Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 94 (2016) 39912-39922

ISSN: 2229-712X

Mechanical Equations with Two Almost Complex Structures on Symplectic Geometry

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ARTICLE INFO

Article history:

Received: 9 March 2016;

Received in revised form: 22 April 2016;

Accepted: 29 April 2016;

Keywords

Symplectic geometry,
Lagrangian,
Hamiltonian,
Mechanical system.

ABSTRACT

An almost complex manifold is a smooth manifold accoutered with smooth linear complex structure on each tangent space. Almost complex structures have important applications on symplectic geometry. M is a symplectic manifold such that it accoutred with a closed nondegenerate differential 2-form then it called the symplectic form. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds. In classical mechanics, dynamic movements with Euler-Lagrange and Hamilton equations is found. This article, using two complex structures, is related mechanical systems on symplectic geometry.

1 Introduction

Nowadays, there is comprehensive use in differential geometry. Differential geometry has a lots of different applications in the branches of science. These applications, came into our lives, are used in many areas and the popular science. We can say that differential geometry provides a good working area for studying Lagrangians and Hamiltonian of classical mechanics and field theory. The dynamic equation for moving bodies is obtained for Lagrangian and Hamiltonian mechanics. Considering information, the above, in a lot of articles and books, it is possible to show how differential geometric methods are applied in Lagrangian's and Hamiltonian's mechanics in the below. There are many studies about Lagrangian and Hamiltonian dynamics, mechanics, formalisms, systems and equations. There are real, complex, para-complex and other analogues. It is well-known that Lagrangian and Hamiltonian analogues are very important tools. They have a simple method to describe the model for mechanical systems. Ye developed a general framework for embedded (immersed) J-holomorphic curves and a systematic treatment of the theory of filling by holomorphic curves in 4-dimensional symplectic manifolds [1]. Lisi considered three applications of pseudo-holomorphic curves to problems in Hamiltonian dynamics [2]. Weinstein explained what symplectic geometry is and to describe its role in contemporary mathematics [3]. Audin and Lafontaine introductioned to symplectic geometry and relevant techniques of Riemannian geometry, proofs of Gormov's compactness theorem, an investigation of local properties of holomorphic curves, including positivity of intersections, and applications to Lagrangian embeddings problems [4].

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McDuff devoted to proving some of the main technical results about J-holomorphic curves which make them such a powerful tool when studying the geometry of symplectic 4-manifolds [5]. The models about mechanical systems are given as follows: Kasap and Tekkoyun obtained Lagrangian and Hamiltonian formalism for mechanical systems using para/pseudo-Kähler manifolds [6]. Kasap examined Weyl-Euler-Lagrange and Weyl-Hamilton equations on $\mathbb{R}_n^{2n}$ which is a model of tangent manifolds of constant W-Sectional curvature [7]. Tekkoyun submitted paracomplex analogue of the Euler-Lagrange equations was obtained in the framework of para-Kählerian manifold and the geometric results on a paracomplex mechanical systems were found [8]. Kasap submitted Weyl-Euler-Lagrange equations of motion on flat manifold [9]. Tekkoyun and Celik present a new analogue of Euler-Lagrange and Hamilton equations on an almost K¨ahler model of a Finsler manifold [10].

2 Riemannian Geometry

Riemannian geometry is the branch of differential geometry such that studies Riemannian manifolds, smooth manifolds with a Riemannian metric, i.e. with an inner product on the tangent space at each point. It has varies smoothly from point to point. This gives, in particular, local notions of angle, length of curves, surface area, and volume. From those some other global quantities can be derived by integrating local contributions. Any smooth manifold admits a Riemannian metric, which often helps to solve problems of differential topology. It also serves as an entry level for the more complicated structure of pseudo-Riemannian manifolds, which (in four dimensions) are the main objects of the theory of general relativity. Other generalizations of Riemannian geometry include Finsler geometry.

3 The Theory of J-Holomorphic Curves

A J-holomorphic curve is a smooth map from a Riemann surface into an almost complex manifold that satisfies the Cauchy--Riemann equation. Introduced in 1985 by Gromov, pseudo-holomorphic curves have since revolutionized the study of symplectic manifolds. The theory of J-holomorphic curves is one of the new techniques which have recently revolutionized the study of symplectic geometry, making it possible to study the global structure of symplectic manifolds J [11]. A (real) curve in a manifold M is a path in M such that it is the image of a map $f:U\rightarrow M$ where U is a lower range of the real line $\mathbb{R}$. A J-holomorphic curve in an almost complex manifold (M,J) is the complex analog of this. It has one complex dimension (but 2 real dimensions) and is the image of a complex map $f:\sum\rightarrow M$ from some complex curve $\sum$ into (M,J). Here we shall take the domain to be either a 2-dimensional disc D (consisting of a circle in the plane together with its interior) or the 2-sphere $S^2=\mathbb{C}-\{\infty\}$, which we shall think of as the complex plane $\mathbb{C}$ completed by adding a point at ∞.The images $f(\sum)$ of such maps have very nice properties. In particular, their area with respect to the associated metric g_J equals their symplectic area. J-holomorphic curves are so-called g_J-minimal surfaces. Thus we can say of them as the complex analog of a real geodesic. A geodesic in a Riemannian manifold (M,g_J) is a path that minimizes the length between any two of its points such that provided these are sufficiently close. The metric area of a surface is a measure of its energy [5].

4 Almost (para)-Complex and Tangent Structures

Definition 1: Let M be a differentiable manifold of dimension 2n, and suppose J is a differentiable vector bundle isomorphism $J:TM\rightarrow TM$ such that $J_x:T_xM\rightarrow T_xM$ is a complex structure for T_xM, i.e. $J^2=-I$ where I is the identity (unit) operator and . $J^2=J\circ J$ on V. Then J is called an almost-complex structure for the differentiable manifold M.

Definition 2: Let be V a vector space over $\mathbb{R}$. Recall that a paracomplex structure on V is a linear operator J on V such that $J^2=I$, where $J^2=J\circ J$, and I is the identity (unit) operator on V. A prototypical example of a paracomplex structure is given by the map $J:V\rightarrow V$, where $V=\mathbb{R}^n\oplus\mathbb{R}^n$. An almost paracomplex structure on M a manifold is a differentiable map $J:TM\rightarrow TM$ on the tangent bundle TM of M such that J preserves each fiber. A manifold with a fixed almost paracomplex structure is called an almost paracomplex manifold.

Definition 3: Let be V a vector space over $\mathbb{R}$. Recall that a tangent(exact) structure on V is a linear operator J on V such that $J^2=0$, where $J^2=J\circ J$, and I is the identity operator on V.

5 Complex Manifold

A manifold with a fixed almost complex structure is called an almost complex manifold. A complex manifold for differential geometry is a manifold with an atlas of charts to the open unit disk in $\mathbb{C}^n$ such that the transition maps are holomorphic. The term complex manifold is comprehensive used to mean a complex manifold such that it can be specified as an integrable complex manifold and an almost (para) complex manifold.

6 Symplectic Manifold

Symplectic geometry is an even dimensional geometry. It lives on even dimensional spaces, and measures the sizes of 2-dimensional objects rather than the 1-dimensional lengths and angles that are familiar from Euclidean and Riemannian geometry. It is naturally associated with the field of complex rather than real numbers.

M symplectic manifold is a smooth manifold such that it equipped with a closed nondegenerate differential 2-form ω The study of symplectic manifolds is called symplectic geometry or symplectic topology. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds. For example, in the Hamiltonian formulation of classical mechanics, which provides one of the major motivations for the field, the set of all possible configurations of a system is modeled as a manifold, and this manifold's cotangent bundle describes the phase space of the system. called the symplectic form. The study of symplectic manifolds is called symplectic geometry or symplectic topology. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds. For example, in the Hamiltonian formulation of classical mechanics, which provides one of the major motivations for the field, the set of all possible configurations of a system is modeled as a manifold, and this manifold's cotangent bundle describes the phase space of the system.

7 Holomorphic Structures

We can be determined $\mathbb{R}^4$ with $\mathbb{C}^2$ in the usual way. We consider of filling by holomorphic curves (especially disks) in symplectic 4-manifolds. M be a 4-dimensional almost complex manifold with almost complex structure J, and S a 2-dimensional submanifold in M such that is $S \subset M$. A complex point of S is a point p such that the tangent plane T_pS is complex, i.e. $J(T_pS)=TpS$. If $T_pS \cap J(T_pS)=\{0\}$; then p is called a real point. Note that points of S are either complex or real. The corresponding almost complex structure J is given by

$$\begin{aligned}
&\text{(i)} && J((\partial/(\partial x_1)))=(\partial/(\partial x_2)),\\
&\text{(ii)} && J((\partial/(\partial x_2)))=-(\partial/(\partial x_1)),\\
&\text{(iii)} && J((\partial/(\partial x_3)))=(\partial/(\partial x_4)),\\
&\text{(iv)} && J((\partial/(\partial x_4)))=-(\partial/(\partial x_3)). && (1)
\end{aligned}$$

The above structures were taken from [1, 5].

Theorem 1. The corresponding almost complex structure J is given on symplectic manifold. If J is integrable, we can choose local complex coordinates on the goal space M of the form $\mathbf{z_1=x_1+ix_2,\ z_2=x_3+ix_4}$ ($i^2=-1$) so that at each point x the linear transformation $J=J_x$ acts on the tangent vectors $(\partial/(\partial x_j))$ by "multiplication by i": namely. If J is defined as a complex structure on the manifold M then structures at (1) are almost complex structures ($J^2=-I$ and $J^{*2}=-I$).

Proof: Let's choose a holomorphic structure J in the complex space of (1).

$$\begin{aligned}
&\text{(i)} && J^2((\partial/(\partial x_1)))=(\partial/(\partial x_2))=J((\partial/(\partial x_2)))=-(\partial/(\partial x_1)),\\
&\text{(ii)} && J^2((\partial/(\partial x_2)))=J(-(\partial/(\partial x_1)))=-(\partial/(\partial x_2)),\\
&\text{(iii)} && J^2((\partial/(\partial x_3)))=J((\partial/(\partial x_4)))=-(\partial/(\partial x_3)),\\
&\text{(iv)} && J^2((\partial/(\partial x_4)))=J(-(\partial/(\partial x_3)))=-(\partial/(\partial x_4)). && (2)
\end{aligned}$$

As above J^2 is -I. J^* holomorphic structures on the cotangent space for Hamilton equations are

(i) $J^{*2}(dx_1)= J^*(dx_2)=-dx_1$,

(ii) $J^{*2}(dx_2)= J^*(-dx_1)=-dx_2$,

(iii) $J^{*2}(dx_3)= J^*(dx_4)=-dx_3$,

(iv) $J^{*2}(dx_4)= J^*(-dx_3)=-dx_4$. (3)

As can be seen from above, $J^{*2}= J^*\circ J^*=-I$, structures are complex.

8 Lagrangian and Hamiltonian Formalism

Definition 4. [12-14] Let M be an n-dimensional manifold and TM its tangent bundle with canonical projection τ_M:TM→M. TM is called the phase space of velocities of the base manifold M. Let L:TM→R be a differentiable function on TM called the Lagrangian function. We consider the closed 2-form on TM given by $\Phi_L=-d(\mathbf{d}_J L)$ and $\mathbf{d}_J=J((\partial/(\partial x_i)))dx_i$. Consider the equation

$$i_X\Phi_L=dE_L. \quad (4)$$

Then X is a vector field, we shall see that (4) under a certain condition on X is the intrinsically expression of the Euler-Lagrange equations of motion. This equation is named as Lagrange dynamical equation. We shall see that for motion in a potential, $E_L=V(L)-L$ is an energy function and V=J(X) a Liouville vector field. Here dE_L denotes the differential of E. The triple (TM,Φ_L,X) is known as Lagrangian system on the tangent bundle TM. If it is continued the operations on (4) for any coordinate system $(q^i(t),p_i(t))$, infinite dimension Lagrange's equation is obtained the form below:

$$(d/(dt))(((\partial L)/(\partial\dot{q}^i)))=((\partial L)/(\partial q^i)),\ \ (dq^i)/(dt)=\dot{q}^i\ ,\ \ i=1,...,n. \quad (5)$$

Definition 5. Let M be the base manifold and its cotangent manifold T^*M. By a symplectic form we mean a 2-form Φ on T^*M such that:

(i) Φ is closed , that is, dΦ=0; (ii) for each $z\in T^*M$, $\Phi_z:T_zT^*M\times T_z T^*M\to\mathbb{R}$ is weakly nondegenerate. If Φ_z in (ii) is nondegenerate, we speak of a strong symplectic form. If (ii) is dropped we refer to Φ as a presymplectic form. Now let (T^*M,Φ) us take as a symplectic manifold. A vector field X_H: $T^*M\to TT^*M$ is called Hamiltonian vector field if there is a C^1 Hamiltonian function H: $T^*M\to\mathbb{R}$ such that Hamilton dynamical equation is determined by

$$i_{X_H}\Phi=dH. \quad (6)$$

We say that X_H is locally Hamiltonian vector field if $iX_H\Phi$ is closed and where Φ shows the canonical symplectic form so that $\Phi=-d\Omega$, $\Omega=J^*(\omega)$, J^* a dual of J, ω a 1-form on T^*M. The triple (T^*M,Φ,X_H) is named Hamiltonian system which it is defined on the cotangent bundle T^*M. From the local expression of X_H we see that $(q^i(t),p_i(t))$ is an integral curve of X_H if Hamilton's equations are expressed as follows:

$$\dot{q}^i=(\partial H)/(\partial p_i),\ \dot{p}^i=-(\partial H)/(\partial q^i). \tag{7}$$

The Euler-Lagrangian and Hamiltonian mechanics use quantum physics, optimal control, biology and fluid dynamics [15-16]. These equations are very important to explain the rotational spatial mechanical-physical problems. They has been used in solving problems in different physical area.

9 Lagrangian Mechanical Systems

In this part, we get Euler-Lagrange equations for quantum and classical mechanics on symplectic manifolds. Firstly, take J as the local basis element on symplectic manifolds and (x_1,x_2,x_3,x_4) be its coordinate functions on symplectic manifolds, let ξ be the vector field decided by

$$\xi=\sum_{i=1}^{4}X_i(\partial/(\partial x_i)),\ X_i=\dot{x}_i. \tag{8}$$

The vector field described by

$$V=J(\xi)=X_1(\partial/(\partial x_2))-X_2(\partial/(\partial x_1))+X_3(\partial/(\partial x_4))-X_4(\partial/(\partial x_3)), \tag{9}$$

is said to be Liouville vector field on symplectic manifolds. The symplectic manifolds form is the closed 2-form which is given by $\Phi_L=-d(\mathbf{d}_J L)$ such that

$$\mathbf{d}_J=(\partial/(\partial x_2))dx_1-(\partial/(\partial x_1))dx_2+(\partial/(\partial x_4))dx_3-(\partial/(\partial x_3))dx_4,\ F(M)\to\wedge^1M. \tag{10}$$

Then we have

$$\begin{aligned}\Phi_L=&-((\partial^2L)/(\partial x_1\partial x_2))dx_1\wedge dx_1+((\partial^2L)/(\partial x_1\partial x_1))dx_1\wedge dx_2-((\partial^2L)/(\partial x_1\partial x_4))dx_1\wedge dx_3\\&+((\partial^2L)/(\partial x_1\partial x_3))dx_1\wedge dx_4\}-((\partial^2L)/(\partial x_2\partial x_2))dx_2\wedge dx_1+((\partial^2L)/(\partial x_2\partial x_1))dx_2\wedge dx_2\\&-((\partial^2L)/(\partial x_2\partial x_4))dx_2\wedge dx_3+((\partial^2L)/(\partial x_2\partial x_3))dx_2\wedge dx_4-((\partial^2L)/(\partial x_3\partial x_2))dx_3\wedge dx_1\\&+((\partial^2L)/(\partial x_3\partial x_1))dx_3\wedge dx_2-((\partial^2L)/(\partial x_3\partial x_4))dx_3\wedge dx_3+((\partial^2L)/(\partial x_3\partial x_3))dx_3\wedge dx_4\\&-((\partial^2L)/(\partial x_4\partial x_2))dx_4\wedge dx_1+((\partial^2L)/(\partial x_4\partial x_1))dx_4\wedge dx_2-((\partial^2L)/(\partial x_4\partial x_4))dx_4\wedge dx_3\\&+((\partial^2L)/(\partial x_4\partial x_3))dx_4\wedge dx_4.\end{aligned} \tag{11}$$

And then we calculate

$i_{\xi}\Phi_L=\Phi_L(\xi)$

$$
\begin{aligned}
&=-((\partial^2 L)/(\partial x_1\partial x_2))X_1dx_1+((\partial^2 L)/(\partial x_1\partial x_2))X_1dx_1+((\partial^2 L)/(\partial x_1\partial x_1))X_1dx_2-((\partial^2 L)/(\partial x_1\partial x_4))X_1dx_3\\
&+((\partial^2 L)/(\partial x_1\partial x_3))X_1dx_4+((\partial^2 L)/(\partial x_2\partial x_2))X_1dx_2+((\partial^2 L)/(\partial x_3\partial x_2))X_1dx_3+((\partial^2 L)/(\partial x_4\partial x_2))X_1dx_4\\
&-((\partial^2 L)/(\partial x_1\partial x_1))X_2dx_1-((\partial^2 L)/(\partial x_2\partial x_2))X_2dx_1+((\partial^2 L)/(\partial x_2\partial x_1))X_2dx_2-((\partial^2 L)/(\partial x_2\partial x_1))X_2dx_2\\
&-((\partial^2 L)/(\partial x_2\partial x_4))X_2dx_3+((\partial^2 L)/(\partial x_2\partial x_3))X_2dx_4-((\partial^2 L)/(\partial x_3\partial x_1))X_2dx_3-((\partial^2 L)/(\partial x_4\partial x_1))X_2dx_4\\
&+((\partial^2 L)/(\partial x_1\partial x_4))X_3dx_1+((\partial^2 L)/(\partial x_2\partial x_4))X_3dx_2-((\partial^2 L)/(\partial x_3\partial x_2))X_3dx_1+((\partial^2 L)/(\partial x_3\partial x_1))X_3dx_3\\
&-((\partial^2 L)/(\partial x_3\partial x_4))X_3dx_3+((\partial^2 L)/(\partial x_3\partial x_4))X_3dx_3+((\partial^2 L)/(\partial x_3\partial x_3))X_3dx_4+((\partial^2 L)/(\partial x_4\partial x_4))X_3dx_4\\
&-((\partial^2 L)/(\partial x_1\partial x_3))X_4dx_1-((\partial^2 L)/(\partial x_2\partial x_3))X_4dx_2-((\partial^2 L)/(\partial x_3\partial x_3))X_4dx_3-((\partial^2 L)/(\partial x_4\partial x_2))X_4dx_1\\
&+((\partial^2 L)/(\partial x_4\partial x_1))X_4dx_2+((\partial^2 L)/(\partial x_4\partial x_4))X_4dx_3-((\partial^2 L)/(\partial x_4\partial x_3))X_4dx_4+((\partial^2 L)/(\partial x_4\partial x_3))X_4dx_4.
\end{aligned}
\tag{12}
$$

Energy function and its differential are like the following:

$$E_L=V(L)-L=X_1((\partial L)/(\partial x_2))-X_2((\partial L)/(\partial x_1))+X_3((\partial L)/(\partial x_4))-X_4((\partial L)/(\partial x_3))-L, \tag{13}$$

And

$dE_L=(\sum^4_{i=1}(\partial/(\partial x_i))dx_i)(X_1((\partial L)/(\partial x_2))-X_2((\partial L)/(\partial x_1))+X_3((\partial L)/(\partial x_4))-X_4((\partial L)/(\partial x_3))-L)$

$$
\begin{aligned}
&=((\partial^2 L)/(\partial x_1\partial x_2))X_1dx_1+((\partial^2 L)/(\partial x_2\partial x_2))X_1dx_2+((\partial^2 L)/(\partial x_3\partial x_2))X_1dx_3+((\partial^2 L)/(\partial x_4\partial x_2))X_1dx_4\\
&-((\partial^2 L)/(\partial x_1\partial x_1))X_2dx_1-((\partial^2 L)/(\partial x_2\partial x_1))X_2dx_2-((\partial^2 L)/(\partial x_3\partial x_1))X_2dx_3-((\partial^2 L)/(\partial x_4\partial x_1))X_2dx_4\\
&+((\partial^2 L)/(\partial x_1\partial x_4))X_3dx_1+((\partial^2 L)/(\partial x_2\partial x_4))X_3dx_2+((\partial^2 L)/(\partial x_3\partial x_4))X_3dx_3+((\partial^2 L)/(\partial x_4\partial x_4))X_3dx_4\\
&-((\partial^2 L)/(\partial x_1\partial x_3))X_4dx_1-((\partial^2 L)/(\partial x_2\partial x_3))X_4dx_2-((\partial^2 L)/(\partial x_3\partial x_3))X_4dx_3-((\partial^2 L)/(\partial x_4\partial x_3))X_4dx_4\\
&-((\partial L)/(\partial x_1))dx_1-((\partial L)/(\partial x_2))dx_2-((\partial L)/(\partial x_3))dx_3-((\partial L)/(\partial x_4))dx_4.
\end{aligned}
\tag{14}
$$

If we use $i_X\Phi_L=dE_L$ then we obtain the equations given by

$$-(X_1(\partial/(\partial x_1))+X_2(\partial/(\partial x_2))+X_3(\partial/(\partial x_3))+X_4(\partial/(\partial x_4)))(((\partial L)/(\partial x_2)))dx_1+((\partial L)/(\partial x_1))dx_1$$
$$+(X_1(\partial/(\partial x_1))+X_2(\partial/(\partial x_2))+X_3(\partial/(\partial x_3))+X_4(\partial/(\partial x_4)))(((\partial L)/(\partial x_1)))dx_2+((\partial L)/(\partial x_2))dx_2$$
$$-(X_1(\partial/(\partial x_1))+X_2(\partial/(\partial x_2))+X_3(\partial/(\partial x_3))+X_4(\partial/(\partial x_4)))(((\partial L)/(\partial x_4)))dx_3+((\partial L)/(\partial x_3))dx_3$$
$$+(X_1(\partial/(\partial x_1))+X_2(\partial/(\partial x_2))+X_3(\partial/(\partial x_3))+X_4(\partial/(\partial x_4)))=0. \tag{15}$$

Considering the curve $\alpha=(x_1,x_2,x_3,x_4)$, an integral curve of ξ, i.e. $\xi(\alpha(\mathbf{t}))=((\partial\alpha)/(\partial\mathbf{t}))$, we can find the equations as follows:

$$-\xi(((\partial L)/(\partial x_2)))dx_1+((\partial L)/(\partial x_1))dx_1+\xi(((\partial L)/(\partial x_1)))dx_2+((\partial L)/(\partial x_2))dx_2$$
$$-\xi(((\partial L)/(\partial x_4)))dx_3+((\partial L)/(\partial x_3))dx_3+\xi(((\partial L)/(\partial x_3)))dx_4+((\partial L)/(\partial x_4))dx_4=0, \tag{16}$$

and

$$-(\partial/(\partial\mathbf{t}))(((\partial L)/(\partial x_2)))+((\partial L)/(\partial x_1))=0,\ (\partial/(\partial\mathbf{t}))(((\partial L)/(\partial x_1)))+((\partial L)/(\partial x_2))=0,$$
$$-(\partial/(\partial\mathbf{t}))(((\partial L)/(\partial x_4)))+((\partial L)/(\partial x_3))=0,\ (\partial/(\partial\mathbf{t}))(((\partial L)/(\partial x_3)))+((\partial L)/(\partial x_4))=0. \tag{17}$$

or

dif1. $-(\partial^2 L)/(\partial\mathbf{t}\partial x_2)+(\partial L)/(\partial x_1)=0,$

dif2. $(\partial^2 L)/(\partial\mathbf{t}\partial x_1)+(\partial L)/(\partial x_2)=0,$

dif3. $-(\partial^2 L)/(\partial\mathbf{t}\partial x_4)+(\partial L)/(\partial x_3)=0,$

dif4. $(\partial^2 L)/(\partial\mathbf{t}\partial x_3)+(\partial L)/(\partial x_4)=0.$

(18)

such that these equations (18) are called Euler-Lagrange Equations constructed on symplectic manifolds and thus the triple (M,Φ_L,ξ) is named as a mechanical system on symplectic manifolds (M,g,J).

10 Hamiltonian Mechanical Systems on Symplectic Manifolds

Here, we present Hamilton equations and Hamiltonian mechanical systems for quantum and classical mechanics constructed on symplectic manifolds. Let (M,g,J^*) be the on symplectic manifolds. Suppose that J^* is the structures, Ω is a Liouville form and ω is a 1-form on symplectic manifolds. Consider a 1-form such that $\omega=(1/2)(-x_2dx_1+x_1dx_2+dt)$. Then we obtain the Liouville form as follows:

$$\Omega=J^*(\omega)=(1/2)(X_1dx_2-X_2dx_1+X_3dx_4-X_4dx_3).$$

It is well known that if Φ is a closed on symplectic manifolds (M,g,J^*), then Φ is also a symplectic structure on (M,g,J^*). Therefore the 2-form Φ indicates the canonical symplectic form and derived from the 1-form Ω to find to mechanical equations. Then the 2-form is calculated as $\Phi=dx_2\wedge dx_1+dx_4\wedge dx_3$. Take a vector field X_H so that called to be Hamiltonian vector field associated with Hamiltonian energy H and determined by

$$X_H=\sum_{i=1}^{4}X_i(\partial/(\partial x_i)). \tag{20}$$

So, we have

$$iX_H\Phi=\Phi(X_H)=-X_1dx_2+X_2dx_1-X_3dx_4+X_4dx_3. \tag{21}$$

Furthermore, the differential of Hamiltonian energy H is obtained by

$$dH=\sum_{i=1}^{4}((\partial H)/(\partial x_i))dx_i. \tag{22}$$

From $iX_H\Phi=dH$, the Hamiltonian vector field is found as follows:

$$X_H=-((\partial H)/(\partial x_2))(\partial/(\partial x_1))+((\partial H)/(\partial x_1))(\partial/(\partial x_2))-((\partial H)/(\partial x_4))(\partial/(\partial x_3))+((\partial H)/(\partial x_3))(\partial/(\partial x_4)). \tag{23}$$

Consider the curve and its velocity vector $\alpha:I\subset\mathbb{R}\rightarrow M$, $\alpha(t)=(x_1,x_2,x_3,x_4)$, $\alpha(t)=((\partial\alpha)/(\partial t))=(((dx_1)/(dt)),((dx_2)/(dt)),((dx_3)/(dt)),((dx_4)/(dt)).)$ such that an integral curve of the Hamiltonian vector field X_H, i.e., $X_H(\alpha(t))=((\partial\alpha)/(\partial\, \mathbf{t}))$, $\mathbf{t}\in I$, t shows the time. Then, we can be find the following equations;

$$\begin{aligned} &\text{dif5.}\quad (dx_1)/(d\mathbf{t})=-(\partial H)/(\partial x_2),\\ &\text{dif6.}\quad (dx_2)/(d\mathbf{t})=(\partial H)/(\partial x_1),\\ &\text{dif7.}\quad (dx_3)/(d\mathbf{t})=-(\partial H)/(\partial x_4),\\ &\text{dif8.}\quad (dx_4)/(d\mathbf{t})=(\partial H)/(\partial x_3). \end{aligned} \tag{24}$$

Hence, the equations introduced in (24) are named Hamilton equations on symplectic manifolds and then the triple (M,Φ,X_H) is said to be a Hamiltonian mechanical system on (M,g,J^*). The Lagrange and Hamiltonian mechanical equations derived on a generalized on symplectic manifolds are suggested to deal with problems in electrical, magnetically and gravitational fields for quantum and classical mechanics of physics.

11 Equations Solution

These found (18) and (24) are partial differential equation depending on symplectic manifolds. We can solve these equations using symbolic computation program. For example, we choose as special case of $x_1(\mathbf{t})=\sin(t)$, $x_2(\mathbf{t})=\cos(t)$, $x_3(\mathbf{t})=\sin(t)$, $x_4(\mathbf{t})=\cos(t)$ for (24) and the solution of the system (24) is as follows:

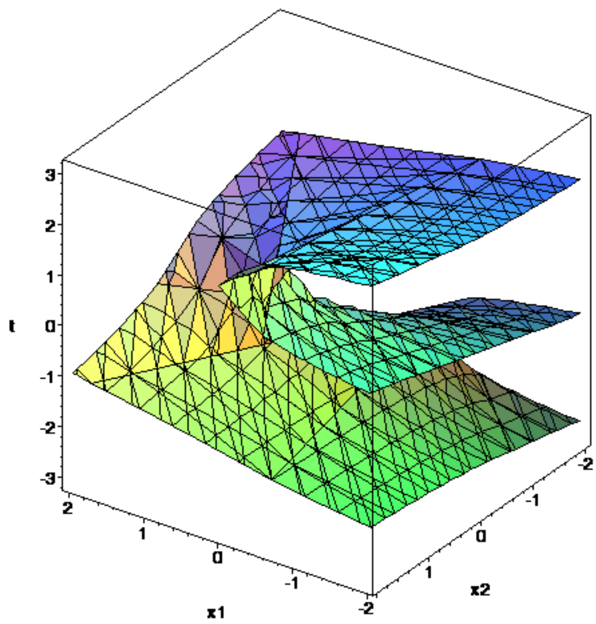
$$H(x_1,x_2,x_3,x_4,t)=(-x_2-x_4+F_1(t))*\cos(t)-\sin(t)*(x_1+x_3). \tag{25}$$

It is well-known that the location of each object in space is represented by three dimensions in space. Also, the dimensions are time, position, mass, and so forth which are represented by higher dimensions. The number of dimensions of (25) will be reduced to three and behind the graphics will be drawn.

Example 1. First, implicit function at (25) will be selected as special $F_1(t)=t$, $x_3=0$, $x_4=0$. After the figure of (25) has been drawn for the geodesics of the movement of objects in the space.

$$H(x_1,x_2,x_3,x_4,t)=(-x_2+t)*\cos(t)-x_1*\sin(t). \tag{26}$$

The graph of the equation (26) is as follows:



(Graph 1)

12 Acknowledgements

This work, Bilecik Sheikh Edebali University organized the 12th National Symposium on Geometry 23-26 June 2014, presented as an oral presentation by the authors.

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