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# PRIME (1,0) Rings

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**ABSTRACT**

A left alternative ring R satisfying (x,y,z) + (y,z,x) +(z,x,y) = 0 is (1,0) ring. Where as if R satisfies (R,(R,R))=0 then R is called a strongly (1,0) ring. All (1,0) rings are not strongly (1,0) rings. But strongly (1,0) rings are (1,0) . In this paper, it will be proved that a prime (1,0) ring of characteristic $\neq 2,3$ is either associative or strongly (1,0) ring.



**Introduction**

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Sterling[3] proved that a prime (-1,1) ring of characteristic $\neq 2,3$ with an idempotent e $\neq 0,1$ is associative. To prove associativity in prime (1,0) rings with idempotent e $\neq 0,1$, Paul [4] assumed the additional identity ((e,x),e,e) =0. E. Kleinfeld proved [1] a semi-prime right alternative ring of characteristic $\neq 2,3$ with (a,(b,a))=0 is strongly (-1,1) ring. In this paper without any additional assumpotion we prove that a prime (1,0) ring of characteristic $\neq 2,3$ is either associative or strongly (1,0) ring.

A non-associative ring R is called (1,0) ring if

$\overline{A}(x,y,z) \equiv (x,y,z) + (y,z,x) + (z,x,y) = 0.$ …………… (1)

$\overline{B}(x,y,z) \equiv (x,y,z) + (y,x,z) = 0.$ …………… (2)

for all x,y,z in R, where the associator (x,y,z) = xy.z - x.yz. The commutator (x,y)=xy –yx . If there exists a positive integer n such that na = 0 for every element a of the ring R, the smallest such positive integer is called the characteristic of R.

A non-associative ring is said to be strongly (1,0) ring if it satisfies (x,y,z) + (y,x,z) = 0 and ((R,R),R) = 0.

A ring R is called prime, if whenever A and C are ideals of the ring such that AC = 0, then either A = 0 or C = 0. Throughout this paper R represents a (1,0) ring of characteristic $\neq 2,3$ .

We consider two ideals A and C of R, where A is the ideal generated by

{(x,y,z)\ x,y,z $\in$ R} and C is the ideal generated by double commutator, {((x,y),z)\x,y,z $\in$ R}.

**2.Preliminaries**: The following identities hold good in left alternative rings[4]

$\overline{C}(x,y,z) \equiv (xy,y,z) - y(x,y,z)=0.$ ……………..(3)

$\overline{D}(w,x,y,z) \equiv (wx,y,z) + (wy,x,z) - x(w,y,z) - y(w,x,z) = 0.$ …………… (4)

$\overline{E}(x,y,z) \equiv (x^2,y,z) - (x,xy+yx,z) = 0.$

The identity known as Teichmuller identity holds in any arbitrary ring:

$\overline{F}(w,x,y,z) = (wx,y,z) - (w,xy,z) + (w,x,yz) - w(x,y,z) - (w,x,y)z = 0.$ ……………. (5)

In [2] Subhashini established that in (1,0) ring, the associator commutes with every element of R. This is equation (18) in [2]. We take that identity here.

(R,(R,R,R)) = 0. …………….. (6)

**Lemma:** In R, (r,w(x,y,z)) =0, for all x,y,z,w,r $\in$ R.

**Proof:** Let r be an arbitrary element of R. Commute equations (3) , (4) and (5) with r, and then apply equation (6). We obtain

(r,y(x,y,z)) = 0. ……………..(7)

(r,x(w,y,z)) = - (r,y(w,x,z)) ……………..(8)

(r,w(x,y,z)) = - (r,(w,x,y)z). From (6) ,this equation become

(r,w(x,y,z)) = - (r,z(w,x,y)). …………..(9)

Linearize equation (7)

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(r,w(x,y,z) = - (r,y(x,w,z)) ……………(10)

Permutating cyclically w,z,y in (9) and then apply (10) and (2), we get (r,w(x,y,z)) = - (r,z(w,x,y)) = (r,y(z,w,x)) = -(r,w(z,y,x)) = -(r,w(y,z,x)). Again use (2) , (10) and again (2) ,we have –(r,z(w,x,y)) = (r,z(x,w,y)) = - (r,w(x,z,y)) = (r,w(z,x,y)). Therefore

(r,w(x,y,z)) = (r,w(y,z,x)) = (r,w(z,x,y)) …………….. (11)

Multiply equation (1) by w and commute with r and apply (11). Then 3(r,w(x,y,z)) = 0, since Characteristic $\neq 3$ , we have

(r,w(x,y,z)) = 0. ……………. (12)

To prove the required result , it is necessary to prove some other identities .

The semi Jacobi identity (xy,z) – x(y,z) – (x,z)y – (x,y,z) + (x,z,y) – (z,x,y) = 0 holds good in any ring.

In a (1,0) ring this identity becomes

$\overline{G}$ (x,y,z) = (xy,z) – x(y,z) – (x,z)y – 2(z,x,y) – (x,y,z) = 0.

$\overline{F}$ (w,x,y,z) - $\overline{D}$ (w,x,y,z) gives

$\overline{H}$ (w,x,y,z) ≡((w,x),y,z) + (w,x,yz) – y(w,x,z) – (w,x,y)z = 0.

And $\overline{H}$ (w,x,y,z) - $\overline{H}$ (w,x,z,y) - $\overline{A}$ ((w,x),y,z)) gives

$\overline{I}$ (w,x,y,z) ≡(w,x,(y,z)) – (y,z,(w,x)) + (z,(w,x,y)) – (y,(w,x,z)) =0.

Now from (3) $\overline{I}$ (w,x,y,z) ≡(w,x,(y,z)) – (y,z,(w,x)) + (z,(w,x,y)) – (y,(w,x,z)) =0 becomes

(w,x,(y,z)) = (y,z,(w,x)). …………….. (13)

Let us define U = $\{u \in R/(u,R)=0\}$. $\overline{G}$ (x,x,u) gives -2(u,x,x) = 0. Because of characteristic $\neq 2$

(u,x,x) = 0. Linearization of this gives

$\overline{J}$ (u,x,y) ≡(u,x,y) + (u,y,x) = 0.

-$\overline{G}$ (x,y,u) - 2 $\overline{J}$ (u,x,y) = 0 gives

$\overline{K}$ (x,y,u) ≡(x,y,u) – 2(u,y,x) = 0.

The combination of -$\overline{G}$ (u,x,y) - 2 $\overline{B}$ (u,x,y) + 2 $\overline{J}$ = 0 gives

$\overline{L}$ (u,x,y) ≡3(u,x,y) – (ux,y) + u(x,y) = 0. ………………. (14)

***Theorem:*** If R is a prime (1,0) ring of characteristic $\neq 2,3$ , then R is either associative or strongly (1,0) ring.

***Proof:*** Put u = (R,R,R) is an arbitrary associator in (14) and apply equation (6) , then we have 3((R,R,R),x,y) = - (R,R,R)(x,y). In this equation put y = (R,R) an arbitrary commutator. Now this becomes

3((R,R,R),x,(R,R)) = - (R,R,R) (x,(R,R)) ……………….. (15)

Put y = (R,R,R) an arbitrary associator in equation (13) and apply (6) , we obtain

((R,R,R),z,(R,R)) = 0 ……………….. (16)

From (15) and (16) (R,R,R) (x,(R,R)) =0 or AC = 0. Since R is prime , either A = 0 or C = 0. If A = 0 implies R is associative and if C = 0 implies R is strongly (1,0) ring.

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