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Peterson Graph and its Hypergraph

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ABSTRACT

In this paper we produce a hypergraph using the Peterson hypergraph. We prove that two vertices in this hypergraph are adjacent if and only if they are not adjacent in the Peterson graph. We find maximum independent sets, maximum stable sets, domination number and H-domination number of this hypergraph.

1.Introduction

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Hypergraph arise naturally in graph theory. They arise from different mathematical structure. In fact Hypergraph are natural extension of graphs. Many concepts for graphs can be extended to hypergraphs. For an excellent treatment of hypergraphs the reader can be the book of C. Berge [1].

In this paper we produce a hypergraph from the Peterson graph and we discover some sets like stable sets, Independent sets for this hypergraph. We also find the minimum number and maximum number of these sets.

We prove that two vertices in this hypergraph are adjacent if and only if they are not adjacent in the Peterson graph. We prove that the domination number of this hypergraph is 2 while the domination number of Peterson graph is 3. The independence number of this hypergraph is 2 and stability number of this hypergraph is 7.

2. Preliminaries

For the terminology and notations related to graphs one can refer to any book on graphs.

A hypergraph G is an order pair (V(G),E(G)) where V(G) is a finite set and E(G) is a collection of nonempty subsets of V(G). We make the following conventions for the hypergraphs.

(1) If $x, y \in V(G)$ then there is at most one $e \in V(G)$ such that $x, y \in e$.

(2) If $e_1, e_2 \in E(G)$ then $e_1 \not\subset e_2$ and $e_2 \not\subset e_1$.

The members of V(G) are called the vertices of G and member of E(G) are called the edges of G.

Two vertices x and y are said to be adjacent vertices in the hypergraph G if there is an edge e such that $x, y \in e$.

Definitions:

Definition 2.1 (Adjacency Graphs)[1]

Let G be a hypergraph whose vertex is V(G). The adjacency graph of this hypergraph is the graph G' for which V(G')=V(G) and two elements $(x, y) \in V(G')$ are adjacent in G' if and only if x and y are adjacent vertices of the hypergraph G.

For example Consider the hypergraph G whose vertex set is {0,1,2,3,4} and edges are e_1={0,1,2} and e_2={0,3,4} then the adjacency graph of this hypergraph is as follows

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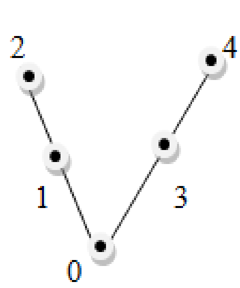

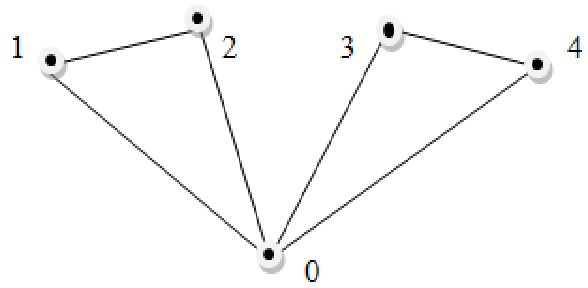

Definition 2.2 :(Dominating set, Domination number)[1]

Let G be a hypergraph. A set S$\subseteq$V(G) is said to be a dominating set if for every vertex v in V(G) \ S, there is a vertex u in S such that v is adjacent to u.

Let G be a hypergraph. A dominating set with minimum cardinality is called minimum dominating set or a γ-set. The cardinality of a minimum dominating set is called the domination number of the hypergraph and it is denoted as γ(G).

Definition 2.3 :(H-dominating set)[2]

Let G is a hypergraph. A set S $\subseteq$V(G) is called H- dominating set if for each vertex v in V(G) \ S, there is an edge e containing v such that e \ {v} is subset of S.

Definition 2.4 (Minimum H-dominating set, H-domination number)[2]

An H- dominating set S of a hypergraph G is said to be a minimum H-dominating set if its cardinality is minimum among all H-dominating sets of G.

The cardinality of a minimum H-dominating set is called the H-domination number of the hypergraph G and its denoted by γ_h. A minimum H-dominating set is also called a γ_h-set of G.

Definition 2.5 (Stable set):[1] Let G be a hypergraph and S$\subseteq$V(G),then S is said to be a stable set if S does not contain any edge of cardinality greater than one.

Definition 2.6 (Maximal Stable set): [1]

Let G be a hypergraph and S$\subseteq$V(G) be an Stable set, then S is said to be an maximal Stable set if S∪{v} is not a Stable set for any vertex v in V(G) \ S.

Definition 2.7: Vertex covering set [1]

A subset S of V(G) is called a vertex covering set if every edge of G has non-empty intersection with S.

A vertex covering set with minimum cardinality is called α_0-set of G.

The cardinality of a minimum vertex covering set of G is called the vertex covering number of G and is denoted as α_0(G).

Definition 2.8: Strong vertex covering set [1]

Let G be a hypergraph and $S \subseteq V(G)$then S is said to be strong vertex covering set if whenever x and y are adjacent in G then $x \in S$ or $y \in S$.

Definition 2.9: Minimum strong vertex covering set [2]

A strong vertex covering set with minimum cardinality is called minimum strong vertex covering set and it is denoted as α_s-set of G.

Definition 2.10: Strong vertex covering number [1]

The cardinality of a minimum strong vertex covering set (α_s-set) is called the strong vertex covering number of the hypergraph G and it is denoted as $\alpha_s(G)$.

Definition 2.11: Independent set [1]

Let G be a hypergraph and $S \subseteq V(G)$ then S is called an independent set of G if whenever x and y belongs to S and $x \neq y$, they are non-adjacent vertices of G.

Definition 2.12: Independence number [1]

The cardinality of a maximum independent set of a hypergraph G is called the independence number of G and it is denoted as $\beta_s(G)$.

Remark 2.13: It is obvious that the compliment of a vertex covering set is a stable set, the compliment of a minimum vertex covering set is a maximum stable set and therefore $\alpha_0(G)+\beta_0(G)=n$, for any hypergraph G with n vertices.

Similarly the compliment of a strong vertex covering set is an independent set, the compliment of a minimum vertex covering set is a maximum independent set and therefore $\alpha_s(G)+\beta_s(G)=n$, for any hypergraph G with n vertices.

3 Main Results

LetX={1,2,3,4,5}and consider the graph G whose vertices are {{1,2},{2,3},{3,4},{4,5},{1,3},{1,4},{1,5},{2,4},{2,5},{3,5}}. Two vertices {i, j} and {a, b} are adjacent in this graph if and only if $\{i,j\}\cap\{a,b\}=\varnothing$. This hypergraph is isomorphic to the well known graph called the Peterson graph.

Therefore we shall call this graph as the Peterson graph and we denote this graph as P.

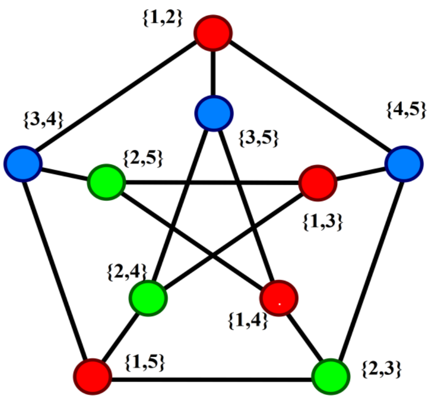

Figure 1

This graph has exactly 5 maximum independent sets which are mentioned below.
M_1={{1,2},{1,3},{1,4},{1,5}},M_2={{2,3},{2,4},{2,5},{2,1}},M_3={{3,4},{3,5},{3,1},{3,2}},
M_4={{4,5},{4,1},{4,2},{4,3}},M_5={{5,1},{5,2},{5,3},{5,4}}.

Observe that any two maximum independent set intersects in exactly one vertex and every vertex is contained in exactly two maximum independent sets.

We define a new hypergraph as follows.

The vertices of this hypergraphs are the vertices of the Peterson graph mentioned above. The edge set of this hypergraph are maximum independent set {M_1, M_2, M_3, M_4, M_5 }. This hypergraph is denoted as H(P).

In this hypergraph every vertex is contained exactly two edges and any two edges intersect in exactly one vertex. If {i, j} and {a, b} are adjacent vertices in this hypergraph then there is an edge M_k which contains {i, j} and {a, b} and therefore $k \in \{i, j\}$ and $k \in \{a,b\}$ that is $\{i, j\} \cap \{a,b\} \neq \varnothing$. Converse is also true.

Thus two vertices {i, j} and {a, b} are adjacent in this graph if and only if $\{i, j\} \cap \{a,b\} \neq \varnothing$.if and only if {i, j} and {a, b} are not adjacent in the Peterson graph.

Thus we have proved that the following proposition.

Proposition 3.1

Any two vertices in H(P) are adjacent if and only if they are not adjacent in the Peterson graph P.

Corollary 3.2

The adjacency graph of H(P) is isomorphic to $\overline{P}$ Where $\overline{P}$ denotes the compliment of the Peterson graph. ■

Proposition 3.3:

The domination number of H(P)=2.

Proof

Let S= {{1, 2}, {3, 4}. Let {i, j} be any vertex of H(P) which is not in S. Since we are using only five symbol from {1, 2, 3, 4, 5}, $i \in \{1,2,3,4\}$ or $j \in \{1,2,3,4\}$. Therefore {i, j} is adjacent to {1, 2} or {i, j} is adjacent to {3, 4}.

Thus S is a minimum dominating set of H(P) and therefore the domination number of H(P)=2. ■

Proposition 3.4

Every edge M_i in H(P) is a dominating set of H(P) but it is not a minimal dominating set of H(P).

Proof

{a, b} be a vertex of H(P) such that $\{a,b\} \notin M_i$ Where M_i={{i ,j}: $j \in \{1,2,3,4,5\} \setminus \{i\}$} then {a, b} is adjacent to {a, i} and {b, i} which are both element if M_i.

Let $\{i, j\} \in M_i$ then {i, j} is adjacent to {i, k}: $\forall k \neq j$. Therefore {i, j} is not a private neighbor of {i, j} with respect to M_i.

Let {j, b} be a vertex which is not in M_i and which is adjacent to {i, j} then {j, b} is also adjacent to another element {i, b} of M_i. Therefore {j, b} is not a private neighbor of {i, j} with respect to M_i.

Thus there is no vertex in H(P) which is a private neighbor of {i, j} with respect to M_i.

Therefore M_i is not a minimal dominating set of H(P). ■

Proposition 3.5

The H-domination number of H(P)=6.

Proof

Let S= V(G) \ M_1 then |S|=6. Let $\{1,k\} \in M_1$ then the elements of M_k \ {1, k} is {k, 2}, {k, 3}, {k, 4} is a subset of S.

Thus S is an H-dominating set of H(P) and note that |S|=6. Let S_1 be a set of vertices having exactly 5 members and let {v_1,v_2,v_3,v_4,v_5} be vertices which are not in S_1. Suppose S_1 is an H-dominating set of H(P). Therefore for every i there is an edge e_i containing v_i such that $e_i \setminus \{v_i\} \subseteq S_1$(i=1, 2, 3, 4, 5). Obviously $e_i \neq e_j$ if $i \neq j$ and $v_i \notin e_j$ if $i \neq j$ also $e_i \cap e_j \subseteq S_1$. Thus e_1,e_2,e_3,e_4,e_5, are all the edges of H(P) and $v_1 \in e_1$ but $v_1 \notin e_j (j = 2,3,4,5)$ This is a contradiction.

Therefore S_1 cannot be an H-dominating set of H(P).

Thus S is an H-dominating set of H(P).

Thus the H-domination number of H(P)=6. ■

Now we prove that the vertex covering number of H(P)=3

Proposition 3.6

The vertex covering number of H(P)=3.

Proof

Let S= {{1, 2}, {3, 4}, {1, 5}}. Obviously $\{1,2\}\in M_1\cap M_2, \{3,4\}\in M_3\cap M_4, \{1,5\}\in M_{5.}$ Thus S is a minimum vertex covering set of H(P). If T is a set of two vertices then at least 3-edges must intersect T in a single vertex which implies that a vertex is contained in 3 edges which is not true. Therefore T cannot be a vertex covering set of H(P). Thus S is a minimum vertex covering set of H(P).

The vertex covering number of H(P)=3■

Corollary 3.7

The stability number of H(P) is 7. ■

Remark

We have mentioned above that the set {{1, 2}, {3, 4}, {1, 5}} is a minimum vertex covering set. Observe that every two element subset gives rise to 3 maximum vertex covering set. Therefore apparently there are 30 vertex covering sets in H(P). However every two elements subset appears thrice in all these sets. Therefore there are exactly 10 minimum vertex covering set. There are exactly 10 maximum stable set in H(P). ■

Now we prove that the independence number of H(P)=2

Proposition 3.8

The independence number of H(P)=2.

Proof

Let I= {{1, 2}, {3, 4}}. Since $\{1,2\}\cap\{3,4\}=\varnothing$.This two vertices are non adjacent vertices in H(P). If I_1 is a set of vertices with at least 3 elements then there are two elements {a, b} and {r, s} in I_1 such that $\{a,b\}\cap\{r,s\}\neq\varnothing$. This means that at least two vertices of I_1 are adjacent if I_1 has at least 3 vertices. So I_1 cannot be an independent set.

Therefore I is a maximum independent subset of H(P).

Therefore Independence number of H(P)=2. ■

Remark 3.9

As mentioned above the set I= {{1, 2}, {3, 4}} is a maximum independent set. Note that this set I involves only 4 symbol namely 1, 2, 3 and 4. From this 4 symbol other two maximum independent sets are as follows:

I_1= {{1, 3}, {2, 4}.

I_2= {{1, 4}, {2, 3}}.

Thus every 4 element subset give rise to 3 distinct maximum independent subset of H(P).Since there are 5 four element subset of {1,2,3,4,5}, there are exactly 15 maximum independent subset of H(P). ■

Corollary 3.10

The strong vertex covering number of H(P)=8.■

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