the example, let X = { a, b, c, d, e } Y = { p, q, r, s, t } and multi fuzzy sets O, U, V, W are defined as follows: O = {< a, 0.9, 0.9, 0.9 >, < b, 0.8, 0.8, 0.8 >, < c, 0, 0, 0 >, < d, 0, 0, 0 >, < e, 0, 0, 0 > }, U = { < a, 0, 0, 0 > ,< b, 0, 0, 0 >, < c, 0.8, 0.8, 0.8 >, < d, 0.7, 0.7, 0.7 >, < e, 0, 0, 0 > }, V = { < a, 0.9, 0.9, 0.9 > ,< b, 0.8, 0.8, 0.8 >, < c, 0.8, 0.8, 0.8 >, < d, 0.7, 0.7, 0.7 >, < e, 0, 0, 0 > }, W = { < a, 0.9, 0.9, 0.9 > ,< b, 0, 0, 0 >, < c, 0.8, 0.8, 0.8 >, < d, 0.7, 0.7, 0.7 >, < e, 0, 0, 0 > }. Let $\Im = \{1_X, 0_X, O, U, V\}$ and $\sigma = \{1_Y, 0_Y, W\}$ be multi fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \Im) \to (Y, \sigma)$ defined by f(a) = p, f(b) = q, f(c) = r, f(d) = s, f(e) = t is multi fuzzy gpr-continuous but not multi fuzzy rw-continuous.

2.22 Theorem

If $f : (X, \Im) \to (Y, \sigma)$ is multi fuzzy rw-continuous, then $f(mfrwcl(A) \subseteq mfcl(f(A))$ for every multi fuzzy set A of X.

Proof

Let A be a multi fuzzy set of X. Then mfcl(f(A)) is a multi fuzzy closed set of Y. Since f is multi fuzzy rw-continuous, f^{-1}(mfcl(f(A))) is multi fuzzy rw-closed in X. Clearly A $\subseteq f^{1}$(mfcl(A)). Therefore mfwcl(A) $\subseteq$ mfwcl(f^{-1}(mfcl(f(A)))) = f^{-1}(mfcl(f(A))). Hence f (mfrwcl(A) $\subseteq$ mfcl(f(A)) for every multi fuzzy set A of X.

2.23 Theorem

If $f : (X, \Im) \to (Y, \sigma)$ is multi fuzzy rg-irresolute and $g : (Y, \sigma) \to (Z, \lambda)$ is multi fuzzy rw-continuous. Then $g \circ f : (X, \Im) \to (Z, \lambda)$ is multi fuzzy rg-continuous.

Proof

Let A is a multi fuzzy closed set in Z, then g^{-1}(A) is multi fuzzy rw-closed in Y, because g is multi fuzzy rw-continuous. Since every multi fuzzy rw-closed set is multi fuzzy rg-closed set, therefore g^{-1}(A) is multi fuzzy rg-closed in Y .Then $(gof)^{-1}(A) = f^{1}(g^{-1}(A))$ is multi fuzzy rg-closed in X, because f is multi fuzzy rg-irresolute. Hence gof: $(X, \Im) \to (Z, \lambda)$ is multi fuzzy rg-continuous.

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