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The Infinite Distance Horizon and the Hyperbolic Inflation in the Hyperbolic Universe

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ABSTRACT

The Universe is globally hyperbolic as we did prove mathematically [S. A. Mabkhout, Phys. Essays **25**, 112 (2012)]. The solution predicts the equation of state of cosmology ($\mathbf{P} = -\rho$). Penrose said if the curvature is not equal to zero, then inflation is out. The hyperbolic universe inflates exponentially produces an accelerated expansion of the universe without cosmological constant or scalar field, as we have shown [S. A. Mabkhout, Phys. Essays 26,422 (2013)]. "In testing the validity of any scientific paradigm, the key criterion is whether measurements agree with what is expected given the paradigm"[1]. The hyperbolic time evolution equation of the universe (the hyperbolic scale factor) fits the observed data and successfully predicts the Planck length (10^{-33} cm) at micro-cosmos scale as well as it predicts the current observed large structure (10^{28} cm) at macro-cosmos scale, thus connecting both General Relativity and Quantum Theory. The distance horizon in the flat universe is 14 Gpc. We found the distance horizon in the hyperbolic universe to be infinitely, thus solves the horizon problem. Although the perspective for nearby objects in hyperbolic space is very nearly identical to Euclidean space (i.e., the universe locally is approximately flat consistent with local observations), the apparent angular size of distant objects falls off much more rapidly, in fact exponentially.The universe is globally hyperbolic.Locally the spacetime is approximately flat, by means of the hyperbolic inflation. Locally any deviations from flatness will be hyperbolically suppressed by the hyperbolic expansion of the scale factor, and the flatness problem is solved.

1. Introduction

"The observable universe consists of the galaxies and other matter that can, in principle, be observed from Earth at the present time because light and other signals from these objects have had time to reach the Earth since the beginning of the cosmological expansion. Assuming the universe is isotropic, the distance to the edge of the observable universe is roughly the same in every direction. That is, the observable universe is a spherical volume (a ball) centered on the observer. Every location in the Universe has its own observable universe, which may or may not overlap with the one centered on Earth"[3].

The word "*observable* simply indicates that it is possible *in principle* for light or other signals from the object to reach an observer on Earth. In practice, we can see light only from as far back as the time of photon decoupling in the recombination epoch. That is when particles were first able to emit photons that were not quickly re-absorbed by other particles. Before then, the Universe was filled with a plasma that was opaque to photons. The surface of last scattering is the collection of points in space at the exact distance that photons from the time of photon decoupling just reach us today. These are the photons we detect today as cosmic microwave background radiation (CMBR)"[4]. However, due to "Hubble's law regions sufficiently distant from us are expanding away from us faster than the speed of light, so that the expansion rate of the Universe continues to accelerate, there is a future visibility limit beyond which objects will *never* enter our observable universe at any time in the infinite future, because light emitted by objects outside that limit would never reach us"[4]."*Comoving distance* and *proper distance* are two closely related distance measures used by cosmologists to define distances between objects. *Proper distance* roughly corresponds to where a distant object would be at a specific moment of cosmological time, which can change over time due to the universe. Comoving *distance* factors out the expansion of the universe, giving a distance that does not change in time due to the expansion of space (though this may change due to other, local factors such as the motion of a galaxy within a cluster). Comoving distance and proper distance are defined to be equal at the present time; therefore, the ratio of proper distance to comoving distance now is 1. At other times, the scale factor differs from 1. The Universe's expansion results in the proper distance changing, while the comoving distance is unchanged by this expansion because it is the proper distance divided by that scale factor. The expanding Universe has an increasing scale factor which explains how constant comoving distances are reconciled with proper distances that increase with time"[5]. The Horizon distance in the flat Universe was found, mathematically, to be 14 Gpc. We did apply the same mathematical method , we had found the horizon distance in the Hyperbolic Universe to be infinitely.

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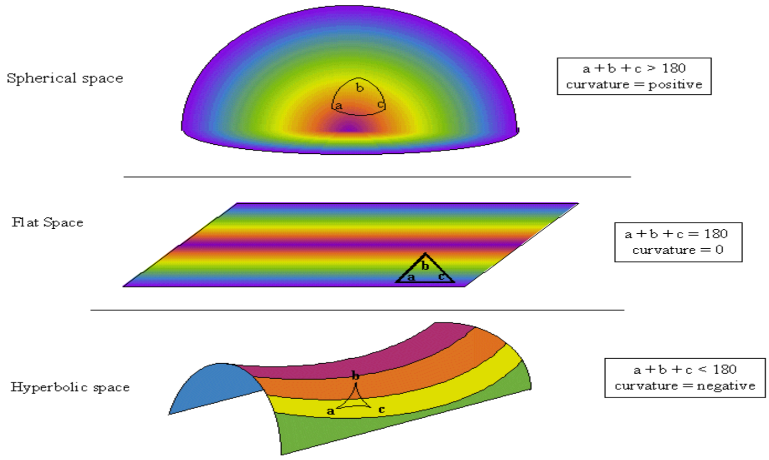

Figure 1. "Curvature" of the spacetime

2. Inflation

Inflation arose as a consequence to the false flat universe paradigm. Consider a photon moving along a radial trajectory in a flat universe. A radial null path obeys [6]

$$0 = ds^2 = -dt^2 + a^2(t)dr^2$$

$$r = \int_{t_e}^{t_0} \frac{dt}{a(t)}$$

For matter dominated component of energy :

$$a \propto t^{2/3}$$

Hubble constant now (at t_0) is

$$H_0 = H(t_0) = \frac{\dot{a}(t_0)}{a(t_0)}$$

So the age of the universe now is

$$t_0 = \frac{2}{3H_0}$$

$$\because H_0 = 72(km/s)/Mpc$$

$$\therefore t_0 = \frac{2}{3H_0} = 9Gyr$$

which is inconsistent compared to the age of the oldest stars whose age is about 12 Gyr in our galaxy. Equations due to the flat universe doesn`t fit the data. The flat universe must be updated by inflation, say. Instead of $R \propto t^{2/3}$, the scale factor should growths exponentially $R \propto e^{Ht}$ In physical cosmology, inflation is the theorized extremely rapid exponential expansion of the early universe by a factor of at least 10^{78} in volume driven by a negative-pressure vacuum energy density. "The inflationary epoch comprises the first part of the electroweak epoch following the grand unification epoch. It lasted from 10^{-36} seconds after the Big Bang to sometime between 10^{-33} and 10^{-32} seconds. Following the inflationary period, the universe continued to expand, but at a slower rate. Inflation proposed to fabricate answers to the classic conundrum of the Big Bang cosmology"[7]:

The horizon problem: "is the problem of determining why the universe appears statistically homogeneous and isotropic in accordance with the cosmological principle"[8]. The cosmic microwave background is the cooled remains of the radiation density from the radiation-dominated phase of the Big Bang. Observations of the "cosmic microwave background show that it is amazingly smooth in all directions, in other words, it is highly isotropic thermal radiation. The temperature of this thermal radiation is 2.73° Kelvin. The variations observed in this temperature across the night sky are very tiny. Radiation can only be so uniform if the photons have been mixed around a lot, or thermalized, through particle collisions. However, this presents a problem for the Big Bang model. Particle collisions cannot move information faster than the speed of light. But in the expanding Universe that we appear to live in, photons moving at the speed of light cannot get from one side of the Universe to the other in time to account for this observed isotropy in the thermal radiation”[9].

“If the cosmic microwave background is at such a uniform temperature, it should mean that the photons have been thermalized through repeated particle collisions. But this presents a problem with causality in an expanding universe. Using the Robertson-Walker metric with k=0, assuming that $a(t) \sim t^m$, the distance a photon could have traveled since the beginning of the Big Bang at t=0 to some other time t_0 is given by the horizon size $r_H(t_0)$

$$r_H(t_0) = \int_0^{t_0} \frac{dt}{a(t)} \sim \int_0^{t_0} t^{-m} dt \sim t^{1-m}\Big|_0^{t_0}$$

$$m < 1 \rightarrow r_H(t_0) \sim \frac{1}{H_0} << \infty$$

The power m is set by the equation of state for the energy source under consideration, so that

$$p = \kappa\rho \rightarrow a(t) \sim t^m, \quad m = \frac{2}{3(1+\kappa)}$$

For a matter or radiation dominated Universe, m=2/3 or 1/2, respectively. Therefore the horizon size is finite, because the integral converges as t -> 0 for m<1, and it is much smaller than necessary to account for the isotropy observed in the cosmic microwave background. To make the horizon integral diverge or grow extremely large would require a Universe that expanded more rapidly than is possible using matter or radiation in the Einstein equations”[10]. “The horizon size represents the “distance a photon can travel as the Universe expands. The horizon size of our Universe today is too small for the isotropy in the cosmic microwave background to have evolved naturally by thermalization”[9]. So that's the horizon problem.

“Both the de Sitter spacetime and the Robertson-Walker spacetime start expanding from a(t) close to zero. But for a spacetime with matter or radiation, a(t) goes to zero when the time t goes to zero, because a(t) goes like a power of t. When the scale factor depends exponentially on time, the scale factor goes to zero when time t goes to minus infinity. Therefore the horizon distance integral can blow up instead of neatly converge and solve the horizon problem”[10].

$$r_H(t_0) = \int_{-\infty}^{t_0} \frac{dt}{a(t)} = \int_{-\infty}^{t_0} e^{-Ht} dt \rightarrow \infty$$

The flatness problem : that the density of matter in the universe was comparable to the critical density necessary for a flat universe. “The Universe as observed today seems to enough energy density in the form of matter and cosmological constant to provide critical density and hence zero spatial curvature. The Einstein equation predicts that any deviation from flatness in an expanding Universe filled with matter or radiation only gets bigger as the Universe expands. So any tiny deviation from flatness at a much earlier time would have grown very large by now. If the deviation from flatness is very small now, it must have been immeasurably small at the start of the part of Big Bang we understand. So why did the Big Bang start off with the deviations from flat spatial geometry being immeasurably small? This is called the flatness problem of Big Bang cosmology. Whatever physics preceded the Big Bang left the Universe in this state. So the physics description of whatever happened before the Big Bang has to address the flatness problem.

Inflationary models also solve the horizon problem. The vacuum pressure accelerates the expansion of space in time so that a photon can traverse much more of space than it could in a spacetime filled with matter. To put it another way, the attractive force of matter on light in some sense slows the light down by slowing down the expansion of space itself. In an inflationary phase, the expansion of space is accelerated by vacuum pressure from the cosmological constant, and light gets farther faster because space is expanding faster. If there were an inflationary phase of our Universe before the radiation-dominated era of the Big Bang, then by the end of the inflationary period, light could have crossed the whole Universe. And so the isotropy of the radiation from the Big Bang would no longer be inconsistent with the finiteness of the speed of light”[9]. “Matter and radiation are gravitationally attractive, so in a maximally symmetric spacetime filled with matter, the gravitational force will inevitably cause any lumpiness in the matter to grow and condense. That's how hydrogen gas turned into galaxies and stars. But vacuum energy comes with a high vacuum pressure, and that high vacuum pressure resists gravitational collapse as a kind of repulsive gravitational force. The pressure of the vacuum energy flattens out the lumpiness, and makes space get flatter, not lumpier, as it expands. .

So one possible solution to the flatness problem would be if our Universe went through a phase where the only energy density presents was a uniform vacuum energy. The maximally symmetric solution to the Einstein equation under those conditions is called de Sitter space and the metric can be written

$$ds^2 = -dt^2 + e^{2Ht}(dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2))$$

$$H = \sqrt{\frac{\Lambda}{3}}$$

In de Sitter cosmology, the Hubble parameter H is constant and related to the cosmological constant as shown. The vacuum energy density is uniform in space and time, so the ratio of the curvature of space to the energy density will decrease exponentially as space expands in time:

$$\rho_\Lambda \sim a^0 \rightarrow \frac{k}{\rho_\Lambda a^2} \sim \frac{k}{a^2} = k\,e^{-2Ht}$$

Any deviations from flatness will be exponentially suppressed by the exponential expansion of the scale factor, and the flatness problem is solved"[10].

Inflationary universe: "matter and radiation are gravitationally attractive, so in a maximally symmetric spacetime filled with matter, the gravitational force will inevitably cause any lumpiness in the matter to grow and condense. That's how hydrogen gas turned into galaxies and stars. But vacuum energy comes with a high vacuum pressure, and that high vacuum pressure resists gravitational collapse as a kind of repulsive gravitational force. The pressure of the vacuum energy flattens out the lumpiness, and makes space get flatter, not lumpier, as it expands. So one possible solution to the flatness problem would be if our Universe went through a phase where the only energy density presents was a uniform vacuum energy. If this phase occurred before the radiation-dominated era, then the Universe could evolve to be extraordinarily flat when the radiation-dominated era began, so extraordinarily flat that the lumpy evolution of the radiation- and matter-dominated periods would be consistent with the high degree of remaining flatness that is observed today. This type of solution to the flatness problem was proposed in the 1980s by cosmologist Alan Guth. The model is called the Inflationary Universe. In the Inflation model, our Universe starts out as a rapidly expanding bubble of pure vacuum energy, with no matter or radiation. After a period of rapid expansion, or inflation, and rapid cooling, the potential energy in the vacuum is converted through particle physics processes into the kinetic energy of matter and radiation. The Universe heats up again and we get the standard Big Bang. So an inflationary phase before the Big Bang could explain how the Big Bang started with such extraordinary spatial flatness that it is still so close to being flat today"[9].

3. How does Inflation Work?.

Not like vacuum energy today, at the very early universe elementary particle interactions themselves could generate an inflationary expansion. "The vacuum energy that drives the rapid expansion in an inflationary cosmology comes from a scalar field that is part of the spontaneous symmetry breaking dynamics of some unified theory particle theory, say, a Grand Unified Theory or string theory. This scalar field is sometimes called the inflaton. The equation of motion for this field in the de Sitter metric above is:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

and the Einstein equation with a scalar field density becomes

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

The conditions for inflationary behavior require that the scalar field time derivatives are small compared to the potential, so that most of the energy of the scalar field is in potential energy and not kinetic energy[11]

$$\dot{\phi}^2 << V(\phi), \quad \left|\ddot{\phi}\right| << \left|3H\dot{\phi}\right|, \left|V'(\phi)\right|$$

4. BICEP2 and Planck space observatory

According to the team at the BICEP2 South Pole telescope, "the detection is at the 5–7 sigma level, so there is less than one chance in two millions of it being a random occurrence. The results were hailed as proof of the Big Bang inflationary theory and its progeny, the multiverse. The BICEP2 team identified a twisty (B-mode) pattern in its maps of polarization of the cosmic microwave background, concluding that this was a detection of primordial gravitational waves. Now, serious flaws in the analysis have been revealed that transform the sure detection into no detection. The search for gravitational waves must begin anew. The problem is that other effects, including light scattering from dust and the synchrotron radiation generated by electrons moving around galactic magnetic fields within our own Galaxy, can also produce these twists"[12]. Two groups of scientists announced that a "tantalizing signal smoking gun evidence of dramatic cosmic expansion just after the birth of the universe was actually caused by something much more mundane: interstellar dust. In the cosmic inflation announcement, which was unveiled in March 2014, scientists with the BICEP2 experiment, claimed to have found patterns in light left over from the Big Bang that indicated that space had rapidly inflated at the beginning of the universe, about 13.8 billion years ago. The discovery also supposedly confirmed the existence of gravitational waves, theoretical ripples in spacetime. Scientists with the European Space Agency said that data from the agency's Planck space observatory has revealed that interstellar dust caused more than half of the signal detected.

BICEP2, Planck and Keck all study the cosmic microwave background (CMB), or light that is left over from the Big Bang, and which can be seen in every direction in the sky. One feature of the CMB that these experiments study is its polarization, or the orientation of the light waves. If inflation did occur when the universe was born, it would have perturbed the fabric of the universe —spacetime — creating what are known as gravitational waves. These waves would have then created swirls in the polarization of the CMB, or what are called B-modes. Thus, the discovery of these B-modes would have meant both confirmation of inflation and evidence of gravitational waves. But the Planck results show that the light from dust is significant over the entire sky — including the region where BICEP2 purportedly observed B-modes. While BICEP2 only sees the sky in one wavelength of light, Planck observes the universe in nine wavelength channels, which help this instrument separate the CMB signal from the background. When the dust is accounted for, the signal identified by BICEP2 becomes too faint to be considered significant"[13].
"Dust is fairly common in the Milky Way, and it can also create B-mode polarization. Because the dust is between us and the

CMB, it can contaminate its B-mode signal. This is sometimes referred to as the foreground problem. To really prove you have evidence of B-mode polarization in the CMB, you must ensure that you've eliminated any foreground effects from your data"[14].

5.Criticisms (The inflationary paradigm is fundamentally untestable and hence scientifically meaningless).

Despite the prediction above, "inflation as described above is far from an ideal theory. It's too hard to stop the inflationary phase. Many of the assumptions that go into the model, such as an initial high temperature phase and a single inflating bubble have been questioned and alternative models have been developed. Today's inflation models have evolved beyond the original assumption of a single inflation event giving birth to a single Universe, and feature scenarios where universes nucleate and inflate out of other universes in the process called eternal inflation"[15].

There are hundreds of models of inflation, each with its own prediction about how fast the universe expanded and no one is much likely than the other. "The BICEP2 incident has also revealed a truth about inflationary theory. The common view is that it is a highly predictive theory. If that was the case and the detection of gravitational waves was the 'smoking gun' proof of inflation, one would think that non-detection means that the theory fails. Such is the nature of normal science. Yet some proponents of inflation who celebrated the BICEP2 announcement already insist that the theory is equally valid whether or not gravitational waves are detected. How is this possible? The answer given by proponents is alarming: the inflationary paradigm is so flexible that it is immune to experimental and observational tests. First, inflation is driven by a hypothetical scalar field, the inflaton, which has properties that can be adjusted to produce effectively any outcome. Second, inflation does not end with a universe with uniform properties, but almost inevitably leads to a multiverse with an infinite number of bubbles, in which the cosmic and physical properties vary from bubble to bubble. The part of the multiverse that we observe corresponds to a piece of just one such bubble. Scanning over all possible bubbles in the multiverse, everything that can physically happen does happen an infinite number of times. No experiment can rule out a theory that allows for all possible outcomes. Hence, the paradigm of inflation is *unfalsifiable* and scientifically meaningless. This may seem confusing given the hundreds of theoretical papers on the predictions of this or that inflationary model. What these papers typically fail to acknowledge is that they ignore the multiverse and that, even with this unjustified choice, there exists a spectrum of other models which produce all manner of diverse cosmological outcomes. Taking this into account, it is clear that the inflationary paradigm is fundamentally *untestable*, and hence scientifically *meaningless"*[12]. "Recent results from the *Planck* satellite combined with earlier observations from WMAP, ACT, SPT and other experiments eliminate a wide spectrum of more complex inflationary models and favor models with a single scalar field, as reported by the *Planck* Collaboration. More important, though, is that all the simplest inflation models are disfavored statistically relative to those with plateau-like potentials. We discuss how a restriction to plateau-like models has three independent serious drawbacks: it exacerbates both the initial conditions problem and the multiverse-unpredictability problem and it creates a new difficulty that we call the inflationary "unlikeliness problem". Finally, we comment on problems reconciling inflation with a standard model Higgs, as suggested by recent LHC results. In sum, we find that recent experimental data disfavors all the best-motivated inflationary scenarios and introduces new, serious difficulties that cut to the core of the inflationary paradigm. Forthcoming searches for B-modes, non-Gaussianity and new particles should be decisive"[16].

David Parkinson at the University of Queensland in Australia and his colleagues decided to look at the nature of those apparent gravitational waves to see if they were the type of waves predicted by inflation. And they weren't. Counter to what the BICEP2 collaboration said initially, Parkinson's analysis suggests the BICEP2 results actually rule out any reasonable form of inflationary theory. Most inflationary models require that as you look at larger and larger scales of the universe, you should see stronger and stronger gravitational waves. Cosmologists call that a "gravitational wave spectrum". "What inflation predicted was actually the reverse of what we found," says Parkinson. How many inflationary models does it rule out? "Most of them, to be honest"[17].

"Cosmic inflation is dead, long live cosmic inflation! Nobel laureate Brian Schmidt at the Australian National University in Canberra, who has been critical of the theory of inflation, says he expects that further analysis will confirm that no gravitational waves were observed at all. But on the other hand, if BICEP2 is shown to be correct, it's exciting, says Schmidt. And it does potentially break standard inflation and therefore you are testing inflation and showing its wrong"[17]. Paul Steinhardt of Princeton University, who helped develop inflationary theory but is now scathing of it, says this is potentially a blow for the theory, but that it pales in significance with inflation's other problems. Steinhardt says the idea that inflationary theory produces any observable predictions at all – even those potentially tested by BICEP2 – is based on a simplification of the theory that simply does not hold true. "The deeper problem is that once inflation starts, it doesn't end the way these simplistic calculations suggest," he says. "Instead, due to quantum physics it leads to a multiverse where the universe breaks up into an infinite number of patches. The patches explore all conceivable properties as you go from patch to patch. So that means it doesn't make any sense to say *what* inflation predicts, except to say it predicts *everything*. Steinhardt says the point of inflation was to explain a remarkably simple universe. "So the last thing in the world you should be doing is introducing a multiverse of possibilities to explain such a simple thing," he says. "I think it's telling us in the clearest possible terms that we should be able to understand this and when we understand it it's going to come in a model that is extremely simple and compelling. And we thought inflation was it – but it isn't."[18]

6. The distance horizon in a flat universe

Consider a photon moving along a radial trajectory in a flat universe. A radial null path obeys

$$o = ds^2 = -dt^2 + a^2(t)dr^2$$

$$r = \int_{t_e}^{t_0} \frac{dt}{a(t)}$$

For matter dominated component of energy :

$$R \propto t^{2/3}$$

Hubble constant now (at t_0) is

$$H_0 = H(t_0) = \frac{\dot{a}(t_0)}{a(t_0)}$$

So the age of the universe now is

$$t_0 = \frac{2}{3H_0}$$

$$\because H_0 = 72(km/s)/Mpc$$

$$\therefore t_0 = \frac{2}{3H_0} = 9Gyr$$

which is inconsistent compared to the age of the oldest stars whose age is about 12 Gyr in our galaxy. Equations due to the flat universe doesn`t fit the data. The physical distance to the horizon-in flat FRW model- at the time of observations is

$$d_H(t) = a(t)r_H = a(t)\int_0^t \frac{dt'}{a(t')}$$

$$\because a(t) \propto t^{2/3}$$

$$\therefore d_H(t) = a(t)r_H = t^{2/3}\int_0^t \frac{dt'}{t'^{2/3}} = 3t$$

The present horizon size of a matter dominated flat universe

$$d_{horizon}(t_0) = 3t_0 = 3\left(\frac{2}{3}t_H\right)$$

$$d_{horizon}(t_0) = 2t_H \approx 8Gpc$$

The discrepancy between this number and the 14 Gpc (observed radius in principle) is due to the presence of the significant vacuum energy (dark energy). Note that Hubble radius

$$H_0^{-1} = 4.2 \times 10^3 Mpc.$$

7. The Hyperbolic Spacetime

To obtain the dynamical equation of cosmology, we should combine Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}$$

with the isotropic homogeneous Robertson- Walker's line-element:

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

to get Friedmann`s equations:

$$\dot{R}^2 + k = (8\pi/3)\rho R^2 \qquad (1)$$

$$2R\ddot{R} + \dot{R}^2 + k = -8\pi p \qquad (2)$$

Where p is the pressure and ρ is the energy density of the cosmological fluid and k is the curvature.

Method of solution:

(i) Now we shall solve the differential equation (1) by separating the variables. We assume the Big Bang Model as an initial condition (i.e. R=0 when t=0).

$$\dot{R}^2 + k = (8\pi/3)\rho R^2$$

$$\dot{R}^2 = (8\pi/3)\rho R^2 - k$$

$$\dot{R} = \sqrt{\left(8\pi\rho/3\right)R^2 - k}$$

$$dR/\sqrt{\left(8\pi\rho/3\right)R^2 - k} = dt$$

$$dR/\sqrt{R^2 - 3k/8\pi\rho} = \sqrt{8\pi\rho/3}\,dt$$

Differential equation (1) allows one to deal with ρ as a parameter since it's not an explicit function of t , so Eq. (1) can be solved for any chosen fixed value , ρ_j ,from the stream of the various values of the parameter ρ :

$$\rho_1, \rho_2, \ldots, \rho_{planck}, \ldots, \rho_j, \ldots, \rho_{now}$$

By means of the mean value theorem, we assume approximately that ρ_j evolves to the fixed physical value ρ_j exactly simultaneously associated to the state (t_j, R_j)since ρ_j is not defined and not continuous at the point of singularity $t = 0$, put

$$\rho(\mathrm{c}) = \rho_{\mathrm{j}} : 0 < \mathrm{c} \le \mathrm{t}$$

$$\int_0^R dr/\sqrt{r^2 - 3k/8\pi\rho_{\mathrm{j}}} = \sqrt{8\pi\rho_{\mathrm{j}}/3}\int_0^t dt$$

We get

$$\cosh^{-1}(R/\sqrt{3k/8\pi\rho_{\mathrm{j}}}) - \cosh^{-1}0 = t\sqrt{8\pi\rho_{\mathrm{j}}/3} \qquad (3)$$

Now we use complex analysis as follows[19]:

$$\cosh^{-1}x = \ln(x \pm \sqrt{x^2 - 1})$$

$$\cosh^{-1}0 = \ln\pm\sqrt{-1} = \ln\sqrt{-1}, or, \cosh^{-1}0 = \ln -\sqrt{-1}$$

$$\cosh^{-1}0 = (1/2)\ln(-1), or, \cosh^{-1}0 = \ln(-1) + (1/2)\ln(-1)$$

$$\because e^{ip} = -1$$

$$\backslash\ \ln(-1) = ip$$

$$\backslash\ \cosh^{-1}0 = ip/2, or$$

$$\cosh^{-1}0 = 3ip/2$$

Substitute the first value $\cosh^{-1}0 = ip/2$ in equation (3), we get:

$$R(t) = \sqrt{3k/8pr_{j}}.\cosh(t\sqrt{8pr_{j}/3} + pi/2)$$

$$R(t) = \sqrt{3k/8pr_{j}}.(\cosh t\sqrt{8pr_{j}/3}.\cosh(pi/2) + \sinh(pi/2).\sinh t\sqrt{8pr_{j}/3})$$

$$R(t) = \sqrt{3k/8pr_{j}}.(\cosh t\sqrt{8pr_{j}/3}.\cos(p/2) + i\sin(p/2).\sinh t\sqrt{8pr_{j}/3})$$

$$R(t) = i\sqrt{3k/8pr_{j}}.\sinh t\sqrt{8pr_{j}/3}$$

Since the function $r(t)$ is always positive, so is any chosen fixed value r_j. A simple analysis shows that the R(t) scale solution represented in the last equation is complex if k is positive, negative if k is negative and vanishes if k is zero. So the first value $\cosh^{-1}0 = ip/2$ is rejected.

Substitute the other value $\cosh^{-1}0 = 3ip/2$ in equation (3), we get:

$$R(t) = \sqrt{3k/8pr_{j}}.\cosh(t\sqrt{8pr_{j}/3} + 3pi/2)$$

$$R(t) = \sqrt{3k/8pr_{j}}(\cosh t\sqrt{8pr_{j}/3}.\cosh(3pi/2) + \sinh(3pi/2)\sinh t\sqrt{8pr_{j}/3})$$

$$R(t) = \sqrt{3k/8pr_{j}}(\cosh t\sqrt{8pr_{j}/3}.\cos(3p/2) + i\sin(3p/2)\sinh t\sqrt{8pr_{j}/3})$$

$$R(t) = -i\sqrt{3k/8pr_{j}}\sinh t\sqrt{8pr_{j}/3}$$

The R(t) scale solution in the last equation is real, positive and non-vanishing if and only if k is negative. Since k is normalized, substitute k=-1, in the last equation, we get: .

$$R(t) = -i\sqrt{3k/8pr_{j}}\sinh t\sqrt{8pr_{j}/3}$$

$$R(t) = -i\sqrt{-3/8pr_{j}}\sinh t\sqrt{8pr_{j}/3}$$

$$R(t) = -i.i\sqrt{3/8pr_{j}}\sinh t\sqrt{8pr_{j}/3}$$

$$R(t) = \sqrt{3/8pr_{j}}\sinh t\sqrt{8pr_{j}/3} \qquad (4)$$

Which mean that $R(t)$ either vanishes if k = 0 or complex if k =1 . Thus, the curvature k must be negative and consequently the universe must be hyperbolic and open. .

Note that the solution represented by Eq. (4) is evaluated only for the values simultaneously associated with r_j, namely (R_j, t_j)

$$R_j = \sqrt{3/8pr_j}\sinh t_j\sqrt{8pr_j/3} \qquad (5)$$

Verification:

The above scale factor can be verified even at the Planck scale and the current scale as follows:

(i)Let us apply equation (5) at Planck scale. To do this we substitute a given Planck time and Planck density in equation (5), while we assume Planck length is unknown.

Note that in geometrical units:-

The speed of light $c = 1$

1 sec = 2.997×10^{10}cm

1 gram = 7.425×10^{-29}cm

1 eV = 1.324×10^{-61}cm.

And we have Planck scale from the following

Planck time = 5.4×10^{-44}s

Planck length = 1.62×10^{-33}cm

Planck mass = 1.2×10^{25}eV/c^2

Planck density = M/V = M/L^3 = $1.2\times10^{25}(\text{eV}/c^2)/(1.6\times10^{-33})^3$

$=1.2\times10^{25}(1.324\times10^{-61}\text{cm})/(1.6\times10^{-33})^3=3.8789\times10^{62}$cm

Recall equation (4) and put k = -1 we get:-

$$R_p = \sqrt{3/8\pi\rho_p}\,\sinh\sqrt{8\pi\rho_p/3}\,t_p$$

$$R_p = \sqrt{3/8\pi\times3.8789\times10^{62}}\,\sinh\sqrt{8\pi\times3.8789\times10^{62}/3}\times5.4\times10^{-44}\times2.997\times10^{10}$$

$= 0.175423 \times 10^{-31} \times \sinh 0.092255888$

$= 0.175423\times10^{-31} \times 0.092386811$

$= 1.62 \times 10^{-33}$ cm = L_p = Planck length.

(ii) The current scale

Note that in geometrical units

1 sec = 2.997×10^{10}cm

1 gram = 7.425×10^{-29}cm

1 yr = 3.16×10^{7} s

The energy density now ρ_{now} $=10^{-31}$ g/cm^3 $=7.425\times10^{-60}$cm^{-2}

The age of the Universe t_{now} $=14\times10^{9}$ yr$=1.32587\times10^{28}$ cm

Substitute the above data in the hyperbolic time evolution equation of the Universe, yields

$$a_j = \sqrt{3/8\pi\rho_j}\,\sinh\left[t_j\sqrt{8\pi\rho_j/3}\right]$$

$$a_{now} = \sqrt{3/8\pi\rho_{now}}\,\sinh\left[t_{now}\sqrt{8\pi\rho_{now}/3}\right]$$

$$a_{now} = \sqrt{3/\left(8\pi\times7.425\times10^{-60}\right)}\times$$

$$\sinh\left[1.32587\times10^{28}\times\sqrt{8\pi\times7.425\times10^{-60}/3}\right]$$

$$a_{now} = 1.6\times10^{29}\times\sinh 0.08287$$

$$a_{now} = 1.3\times10^{28}\,cm$$

8. The Hyperbolic universe possesses an accelerated expansion

We shall see that the solution of equation (1) satisfies the second order differential equation (2) in order to be consistent .We have from the solution of Eq. (1) for any chosen value ρ_j

$$R = \sqrt{\frac{3}{8\pi\rho_j}}\,\sinh t\sqrt{\frac{8\pi\rho_j}{3}}$$

$$\dot{R} = \cosh t\sqrt{\frac{8\pi\rho_j}{3}}$$

$$\ddot{R} = \sqrt{\frac{8\pi\rho_j}{3}}\,\sinh t\sqrt{\frac{8\pi\rho_j}{3}} = \frac{8\pi\rho_j}{3}R$$

Substitute these values in Eq. (2), and k = -1, yields:

$$2R\ddot{R}+\dot{R}^2-1=-8\pi pR^2$$

$$2R\left(\frac{8\pi\rho_j}{3}R\right)+\cosh^2\sqrt{\frac{8\pi\rho_j}{3}}-1=-8\pi pR^2$$

$$2R^2\left(\frac{8\pi\rho_j}{3}\right)+\sinh^2\sqrt{\frac{8\pi\rho_j}{3}}=-8\pi pR^2$$

$$2R^2\left(\frac{8\pi\rho_j}{3}\right)+\frac{8\pi\rho_j}{3}R^2=-8\pi pR^2$$

$$8\pi\rho_j R^2=-8\pi pR^2$$

$$p=-\rho_j$$

The last equation is known as the equation of state of cosmology. The argument of the solution predicts the equation of state of cosmology $p=-\rho_j$. Since the energy density is always positive, the negative pressure implies an accelerated expansion of the universe. Hence equations (1) and (2) are consistent for any chosen fixed value ρ_j of the parameter ρ. The argument of the solution predicts the equation of state $p=-\rho_j$

9. Hyperbolic expansion of the universe possesses a legitimate inflation

We exhibit the hyperbolic structure of the universe that explains the accelerating expansion of the universe without needs for an additional components, dark energy. One explanation for dark energy is that it is a property of space. The simplest explanation for dark energy is that it is simply the "cost of having space": that is, a volume of space has some intrinsic, fundamental energy. Just the ordinary energy density state ρ_j remains in the Hyperbolic Universe to derive the accelerating expansion equivalent to its negative pressure. Hyperbolic Universe involves zero cosmological constant (the vacuum energy). The negative pressure $p=-\rho_j$ is the property of the hyperbolic structure of the Universe. Flat universe dominated by matter is modeled as a zero pressure-dust universe model, and the expansion of the universe would be slowing due to the gravity attraction. Which is incorrect, as we shall see below:

Einstein postulates[13] that the matter dominated universe could be modeled as dust with zero pressure in order to simplify and solves Friedmann`s equations

.

$$\dot{R}^2+k=(8\pi/3)\rho R^2$$

$$2R\ddot{R}+\dot{R}^2+k=0$$

$$2\ddot{R}/R+(8\pi/3)\rho=0$$

$$\ddot{R}=-(8\pi/3)\rho R/2<0$$

$$\therefore\ \ddot{R}<0$$

The pressure less form of Eq. (2) describes a decelerating expansion state of the universe which is described by the energy tensor of matter for dust where $p=0$. We solved the second dynamical equation of cosmology, the space-space component; in it is pressure less form:

$$2R\ddot{R}+\dot{R}^2+k=0$$

$$\because k=-1$$

$$2R\ddot{R}+\dot{R}^2-1=0$$

to be $t=R$, which satisfies the last differential equation.

Substitute $t=R, k=-1$ in the first dynamical equation (2)

$$\dot{R}^2 + k = (8p/3)r R^2$$

$$(1)^2 - 1 = (8p/3)r R^2$$

$$0 = (8p/3)r R^2$$

$$\backslash \ r = 0$$

Hence the zero pressure does not lead to a dusty universe. In fact zero pressure Universe is an empty space, since $r = 0$.

In the presence of pressure, from Eqs. (2) and (1) we can obtain

$$\ddot{R} = -\frac{4p}{3}(r + 3p)R$$

$$\because p = -r$$

$$\backslash \ \ddot{R} = \frac{8p}{3}R > 0$$

which guarantees an accelerating expansion of the universe.

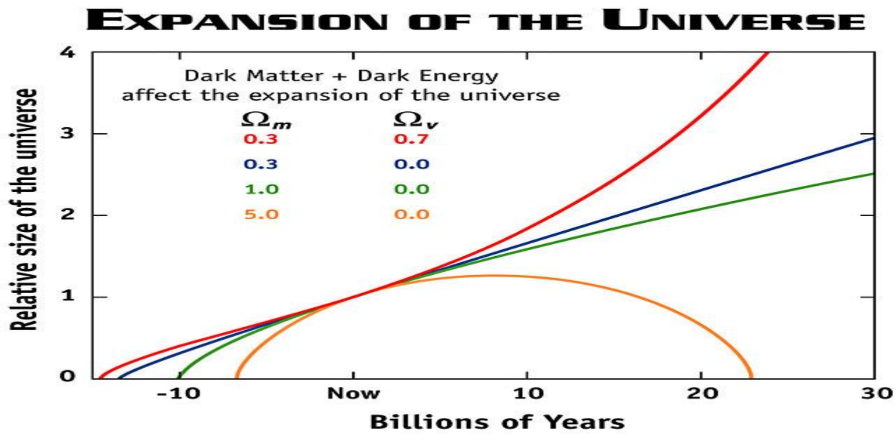

Figure 2. Scientists used to think that the universe was described by the yellow, green, or blue curves. But surprise, it's actually the red *curve* instead.

Newton first law states that the body keep moving with a uniform velocity in straight line. Similarly, the free fall of an object in a flat spacetime is uniform. An accelerated motion is described by a curve. For large structure, the curvature of the spacetime can't be ignored. *It is clear from Fig (2) the expansion of the universe is described by a hyperbolic curve*. The distant objects- e.g. supernovae - were influenced under the curvature of the spacetime. They possess an accelerating free fall due to the curvature of the hyperbolic spacetime, that manifests itself by the equation of the state $p = -r$ which is the property of the hyperbolic structure of the Universe. The Universe is not flat. We did prove that, the universe globally is hyperbolic. The hyperbolic universe doesn't need dark energy to account for the accelerating expansion. The equation of the state $p = -r$ associated with the hyperbolic universe, derives such an accelerated expansion.

Inflation *fabricates* an initial conditions to predict a flat Universe. The flat universe assumption arose due to the local observations which show that the universe is approximately flat restricted to the existence of pointless dark energy and dark matter and illegitimately generalized to the whole *global* geometry of the universe. Although general relativity says space-time locally is approximately flat but that doesn't imply it must be globally flat. Such problematic flat universe paradigm needs inflation to stay alive. The hyperbolic evolution of the Universe exhibits rational and reasonable inflation covers the large structure, isotropic and homogeneity in accordance with the cosmological principle. So, the flatness problem does no longer exist in the Hyperbolic Universe. Rather than the unjustified magical inflation that occurs within sometime between 10^{-33} and 10^{-32} seconds cause the universe expand to an order 10^{50} , we did prove the hyperbolic geometry of the universe. Such a Hyperbolic Universe possesses a reasonable hyperbolic inflation through its whole age. The Hyperbolic Universe inflate, throu gh the hyperbolic time evolution equation below, legitimately to 10^{28} cm, very consistent with the current observable universe

The Hyperbolic Universe scale factor grows exponentially preserve a legitimate inflation covers the current observed large structure. Hence, the horizon problem also does no longer exist, since the backward exponential contraction re-put both sides of the Universe at causal contact. “Penrose said if $k ¹ o$, then inflation is out !” [20]. Instead of unphysical inflation epoch the Hyperbolic Universe grows exponentially preserve a legitimate inflation covers the current observ ed large structure (10^{28} cm)[21].

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10.Equation of the radial motion in the galaxy`s hyperbolic spacetime

Flat rotation curve: “The galaxy rotation problem is the discrepancy between observed galaxy rotation curves and the ones predicted assuming a centrally-dominated mass that follows the luminous material observed. If masses of galaxies are derived solely from the luminosities and the mass-to-light ratios in the disk and core portions of spiral galaxies are assumed to be close to that of stars, the masses derived from the kinematics of the observed rotation do not match”[22].

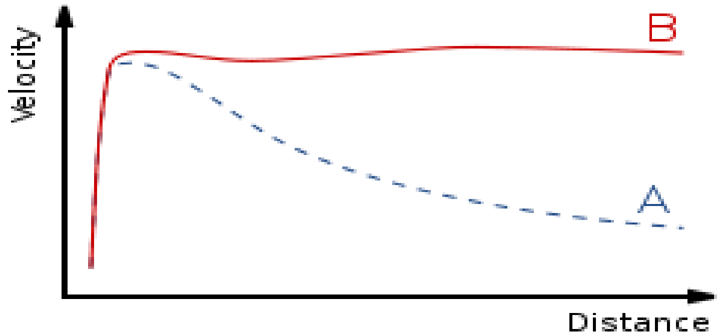

Figure 3. Rotation curve of a typical spiral galaxy: predicted (A) and observed (B). The discrepancy between the curves can be accounted for by adding a dark matter component to the galaxy.

We develop an equation describes the speed up motion in the hyperbolic spacetime and predicts the flat curve. To do this, I will follow the following strategy

1- Seek for an equation $v = f(r)$ such that $v = \lim_{r \to 0} f(r) = 0$

2- $$v = f(r) \xrightarrow{\text{large } r} \sqrt{m\left(\frac{2}{r} - \frac{1}{a}\right)} \xrightarrow{r \to \infty} \sqrt{-\frac{m}{a}}$$

3- I guess the required equation –that fits the data- should be

$$v = f(r) = e^{-\frac{1}{r}} \sqrt{m\left(\frac{2}{r} - \frac{1}{a}\right)},$$

4- The final step in the mathematical problem solving method is to prove the conjecture

$$v = f(r) = e^{-\frac{1}{r}} \sqrt{m\left(\frac{2}{r} - \frac{1}{a}\right)}$$

To find such an equation of the radial motion in the galaxy`s hyperbolic space-time, we proceed as follows: The required modified Schwarzschild spherically symmetric metric will be

$$d\tau^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 d\Omega^2$$

$$d\tau^2 = \left(1 + \nu + (1/2)\nu^2 + \ldots\right)dt^2 - \left(1 + \lambda + (1/2)\lambda^2 + \ldots\right)dr^2 - r^2 d\Omega^2$$

for which the Schwarzschild metric

$$d\tau^2 = \left(e^{\nu}\right)dt^2 - \left(e^{\lambda}\right)dr^2 - r^2 d\Omega^2$$

$$d\tau^2 \approx (1 + \nu)dt^2 - (1 + \lambda)dr^2 - r^2 d\Omega^2$$

is just an approximation . The Ricci tensor

$$0 = R_{tt} = -(1/2)e^{\nu - \lambda}\left(\nu'' + \nu'^2/2 - \nu'\lambda'/2 + 2\nu'/r\right), \ldots(i)$$

$$0 = R_{rr} = (1/2)\left(\nu'' + \nu'^2/2 - \nu'\lambda'/2 + 2\lambda'/r\right), \ldots\ldots\ldots\ldots(ii)$$

$$0 = R_{\theta\theta} = -\left\{1 - \left(e^{-\lambda} r\right)' + e^{-\lambda} r\left(\frac{\nu' + \lambda'}{2}\right)\right\}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

From $R_{tt} = R_{\theta\theta} = 0$ web have $\nu' + \lambda' = 0$, so $\nu + \lambda = k(\text{constant})$

Write simply

$\lambda = -\nu + \log k$. Equation (i) is now just

$$(e^{\nu} r)'' = 0$$

$$e^{\nu} r = -a + br$$

Equation (iii) is

$$(e^{-\lambda} r)' = 1$$

$$(e^{\nu} r)' = k$$

$$\therefore\ b = k$$

We have now the complete solution

$$e^{\lambda} = (1 - 2m/kr)^{-1} \approx \left(e^{-2m/kr}\right)^{-1} = e^{2m/kr}$$

$$e^{\nu} = k(1 - a/kr) = k(1 - 2m/kr) = (k - 2m/r)$$

For radial motion, $d\Omega^2 = 0$. The Schwarzschild metric will be

$$d\tau^2 = e^{\nu} dt^2 - e^{\lambda} dr^2$$

$$d\tau^2 = (k - 2m/r)dt^2 - e^{2m/kr} dr^2$$

The free fall from rest of a star (of mass m and energy E) far from the center possesses[15]

$$\frac{E}{m} = \left(1 - \frac{2m}{r}\right)\frac{dt}{d\tau} = 1$$

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{2m}{r}\right)^2$$

$$\left(\frac{d\tau}{dt}\right)^2 = (k - 2m/r) - e^{2m/kr}\left(\frac{dr}{dt}\right)^2$$

$$(1 - 2m/r)^2 = (k - 2m/r) - e^{2m/kr}\left(\frac{dr}{dt}\right)^2$$

To our purpose for the hyperbolic space- time, the velocity far away from the center would be $V_{\infty} = \sqrt{-m/a}$ and consequently $k = 1 - m/a$

$$\left(1 - 4m/r + (2m/r)^2\right) = (1 - 2m/r - m/a) - e^{2m/\left[(1-m/a)r\right]}V^2$$

neglect the term $(2m/r)^2$ and rearrange

$$(1 - 4m/r) = (1 - 2m/r - m/a) - e^{2m/\left[(1-m/a)r\right]}V^2$$

$$e^{2m/\left[(1-m/a)r\right]}V^2 = (2m/r - m/a)$$

$$V = e^{-m/\left[(1-m/a)r\right]}\sqrt{2m/r - m/a}$$

$$V = e^{-am/[(a-m)r]}\sqrt{2m/r - m/a}$$

$$\because -a >> m$$

$$\therefore a - m \approx a$$

$$(1 - m/a)(km/s) \approx 1(km/s)$$

$$V = e^{-m/r}\sqrt{2m/r - m/a}$$

Example

A typical galaxy of ordinary enclosed mass (Milky way or Andromeda)

$$M = 10^{11} M_{\circ} = 10^{11} \times 2 \times 10^{30} kg$$

$$m = 10^{11} \times 2 \times 10^{30} \times 7.4 \times 10^{-31} km$$

$$m = 1.5 \times 10^{11} km$$

$$m = 1.5 \times 10^{11} km \times (s/s)$$

$$m = 1.5 \times 10^{11} km \times (3 \times 10^{5} km/s)$$

$$m = 4.5 \times 10^{16} (km^2/s)$$

$$210 = \sqrt{m/-a}$$

$$(210)^2 = m/-a$$

$$V = e^{m/r(kpc)}\sqrt{2m/r(kpc) - m/a}$$

$$V = e^{-4.5 \times 10^{16}(km^2/s)/(1(km/s) \times r(3.1 \times 10^{16} km))} \times$$

$$\sqrt{9 \times 10^{16}(km^2/s)/(1(km/s) \times r(3.1 \times 10^{16} km)) + 210^2}$$

$$V = e^{-1.45}\sqrt{3/r + 210^2}$$

The curve of the last equation is drown by visual mathematics program:

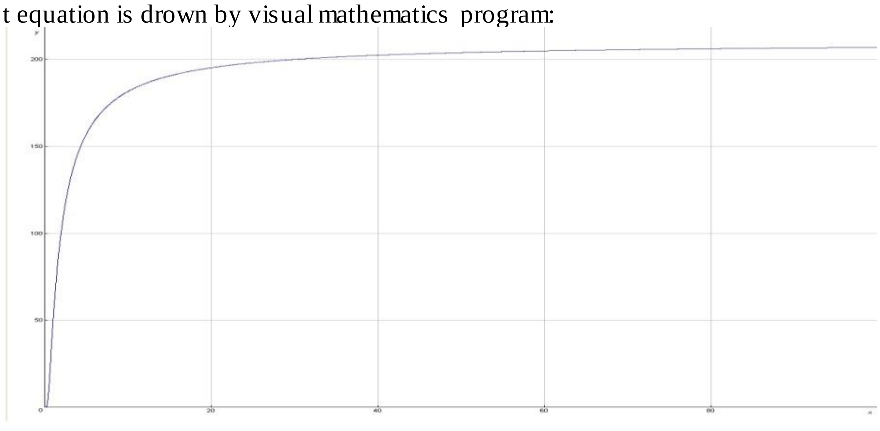

Figure 4. The curve describes the motion of a star in the Milky way (or Andromeda) galaxy. The vertical axis represents the velocity, while the horizontal axis represents the distance from the center of the galaxy.

A typical cluster of galaxies of ordinary enclosed mass

$$M = 10^{14} M_{\odot} = 10^{14} \times 2 \times 10^{30} kg$$

$$m = 10^{14} \times 2 \times 10^{30} \times 7.4 \times 10^{-31} km$$

$$m = 1.5 \times 10^{14} km$$

$$m = 1.5 \times 10^{14} km \times (3 \times 10^{5} km/s)$$

$$m = 4.5 \times 10^{19} (km^2/s)$$

$$1000 = \sqrt{m/-a}$$

$$(1000)^2 = m/-a$$

$$V = e^{-m/(r\mathrm{M}pc)} \sqrt{2m/r(10\mathrm{M}pc) - m/a}$$

$$V = e^{-4.5 \times 10^{19}/(3.1 \times 10^{19} \times r)} \sqrt{2 \times 4.5 \times 10^{19}/(3.1 \times 10^{19} \times r) + (1000)^2}\,(km/s)$$

$$V = e^{-1.45/r} \sqrt{3/r + (1000)^2}$$

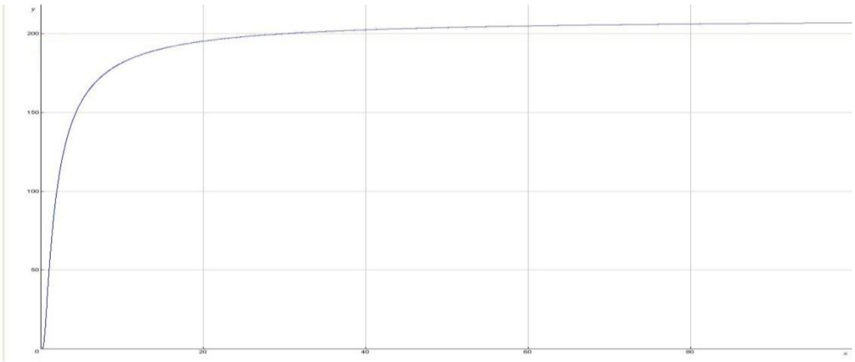

Figure 5. The curve describes the motion of a cluster of galaxies. The vertical axis represents the velocity, while the horizontal axis represents the distance from the center of the cluster.

"The dark matter halo is nothing but instead of it we have a cell of same hyperbolic negative curvature as the negative curvature of the whole Hyperbolic Universe. Virial theorem $(M = V^2R/G)$ does no longer hold for Non-Euclidian space. We developed the equation of motion in the hyperbolic space-time : $V = e^{-m/r}\sqrt{m(2/r - 1/a)}$, that describes the speed up motion in the hyperbolic space-time and predicts the flat curve. Farther away from the center the exponential factor $e^{-1/r}$ drops to one. Galaxies furthest away from the center are moving fastest until they reached large distance from the center the space-time turns flat and they possessed hyperbolic trajectory: $V = \sqrt{m(2/r - 1/a)}$, according to Vallado theorem, with constant speed called hyperbolic excess velocity: $V_{\infty} = \sqrt{-m/a}$ that can solve the flat rotation curve problem, where a is the negative semi major axis of orbit`s hyperbola"[2].

11. Look-Back Time

"Because the Universe is expanding the distance to a galaxy is not very well defined. Because of this ambiguity, astronomers prefer to work in terms of a look-back time, which is simply how long ago an object emitted the radiation we see today. Astronomers talk frequently about redshifts and sometimes about look-back times, but they hardly ever talk of distances to high-redshift objects. The redshift is the only unambiguously measured quantity. Statements about derived quantities, such as distances and look-back times, all require that we make specific assumptions about how the universe has evolved with time. For nearby sources, the look-back time is numerically equal to the distance in light-years. However, for more distant objects, the look-back time and the present distance in light–years differ because of the expansion of the universe, and the divergence increase dramatically with increasing redshift"[23].

12. The distance horizon in a hyperbolic universe.

Comoving distance "is the distance between two points measured along a path defined at the present cosmological time. For objects moving with the Hubble flow, it is deemed to remain constant in time. The comoving distance from an observer to a distant object (e.g. galaxy) can be computed by the following formula:

$$c = \int_{t_e}^{t_0} \frac{dt'}{a(t')}$$

where $a(t')$ is the scale factor, t_e is the time of emission of the photons detected by the observer, t is the present time, and c is the speed of light in vacuum. Despite being an integral over time, this does give the distance that *would* be measured by a hypothetical tape measure at *fixed* time t, i.e. the "proper distance" as defined below, divided by the scale factor $a(t)$ at that time"[24].

The isotropic homogeneous Robertson- Walker's line-element:

$$d\tau^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

For the hyperbolic spacetime

$$k = -1$$

$$d\tau^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)$$

For radial null trajectory

$$0 = -dt^2 + a^2(t)\left(\frac{dr^2}{1+r^2}\right)$$

$$\frac{dr}{\sqrt{1+r^2}} = \frac{dt}{a(t)}$$

The physical radius to the horizon-in hyperbolic universe FRW model- at the time of observations is

$$\int_0^r \frac{dr'}{\sqrt{1+r'^2}} = \int_0^t \frac{dt'}{a(t')} \qquad (6)$$

We have the scale factor in the hyperbolic universe

$$a(t) = \sqrt{3/8\pi\rho_\varphi}\,\sinh t\sqrt{8\pi\rho_\varphi/3}$$

Substitute this scale factor in Eq.(6), we get

$$\int_0^r \frac{dr'}{\sqrt{1+r'^2}} = \int_0^t \frac{dt'}{\sqrt{3/8\pi\rho_\varphi}\,\sinh t'\sqrt{8\pi\rho_\varphi/3}}$$

$$\int_0^r \frac{dr'}{\sqrt{1+r'^2}} = \sqrt{8\pi\rho_\varphi/3}\int_0^t \frac{dt'}{\sinh t'\sqrt{8\pi\rho_\varphi/3}}$$

$$\int_0^r \frac{dr'}{\sqrt{1+r'^2}} = \sqrt{8\pi\rho_\varphi/3}\int_0^t coseh t'\sqrt{8\pi\rho_\varphi/3}\,dt'$$

$$\sin h^{-1} r'\Big|_0^r = \coth^{-1}\cosh t'\sqrt{8\pi\rho_\varphi/3}\Big|_0^t$$

$$\therefore \sin h^{-1} r - \sin sh^{-1} 0$$

$$= \coth^{-1}\cosh t\sqrt{8\pi\rho_\varphi/3} - \coth^{-1}\cosh 0$$

$$\therefore \ln\left|r \pm \sqrt{1+r^2}\right| - \ln\left|\pm 1\right|$$

$$= \frac{1}{2}\ln\left|\frac{\cosh t\sqrt{8pr_j/3}+1}{\cosh t\sqrt{8pr_j/3}-1}\right| - \frac{1}{2}\ln\left|\frac{\cosh 0+1}{\cosh 0-1}\right|$$

$$\therefore \ln\left|\frac{\left|r \pm \sqrt{1+r^2}\right|}{\pm 1}\right|$$

$$= \ln\left|\frac{\sqrt{(\cosh t\sqrt{8pr_j/3}+1)/(\cosh t\sqrt{8pr_j/3}-1)}}{\sqrt{(\cosh 0+1)/(\cosh 0-1)}}\right|$$

$$\therefore r \pm \sqrt{1+r^2} = \frac{\sqrt{(\cosh t\sqrt{8pr_j/3}+1)/(\cosh t\sqrt{8pr_j/3}-1)}}{\sqrt{(2)/(0)}}$$

$$\therefore r \pm \sqrt{1+r^2} = \frac{\sqrt{(\cosh t\sqrt{8pr_j/3}+1)/(\cosh t\sqrt{8pr_j/3}-1)}}{\infty}$$

$$\therefore r \pm \sqrt{1+r^2} = 0$$

$$\therefore r = \mp\sqrt{1+r^2}$$

$$\therefore r^2 = 1+r^2$$

$$\therefore r_{horizon} = \pm \infty$$

The distance horizon in the hyperbolic universe is infinite, simply solves the horizon problem.

13. Conclusion

In testing the validity of any scientific paradigm, the key criterion is whether measurements agree with what is expected given the paradigm[1].

- Our Hyperbolic Universe paradigm agrees with observations and accounts for the acceleration expansion of the universe due to the hyperbolic property $p = -r$ and the flat rotation curve according to the equation $V = e^{-m/r}\sqrt{2m/r - m/a}$.

- Moreover Our mathematical model exactly predicts - through its hyperbolic scale factor - the Planck length at quantum scale and the current large structure of the universe.

- In comparison with other models such as modifying gravity models, fail to agree with observations or those need additional assumptions such as dark energy or dark matter. Our model seems to be aesthetically pleasing and preferred according to the Criteria for scientific method and Occam's razor as stated below:

Criteria for scientific method

(i) The model must fit the data and agree with observations.

(ii) The model must make predictions that allow it to be tested (falsifiable).

(iii) The model should be aesthetically pleasing (Occam's razor)

Occam's razor [25]:

"If you have two theories that both explain the observed facts, then you should use the simplest until more evidence comes along"

"The simplest explanation for some phenomenon is more likely to be accurate than more complicated explanations."

"If you have two equally likely solutions to a problem, choose the simplest."

"The explanation requiring the fewest assumptions is most likely to be correct."

- Despite the prediction above, inflation as described above is far from an ideal theory. It's too hard to stop the inflationary phase. Many of the assumptions that go into the model, such as an initial high temperature phase and a single inflating bubble have been questioned and alternative models have been developed. Today's inflation models have evolved beyond the original assumption of a single inflation event giving birth to a single Universe, and feature scenarios where universes nucleate and inflate out of other universes in the process called eternal inflation.

- Instead of the unphysical inflation, the Hyperbolic universe possesses a legitimate hyperbolic inflation cover the current observed large structure.

-Although the perspective for nearby objects in hyperbolic space is very nearly identical to Euclidean space (i.e., the universe locally is approximately flat consistent with local observations, the apparent angular size of distant objects falls off much more rapidly, in fact exponentially. The universe is globally hyperbolic. Locally the spacetime is approximately flat, by means of the hyperbolic inflation. Locally any deviations from flatness will be hyperbolically suppressed by the hyperbolic expansion of the scale factor, and the flatness problem is solved.

-The distance horizon in the hyperbolic universe is infinite, simply solves the horizon problem.

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Fig (2) Earthsky.org, What is dark energy.
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Fig (4) Salah. A. Mabkhout (*2013*) The Big Bang hyperbolic universe neither needs inflation nor dark matter and dark energy. Physics Essays: September 2013, Vol. 26, No. 3, pp. 422-429.
Fig (5) Salah. A. Mabkhout (*2013*) The Big Bang hyperbolic universe neither needs inflation nor dark matter and dark energy. Physics Essays: September 2013, Vol. 26, No. 3, pp. 422-429.