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Fixed Point Theorems on Fuzzy Soft Normed Linear Space

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ABSTRACT

In this paper fixed point theorems on fuzzy soft normed linear space are discussed in a different way. Also the concepts like mapping using set of all soft points, fuzzy soft contraction, S -contraction, R -weakly commuting, etc are defined.

1. Introduction

The concept of fuzzy set theory was first introduced by Zadeh [10] in 1965. The soft set theory was introduced by Molodostov [6] in 1999 using parameters. The combination of fuzzy set theory and soft set theory led Maji etal., [4] in 2001 to introduce fuzzy soft set theory. In [8], a new notion for fuzzy soft norm and fuzzy soft metric are defined and using the notion developed, fuzzy soft contraction, weakly compatible, S -contraction, R -weakly commuting and occasionally weakly commuting are defined in this paper. Some fixed point theorems are proved relating to these concepts.

2. Preliminaries

Definition 2.1

Let X by a vector space over a field $K(K=\mathbb{R})$ and the parameter set E be the real number set $\mathbb{R}$. Let (F,E) be a soft set over X. The soft set (F,E) is said to be a soft vector and denoted by $\tilde{x}_e$ if there is exactly one $e \in E$, such that $F(e)=\{x\}$ for some $x \in X$ and $F(e')=\phi,\ \forall\ e' \in E/\{e\}$.

The set of all soft vectors over $\tilde{X}$ will be denoted by $SV(\tilde{X})$. The set $SV(\tilde{X})$ is called a soft vector space.

Definition 2.2

Let $SV(\tilde{X})$ be a soft vector space. Then a mapping $\|\,.\,\| : SV(\tilde{X}) \to \mathbb{R}^+(E)$ is said to be a soft norm on $SV(\tilde{X})$, if $\|\,.\,\|$ satisfies the following conditions:

1) $\|\tilde{x}_e\| \tilde{\geq} \tilde{0}$ for all $SV(\tilde{X})$ and

$\|\tilde{x}_e\| = \tilde{0} \Leftrightarrow \tilde{x}_e = \tilde{\theta}_0$

2) $\|\tilde{r}.\tilde{x}_e\| = |\tilde{r}|\|\tilde{x}_e\|$ for all $\tilde{x}_e \tilde{\in} SV(\tilde{X})$ for every soft scalar $\tilde{r}$

3) $\|\tilde{x}_e + \tilde{y}_{e'}\| = \|\tilde{x}_e\| + \|\tilde{y}_{e'}\|$ for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SV(\tilde{X})$

The soft vector space $SV(\tilde{X})$ with a soft norm $\|\,.\,\|$ on $\tilde{X}$ is said to be a soft normed linear space and is denoted by $(\tilde{X}, \|\,.\,\|)$.

Definition 2.3

Let X be a linear space over the field F (real or complex) and $*$ is a continuous t-norm. A fuzzy subset N on $X \times \mathbb{R}$, $\mathbb{R}$ - set of all real numbers is called a fuzzy norm on X if and only if for $x, y \in X$ and $c \in F$

1) $\forall\ t \in \mathbb{R}$ with $t \leq 0$, $N(x,t)=0$

2) $\forall\ t \in \mathbb{R}$ with $t > 0$, $N(x,t)=1$ if and only if $x=0$

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3) $\forall\, t \in \mathbb{R}$ with $t > 0$, $\mathrm{N}(cx,t) = \mathrm{N}\left(x, \frac{t}{|c|}\right)$ if $c \neq 0$

4) $\forall\, s, t \in \mathbb{R},\ x, y \in \mathrm{X};\ \mathrm{N}(x+y, t+s) \geq \mathrm{N}(x,t) * \mathrm{N}(y,s)$

5) $\mathrm{N}(x,.)$ is a continuous nondecreasing function of $\mathbb{R}$ and $\lim_{x\to\infty} \mathrm{N}(x,t) = 1$

The triplet $(\mathrm{X}, \mathrm{N}, *)$ will be referred to as a fuzzy normed linear space.

3. Fuzzy soft normed linear space

Definition 3.1

Let $\tilde{\mathrm{X}}$ be an absolute soft linear space over the scalar field K. Suppose $*$ is a continuous t-norm, $\mathbb{R}\left(\mathrm{A}^*\right)$ is the set of all nonnegative soft real numbers and $\mathrm{SSP}(\tilde{\mathrm{X}})$ denote the set of all soft points on $\tilde{\mathrm{X}}$. A fuzzy subset Γ on $\mathrm{SSP}(\tilde{\mathrm{X}}) \times \mathbb{R}\left(\mathrm{A}^*\right)$ is called a fuzzy soft norm on $\tilde{\mathrm{X}}$ if and only if for $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \mathrm{SSP}(\tilde{\mathrm{X}})$ and $\tilde{\mathrm{k}} \tilde{\in} \mathrm{K}$ (where $\tilde{\mathrm{k}}$ is a soft scalar) the following conditions hold

1) $\Gamma(\tilde{x}_e, \tilde{t}) = 0\ \forall\, \tilde{t} \tilde{\in} \mathrm{R}\left(\mathrm{A}^*\right)$ with $\tilde{t} \tilde{\leq} \tilde{0}$

2) $\Gamma(\tilde{x}_e, \tilde{t}) = 1\ \forall\, \tilde{t} \tilde{\in} \mathbb{R}\left(\mathrm{A}^*\right)$ with $\tilde{t} \tilde{>} \tilde{0}$ if and only if $\tilde{x}_e = \tilde{\theta}_0$

3) $\Gamma(\tilde{\mathrm{k}} \odot \tilde{x}_e, \tilde{t}) = \Gamma\left(\tilde{x}_e, \frac{\tilde{t}}{|\tilde{\mathrm{k}}|}\right)$ if $\tilde{\mathrm{k}} \neq \tilde{0}\ \forall\, \tilde{t} \tilde{\in} \mathbb{R}\left(\mathrm{A}^*\right),\ \tilde{t} \tilde{>} \tilde{0}$

4) $\Gamma(\tilde{x}_e \oplus \tilde{y}_{e'}, \tilde{t} \oplus \tilde{s}) \tilde{\geq} \Gamma(\tilde{x}_e, \tilde{t}) * \Gamma(\tilde{y}_e, \tilde{s}),\ \forall \tilde{s}, \tilde{t} \tilde{\in} \mathbb{R}\left(\mathrm{A}^*\right),\ \tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \mathrm{SSP}(\tilde{\mathrm{X}})$

5) $\Gamma(\tilde{x}_e, .)$ is a continuous nondecreasing function of $\mathbb{R}\left(\mathrm{A}^*\right)$ and $\lim_{\tilde{t}\to\infty} \Gamma(\tilde{x}_e, \tilde{t}) = 1$

The triplet $(\tilde{\mathrm{X}}, \Gamma, *)$ will be referred to as a fuzzy soft normed linear space.

Definition 3.2

Let $(\tilde{\mathrm{X}}, \Gamma, *)$ be a fuzzy soft normed linear space and $\tilde{t} \tilde{>} \tilde{0}$ be a soft real number. We define an open ball, a closed ball and a sphere with centre at $\tilde{x}_{e_1}$ and radius α as follows

$$\mathrm{B}\left(\tilde{x}_{e_1}, \alpha, \tilde{t}\right) = \left\{\tilde{y}_{e_2} \tilde{\in} \mathrm{SSP}(\tilde{\mathrm{X}}) : \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right) \tilde{>} 1 - \alpha\right\}$$

$$\overline{\mathrm{B}}\left(\tilde{x}_{e_1}, \alpha, \tilde{t}\right) = \left\{\tilde{y}_{e_2} \tilde{\in} \mathrm{SSP}(\tilde{\mathrm{X}}) : \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right) \tilde{\geq} 1 - \alpha\right\}$$

$$\mathrm{S}\left(\tilde{x}_{e_1}, \alpha, \tilde{t}\right) = \left\{\tilde{y}_{e_2} \tilde{\in} \mathrm{SSP}(\tilde{\mathrm{X}}) : \Gamma\left(\tilde{x}_{e_1} - \tilde{y}_{e_2}, \tilde{t}\right) = 1 - \alpha\right\}$$

$\mathrm{SFS}\left(\mathrm{B}\left(\tilde{x}_{e_1}, \alpha, \tilde{t}\right)\right)$, $\mathrm{SFS}\left(\overline{\mathrm{B}}\left(\tilde{x}_{e_1}, \alpha, \tilde{t}\right)\right)$ and $\mathrm{SFS}\left(\mathrm{S}\left(\tilde{x}_{e_1}, \alpha, \tilde{t}\right)\right)$ are called a fuzzy soft open ball, a fuzzy soft closed ball and a fuzzy soft sphere respectively with centre $\tilde{x}_{e_1}$ at and radius α .

Definition 3.3

A mapping $\Delta : \mathrm{SSP}(\tilde{\mathrm{X}}) \times \mathrm{SSP}(\tilde{\mathrm{X}}) \times \mathbb{R}\left(\mathrm{A}^*\right) \to [0,1]$ is said to be a fuzzy soft metric on the soft set $\tilde{\mathrm{X}}$ if Δ satisfies the following conditions

1) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = 0$, for all $\tilde{t} \tilde{\leq} \tilde{0}$

2) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = 1$, for all $\tilde{t} \tilde{>} \tilde{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$

3) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{t}\right) = \Delta\left(\tilde{y}_{e_2}, \tilde{x}_{e_1}, \tilde{t}\right)$

4) $\Delta\left(\tilde{x}_{e_1}, \tilde{z}_{e_3}, \tilde{s} \oplus \tilde{t}\right) \tilde{\geq} \Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{s}\right) * \Delta\left(\tilde{y}_{e_2}, \tilde{z}_{e_3}, \tilde{t}\right)$ for all $\tilde{t}, \tilde{s} \tilde{>} \tilde{0}$

5) $\Delta\left(\tilde{x}_{e_1}, \tilde{y}_{e_2}, .\right) : (0, \infty) \to [0,1]$ is continuous

The soft set $\tilde{\mathrm{X}}$ with a fuzzy soft metric Δ is called a fuzzy soft metric space and denoted by $(\tilde{\mathrm{X}}, \Delta, *)$.

Definition 3.4

Let be a sequence $\{\tilde{x}^n_{e_j}\}$ of soft vectors in a fuzzy soft normed linear space $(\tilde{X},\Gamma,*)$. Then the sequence converges to $\tilde{x}^0_{e_j}$ with respect to fuzzy soft norm Γ if $\Gamma(\tilde{x}^n_{e_j}-\tilde{x}^0_{e_j},\tilde{t})\tilde{\geq}1-\alpha$ for every $n\geq n_0$ and $\alpha\in(0,1]$ where n_0 is a positive integer and $\tilde{t}\tilde{>}\tilde{0}$.

Or

$\lim_{n\to\infty}\Gamma(\tilde{x}^n_{e_j}-\tilde{x}^0_{e_j},\tilde{t})=1$, as $\tilde{t}\to\infty$

Similarly if $\lim_{n\to\infty}\Delta(\tilde{x}^n_{e_j},\tilde{x}^0_{e_j},\tilde{t})=1$, as $\tilde{t}\to\infty$ then $\{\tilde{x}^n_{e_j}\}$ is a convergent sequence in fuzzy soft metric space $(\tilde{X},\Delta,*)$.

Definition 3.5

A sequence $\{\tilde{x}^n_{e_j}\}$ in a fuzzy soft normed linear space $(\tilde{X},\Gamma,*)$ is said to be a Cauchy sequence with respect to the fuzzy soft norm Γ if $\Gamma(\tilde{x}^n_{e_j}-\tilde{x}^m_{e_j},\tilde{t})\tilde{\geq}1-\alpha$ for every $n,m\geq n_0$ and $\alpha\in(0,1]$ where n_0 is a positive integer and $\tilde{t}\tilde{>}\tilde{0}$.

Or

$\lim_{n,m\to\infty}\Gamma(\tilde{x}^n_{e_j}-\tilde{x}^m_{e_j},\tilde{t})=1$, as $\tilde{t}\to\infty$

Similarly if $\lim_{n\to\infty}\Delta(\tilde{x}^n_{e_j},\tilde{x}^0_{e_j},\tilde{t})=1$ as $\tilde{t}\to\infty$ then $\{\tilde{x}^n_{e_j}\}$ is a Cauchy sequence in fuzzy soft metric space $(\tilde{X},\Delta,*)$.

Definition 3.6

Let $SSP(\tilde{X})$ and $SSP(\tilde{Y})$ be set of all soft points on soft normed linear spaces $\tilde{X}$ and $\tilde{Y}$ respectively also let E and E' be the corresponding parameter sets. The map from the soft point $\tilde{x}_e$ on $\tilde{X}$ to the soft point $T(\tilde{x}_e)$ on $\tilde{Y}$ is denoted as $T:SSP(\tilde{X})\to SSP(\tilde{Y})$.

Definition 3.7

Let $(\tilde{X},\Gamma,*)$ be a fuzzy soft normed linear space. $\bar{B}(\tilde{x}_{e_1},\alpha,\tilde{t})=\{\tilde{y}_{e_2}\tilde{\in}SSP(\tilde{X}):\Gamma(\tilde{x}_{e_1}-\tilde{y}_{e_2},\tilde{t})\tilde{\geq}1-\alpha\}$ is said to be a fuzzy soft closed ball centered at $\tilde{x}_{e_1}$ of radius α with respect to $\tilde{t}$ if and only if any sequence $\{\tilde{x}_{e_n}\}$ in $\bar{B}(\tilde{x}_{e_1},\alpha,\tilde{t})$ converges to $\tilde{y}_{e_2}\tilde{\in}\bar{B}(\tilde{x}_{e_1},\alpha,\tilde{t})$.

Definition 3.8

Let $(\tilde{X},\Gamma,*)$ be a fuzzy soft normed linear space. The mapping $T:SSP(\tilde{X})\to SSP(\tilde{X})$ is said to be fuzzy soft contraction if there exists $c\in(0,1]$ such that T satisfies $c\Gamma(T(\tilde{x}_e),T(\tilde{y}_{e'}),\tilde{t})\tilde{\geq}\Gamma(\tilde{x}_e,\tilde{y}_{e'},\tilde{t})$.

Definition 3.9

Let $(\tilde{X},\Delta,*)$ be a fuzzy soft metric space and $T,S:SSP(\tilde{X})\to SSP(\tilde{X})$. The map T is called S–contraction if there exists $\alpha\in(0,1]$ such that $\Delta(T(\tilde{x}_e),T(\tilde{y}_{e'}),\tilde{t})\tilde{\leq}\alpha\,\Delta(S(\tilde{x}_e),S(\tilde{y}_{e'}),\tilde{t})$ for all $\tilde{x}_e,\tilde{y}_{e'}\tilde{\in}SSP(\tilde{X})$ holds.

Definition 3.10

Let T and S be self mappings on $SSP(\tilde{X})$. If $T(\tilde{x}_e)=S(\tilde{x}_e)=\tilde{w}_{e'}$ for some $\tilde{x}_e$ in $SSP(\tilde{X})$, then $\tilde{x}_e$ is called coincidence point of and $\tilde{w}_{e'}$ is called point of coincidence of T and S.

Definition 3.11

A pair of maps $\{T,S\}$ is called weakly compatible pair if they commute at coincidence point $T(\tilde{x}_e)=S(\tilde{x}_e)\Rightarrow TS(\tilde{x}_e)=ST(\tilde{x}_e)$.

Definition 3.12

A pair of self mappings $\{T,S\}$ on $SSP(\tilde{X})$ of a fuzzy soft metric space $(\tilde{X},\Delta,*)$ is said to be R-weakly commuting if there exists some $R>0$ such that $\Delta(TS(\tilde{x}_e),ST(\tilde{x}_e),\tilde{t})\tilde{\geq}\Delta(T(\tilde{x}_e),S(\tilde{x}_e),\frac{\tilde{t}}{R})$.

$$\Delta(TS(\tilde{x}_e),ST(\tilde{x}_e),\tilde{t})\tilde{\geq}\Delta(T(\tilde{x}_e),S(\tilde{x}_e),\frac{\tilde{t}}{R})\cdot$$

4. Fixed Point Theorems on Fuzzy Soft Normed Linear Space

Theorem 4.1

Suppose $(\tilde{X},\Gamma,*)$ is a fuzzy Banach space. Let $T:SSP(\tilde{X})\to SSP(\tilde{X})$ be a fuzzy soft contractive mapping on $\bar{B}(\tilde{x}_{e_1},\alpha,\tilde{t})$ with contraction constant $c\in(0,1]$ and $c\Gamma(\tilde{x}_e,T_{up}(\tilde{x}_e),\tilde{t})\tilde{\geq}1-\alpha$. Then there exists a sequence $\{\tilde{x}_{e_n}\}$ in $SSP(\tilde{X})$ such that $\Gamma(\tilde{x}_e,\tilde{x}_{e_n},\tilde{t})\tilde{\geq}1-\alpha$.

Proof

Assume

$$\tilde{x}_{e_1}=T(\tilde{x}_{e_0})$$

$$\tilde{x}_{e_2}=T(\tilde{x}_{e_1})=T(T(\tilde{x}_{e_0}))=T^2(\tilde{x}_{e_0})$$

…

$$\tilde{x}_{e_n}=T(\tilde{x}_{e_{n-1}})=T^n(\tilde{x}_{e_0})$$

By the given condition, for any point $\tilde{x}_{e_0}\tilde{\in}SSP(\tilde{X})$

$$c\Gamma(\tilde{x}_{e_0},T(\tilde{x}_{e_0}),\tilde{t})\tilde{\geq}1-\alpha$$

$$\Gamma(\tilde{x}_{e_0},T(\tilde{x}_{e_0}),\tilde{t})\ \tilde{\geq}\frac{1-\alpha}{c}\tilde{\geq}1-\alpha$$

$$\Gamma(\tilde{x}_{e_0},\tilde{x}_{e_1},\tilde{t})\tilde{\geq}1-\alpha \qquad (1)$$

This implies

$$\tilde{x}_{e_1}\tilde{\in}\bar{B}(\tilde{x}_{e_0},\alpha,\tilde{t})$$

Assume $\tilde{x}_{e_1},\tilde{x}_{e_2},\tilde{x}_{e_3},...\tilde{x}_{e_{n-1}}\tilde{\in}\bar{B}(\tilde{x}_{e_0},\alpha,\tilde{t})$

To claim that $\tilde{x}_{e_n}\tilde{\in}\bar{B}(\tilde{x}_{e_0},\alpha,\tilde{t})$

$$c\Gamma(\tilde{x}_{e_1},\tilde{x}_{e_2},\tilde{t})=c\Gamma(T(\tilde{x}_{e_0}),T(\tilde{x}_{e_1}),\tilde{t})$$

$$\tilde{\geq}\Gamma(\tilde{x}_{e_0},\tilde{x}_{e_1},\tilde{t}) \quad \text{Since T is a fuzzy soft contraction map}$$

$$\tilde{\geq}1-\alpha \quad \text{By (1)}$$

$$\Gamma(\tilde{x}_{e_1},\tilde{x}_{e_2},\tilde{t})\tilde{\geq}\frac{1-\alpha}{c}\tilde{\geq}1-\alpha$$

$$\Gamma(\tilde{x}_{e_1},\tilde{x}_{e_2},\tilde{t})\tilde{\geq}1-\alpha \qquad (2)$$

$$c\Gamma(\tilde{x}_{e_2},\tilde{x}_{e_3},\tilde{t})=c\Gamma(T(\tilde{x}_{e_1}),T(\tilde{x}_{e_2}),\tilde{t})$$

$$\tilde{\geq}\Gamma(\tilde{x}_{e_1},\tilde{x}_{e_2},\tilde{t})$$

$$\tilde{\geq}1-\alpha \quad \text{By (2)}$$

$$\Gamma(\tilde{x}_{e_2},\tilde{x}_{e_3},\tilde{t})\tilde{\geq}\frac{1-\alpha}{c}\tilde{\geq}1-\alpha$$

$$\Gamma(\tilde{x}_{e_2},\tilde{x}_{e_3},\tilde{t})\tilde{\geq}1-\alpha$$

Similarly

$$\Gamma(\tilde{x}_{e_3},\tilde{x}_{e_4},\tilde{t})\tilde{\geq}1-\alpha$$

…

$$\Gamma(\tilde{x}_{e_{n-1}},\tilde{x}_{e_n},\tilde{t})\tilde{\geq}1-\alpha$$

$$\Gamma(\tilde{x}_{e_0},\tilde{x}_{e_n},\tilde{t})\tilde{\geq}\Gamma\left(\tilde{x}_{e_0},\tilde{x}_{e_1},\frac{\tilde{t}}{n}\right)*\Gamma\left(\tilde{x}_{e_1},\tilde{x}_{e_2},\frac{\tilde{t}}{n}\right)*...*\Gamma\left(\tilde{x}_{e_{n-1}},\tilde{x}_{e_n},\frac{\tilde{t}}{n}\right)$$

$$\tilde{\geq}(1-\alpha)*(1-\alpha)*...*(1-\alpha)$$

$$=1-\alpha$$

$$\Gamma(\tilde{x}_{e_0},\tilde{x}_{e_n},\tilde{t})\tilde{\geq}1-\alpha$$

This shows $\tilde{x}_{e_n}\tilde{\in}\bar{B}(\tilde{x}_{e_0},\alpha,\tilde{t})$ and this implies $\{\tilde{x}_{e_n}\}$ converges to $\tilde{x}_{e_0}$.

Uniqueness of $\tilde{x}_{e_0}$ is true directly from the proof limit of a sequence in a fuzzy soft normed linear space if exists is unique.

Theorem 4.2

Let $(\tilde{X},\Gamma,*)$ be a fuzzy soft normed linear space. Let $A,B:SSP(\tilde{X})\to SSP(\tilde{X})$ be a self map satisfying the condition: there exists a $\lambda\in(0,1)$ such that $\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),\tilde{t}\right)\tilde{\geq}1-\tilde{t}\Rightarrow\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\lambda\tilde{t}\right)\tilde{\geq}1-\lambda\tilde{t}$ (1)

for all $\tilde{x}_{e_1},\tilde{y}_{e_2}\tilde{\in}SSP(\tilde{X})$ and for all $\tilde{t}\tilde{>}0$

Also A is a B- contraction. Then

1) For any real number $\varepsilon>0$ there exists $k_0(\varepsilon)\in N$ such that $A(\tilde{x}_{e_1})\to A(\tilde{y}_{e_2})$

2) A and B have unique common fixed point.

Proof (1)

Choose $\tilde{t}=1$, for every $\varepsilon\in(0,1)$ there exists $k_0=k_0(\varepsilon)$ such that for all $k\geq k_0$ and for every $\tilde{x}_{e_1},\tilde{y}_{e_2}\tilde{\in}SSP(\tilde{X})$

$$\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),1\right)\tilde{\geq}0\Rightarrow\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\varepsilon\right)\tilde{\geq}1-\varepsilon \quad (2)$$

It easy to show the same for $\tilde{t}=1+\varepsilon$ and for any real number $\varepsilon>0$.

$$\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),1+\varepsilon\right)\tilde{\geq}1-(1+\varepsilon)\Rightarrow\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\lambda(1+\varepsilon)\right)\tilde{\geq}1-\lambda(1+\varepsilon) \quad (3)$$

Since A is a B- contraction

That is

$$\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\tilde{t}\right)\tilde{\leq}\alpha\,\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),\tilde{t}\right)$$

Since $\alpha\in(0,1]$, take $\alpha=1$.

From (3)

$$\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\lambda(1+\varepsilon)\right)\tilde{\leq}\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),\lambda(1+\varepsilon)\right)$$

Therefore (3) implies

$$\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),\lambda(1+\varepsilon)\right)\tilde{\geq}\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\lambda(1+\varepsilon)\right)\tilde{\geq}1-\lambda(1+\varepsilon)$$

$$\Gamma\left(B(\tilde{x}_{e_1})-B(\tilde{y}_{e_2}),\lambda(1+\varepsilon)\right)\tilde{\geq}1-\lambda(1+\varepsilon)$$

Again by condition (1), the above implies

$$\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\lambda^k(1+\varepsilon)\right)\tilde{\geq}1-\lambda^k(1+\varepsilon) \quad (4)$$

As $k\to\infty$

$$\Gamma\left(A(\tilde{x}_{e_1})-A(\tilde{y}_{e_2}),\varepsilon\right)\tilde{\geq}1-\varepsilon$$

Hence $A(\tilde{x}_{e_1})\to A(\tilde{y}_{e_2})$.

Proof (2)

Suppose $\tilde{z}_{e_j}$ is a fixed point of A and $\tilde{z}_{e_k}$ is a fixed point of B, where $\tilde{z}_{e_j},\tilde{z}_{e_k}\tilde{\in}SSP(\tilde{X})$.

That is

$$A(\tilde{z}_{e_j})=\tilde{z}_{e_j} \text{ and } B(\tilde{z}_{e_k})=\tilde{z}_{e_k} \quad (5)$$

By condition (1)

$$\Gamma\left(B(\tilde{z}_{e_j})-B(\tilde{z}_{e_k}),\tilde{t}\right)\tilde{\geq}1-\tilde{t}\Rightarrow\Gamma\left(A(\tilde{z}_{e_j})-A(\tilde{z}_{e_k}),\lambda\tilde{t}\right)\tilde{\geq}1-\lambda\tilde{t}$$

$$\Gamma\left(B(\tilde{z}_{e_j})-\tilde{z}_{e_k},\tilde{t}\right)\tilde{\geq}1-\tilde{t}\Rightarrow\Gamma\left(\tilde{z}_{e_j}-A(\tilde{z}_{e_k}),\lambda\tilde{t}\right)\tilde{\geq}1-\lambda\tilde{t}$$

Using (4)

$$\Gamma\left(\tilde{z}_{e_j}-A(\tilde{z}_{e_k}),\lambda^k(1+\varepsilon)\right)\tilde{\geq}1-\lambda^k(1+\varepsilon)$$

$$\Gamma\left(\tilde{z}_{e_j}-A(\tilde{z}_{e_k}),\varepsilon\right)\tilde{\geq}1-\varepsilon \text{ as } k\to\infty$$

This implies

$$A(\tilde{z}_{e_k})=\tilde{z}_{e_j}$$

Hence by (5) the fixed points of A and B are same.

Theorem 4.3

Let $(\tilde{X},\Gamma,*)$ be a complete fuzzy soft normed linear space and let $T,S:SSP(\tilde{X})\to SSP(\tilde{X})$ be a pair of continuous self mappings satisfying the following conditions

$$\Gamma\left(\mathrm{T}\left(\tilde{x}_e\right)-\mathrm{T}\left(\tilde{y}_{e'}\right),\tilde{t}\right)\tilde{\geq}1-\lambda+\lambda\Gamma\left(\mathrm{S}\left(\tilde{x}_e\right)-\mathrm{S}\left(\tilde{y}_{e'}\right),\frac{\tilde{t}}{\lambda}\right) \quad (1)$$

and $\lim_{t\to\infty}\Gamma\left(\mathrm{T}^n\left(\tilde{x}_{e_{j_0}}\right)-\mathrm{S}^n\left(\tilde{x}_{e_{j_0}}\right),\tilde{t}\right)=1$ as $n\to\infty$ (2)

for all $\tilde{x}_e,\tilde{y}_{e'}\tilde{\in}\mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$, $\tilde{t}\tilde{>}0$ and $\lambda\in(0,1)$.

Then T and S have a unique common fixed point. Note that S is a fuzzy soft contraction mapping.

Proof

Fix $\tilde{x}_{e_{j_0}}\tilde{\in}\mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$

Such that choose $\left\{\tilde{x}_{e_{j_n}}=\mathrm{T}^n\left(\tilde{x}_{e_{j_0}}\right)\right\}_{n=1}^{\infty}$

Let $m=n+1$, where $n\in\mathrm{N}$

By induction,

If $n=1$

$$\begin{aligned}\Gamma\left(\tilde{x}_{e_{j_2}}-\tilde{x}_{e_{j_1}},\tilde{t}\right)&=\Gamma\left(\mathrm{T}^2\left(\tilde{x}_{e_{j_0}}\right)-\mathrm{T}\left(\tilde{x}_{e_{j_0}}\right),\tilde{t}\right)\\&=\Gamma\left(\mathrm{T}\mathrm{T}\tilde{x}_{e_{j_0}}-\mathrm{T}\tilde{x}_{e_{j_0}},\tilde{t}\right)\\&=\Gamma\left(\mathrm{T}\tilde{x}_{e_{j_1}}-\mathrm{T}\tilde{x}_{e_{j_0}},\tilde{t}\right)\\&\tilde{\geq}1\text{-}\lambda+\lambda\Gamma\left(\mathrm{S}\left(\tilde{x}_{e_{j_1}}\right)-\mathrm{T}\left(\tilde{x}_{e_{j_0}}\right),\frac{\tilde{t}}{\lambda}\right) \qquad \text{By (1)}\\&=1\text{-}\lambda+\lambda\Gamma\left(\tilde{x}_{e_{j_1}}-\tilde{x}_{e_{j_0}},\frac{\tilde{t}}{\lambda}\right)\\&\tilde{\geq}1\text{-}\lambda+\lambda(1\text{-}\lambda)\end{aligned}$$

$$\Gamma\left(\tilde{x}_{e_{j_2}}-\tilde{x}_{e_{j_1}},\tilde{t}\right)\tilde{\geq}1\text{-}\lambda+\lambda(1\text{-}\lambda)$$

If $n=2$

$$\begin{aligned}\Gamma\left(\tilde{x}_{e_{j_3}}-\tilde{x}_{e_{j_2}},\tilde{t}\right)&=\Gamma\left(\mathrm{T}^3\left(\tilde{x}_{e_{j_0}}\right)-\mathrm{T}^2\left(\tilde{x}_{e_{j_0}}\right),\tilde{t}\right)\\&=\Gamma\left(\mathrm{T}^2\mathrm{T}\tilde{x}_{e_{j_0}}-\mathrm{T}\tilde{x}_{e_{j_1}},\tilde{t}\right)\\&=\Gamma\left(\mathrm{T}\mathrm{T}^2\tilde{x}_{e_{j_0}}-\mathrm{T}\tilde{x}_{e_{j_1}},\tilde{t}\right)\\&=\Gamma\left(\mathrm{T}\tilde{x}_{e_{j_2}}-\mathrm{T}\tilde{x}_{e_{j_1}},\tilde{t}\right)\\&\tilde{\geq}1\text{-}\lambda+\lambda\Gamma\left(\mathrm{S}\left(\tilde{x}_{e_{j_2}}\right)-\mathrm{T}\left(\tilde{x}_{e_{j_1}}\right),\frac{\tilde{t}}{\lambda}\right)\\&=1\text{-}\lambda+\lambda\Gamma\left(\tilde{x}_{e_{j_2}}-\tilde{x}_{e_{j_1}},\frac{\tilde{t}}{\lambda}\right)\\&\tilde{\geq}1\text{-}\lambda+\lambda\left[1\text{-}\lambda+\lambda(1\text{-}\lambda)\right]\\&=1\text{-}\lambda+\lambda(1\text{-}\lambda)+\lambda^2(1\text{-}\lambda)\end{aligned}$$

$$\Gamma\left(\tilde{x}_{e_{j_3}}-\tilde{x}_{e_{j_2}},\tilde{t}\right)\tilde{\geq}1\text{-}\lambda+\lambda(1\text{-}\lambda)+\lambda^2(1\text{-}\lambda)$$

Similarly,

$$\begin{aligned}\Gamma\left(\tilde{x}_{e_{j_m}}-\tilde{x}_{e_{j_n}},\tilde{t}\right)&\tilde{\geq}1\text{-}\lambda+\lambda(1\text{-}\lambda)+\lambda^2(1\text{-}\lambda)+\ldots+\lambda^n(1\text{-}\lambda)\\&=(1\text{-}\lambda)\left[1+\lambda+\lambda^2+\ldots+\lambda^n\right]\\&=(1\text{-}\lambda)\left[\frac{1\text{-}\lambda^{n-1}}{1\text{-}\lambda}\right]\\&=1\text{-}\lambda^{n-1}\quad\to1\quad\text{as}\quad n\to\infty\end{aligned}$$

$$\lim_{t\to\infty}\Gamma\left(\tilde{x}_{e_{j_m}}-\tilde{x}_{e_{j_n}},\tilde{t}\right)=1 \qquad \text{as}\quad n\to\infty$$

Hence $\left\{\tilde{x}_{e_{j_n}}\right\}$ is a Cauchy sequence.

Suppose $\tilde{x}_{e_{j_n}}\to\tilde{x}_{e_j}$

By continuity of T

$T^n \tilde{x}_{e_{j_0}} \to T\tilde{x}_{e_j} = \tilde{x}_{e_j}$ {Since T is a self mapping}

Therefore $\tilde{x}_{e_j}$ is a fixed point of T.

Similarly for a given $\tilde{x}_{e_{j_0}} \tilde{\in} SSP(\tilde{X})$, it is easy to prove that $S^n(\tilde{x}_{e_{j_0}})$ converges to $\tilde{y}_{e_j}$.

Now to prove that $\tilde{x}_{e_j} = \tilde{y}_{e_j}$.

Let $H(T)$ be the set of all fixed points of T and $H(S)$ be the set of all fixed points of S.

Since $\tilde{x}_{e_j} \tilde{\in} H(T)$ and $\tilde{y}_{e_j} \tilde{\in} H(S)$

$H(T) \neq \phi$ and $H(S) \neq \phi$

Consider,

$$\Gamma\left(\tilde{x}_{e_j} - \tilde{y}_{e_j}, \tilde{t}\right) = \lim_{\tilde{t}\to\infty} \Gamma\left(T^n \tilde{x}_{e_{j_0}} - S^n \tilde{x}_{e_{j_0}}, \tilde{t}\right)$$
$$\to 1 \qquad \text{as} \quad n \to \infty$$
$$\Gamma\left(\tilde{x}_{e_j} - \tilde{y}_{e_j}, \tilde{t}\right) = 1$$
$$\tilde{x}_{e_j} - \tilde{y}_{e_j} = \tilde{\theta}$$
$$\tilde{x}_{e_j} = \tilde{y}_{e_j}$$

This implies,

T and S have a unique common fixed point.

Theorem 4.4

Let $(\tilde{X}, \Delta, *)$ be a complete fuzzy soft metric space and let A be a self mapping of $SSP(\tilde{X})$ also let S be a continuous self mapping of $SSP(\tilde{X})$. Let the pair $\{A, S\}$ be R–weakly commuting and

1) $S(\tilde{x}_e) \subseteq A(\tilde{x}_e)$

2) $\Delta(A\tilde{x}_e, A\tilde{y}_{e'}, q\tilde{t}) \tilde{\geq} r\left[\min\{\Delta(S\tilde{x}_e, A\tilde{y}_{e'}, \tilde{t}), \Delta(S\tilde{x}_e, A\tilde{x}_e, \tilde{t}), \Delta(A\tilde{x}_e, A\tilde{y}_{e'}, \tilde{t})\}\right]$

for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} SSP(\tilde{X})$ and for all $\tilde{t} > 0$ where $r:[0,1] \to [0,1]$ is a continuous function such that $r(\tilde{t}) > \tilde{t}$ for each $0 \leq \tilde{t} \leq 1$ and $r(\tilde{t}) = 1$ for $\tilde{t} = 1$. Then A and S have unique common fixed point in $SSP(\tilde{X})$.

Proof

Define two sequences $\{\tilde{x}_{e_n}\}$ and $\{\tilde{y}_{e'_n}\}$ in $SSP(\tilde{X})$ as $S\tilde{x}_{e_{2n+1}} = A\tilde{x}_{e_{2n}} = \tilde{y}_{e'_{2n}}$

Applying condition (2), we get

$$\Delta\left(A\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n}}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(S\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n}}, \tilde{t}\right), \Delta\left(S\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n+1}}, \tilde{t}\right), \Delta\left(A\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n}}, \tilde{t}\right)\right\}\right]$$
$$\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right), \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n+1}}, \tilde{t}\right), \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right\}\right]$$
$$= r\left[\min\left\{1, \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n+1}}, \tilde{t}\right), \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right\}\right]$$
$$= r\left[\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right]$$
$$\tilde{>} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)$$
$$\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, q\tilde{t}\right) \tilde{>} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)$$

This implies $\tilde{y}_{e'_{2n+1}} = \tilde{y}_{e'_{2n}}$ for all $n \geq 0$.

Hence $\{\tilde{y}_{e'_{2n}}\}$ is a constant sequence and therefore it is a Cauchy sequence in $SSP(\tilde{X})$.

By completeness of $\tilde{X}$, $\{\tilde{y}_{e'_n}\}$ converges to $\tilde{v}_{e_j}$ in $S(\tilde{x}_E)$.

Using condition (1), $\{\tilde{y}_{e'_n}\}$ also converges to $\tilde{v}_{e_j}$ in $A(\tilde{x}_E)$.

Given $\{A, S\}$ is R–weakly commuting

$$\Delta\left(AS\tilde{x}_{e_{2n+1}}, SA\tilde{x}_{e_{2n+1}}, \tilde{t}\right) \tilde{\geq} \Delta\left(A\tilde{x}_{e_{2n+1}}, S\tilde{x}_{e_{2n+1}}, \frac{\tilde{t}}{R}\right)$$

Since $S\tilde{x}_{e_{2n+1}} = \tilde{y}_{e'_{2n}}$, $A\tilde{x}_{e_{2n+1}} = \tilde{y}_{e'_{2n+1}}$

$\Delta\left(A\tilde{y}_{e'_{2n}}, S\tilde{y}_{e'_{2n+1}}, \tilde{t}\right) \tilde{\geq} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \frac{\tilde{t}}{R}\right)$

On taking limit $n \to \infty$,we get

$\Delta\left(A\tilde{v}_{e_j}, S\tilde{v}_{e_j}, \tilde{t}\right) \tilde{\geq} \Delta\left(\tilde{v}_{e_j}, \tilde{v}_{e_j}, \frac{\tilde{t}}{R}\right) = 1$

$\Delta\left(A\tilde{v}_{e_j}, S\tilde{v}_{e_j}, \tilde{t}\right) = 1$

$A\tilde{v}_{e_j} = S\tilde{v}_{e_j}$

Now to show that $A\tilde{v}_{e_j} = S\tilde{v}_{e_j} = \tilde{v}_{e_j}$

$\Delta\left(AS\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n}}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(S^2\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n}}, \tilde{t}\right), \Delta\left(S^2\tilde{x}_{e_{2n+1}}, AS\tilde{x}_{e_{2n+1}}, \tilde{t}\right), \Delta\left(AS\tilde{x}_{e_{2n+1}}, A\tilde{x}_{e_{2n}}, \tilde{t}\right)\right\}\right]$

$\Delta\left(A\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(S\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right), \Delta\left(S\tilde{y}_{e'_{2n}}, A\tilde{y}_{e'_{2n}}, \tilde{t}\right), \Delta\left(A\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right\}\right]$

As limit $n \to \infty$

$\Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(S\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(S\tilde{v}_{e_j}, A\tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right\}\right]$

$= r\left[\min\left\{\Delta\left(S\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(S\tilde{v}_{e_j}, S\tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(S\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right\}\right]$

$= r\left[\min\left\{\Delta\left(S\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), 1, \Delta\left(S\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right\}\right]$

$= r\left[\Delta\left(S\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right] = r\left[\Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right] \quad \{Since \ S\tilde{v}_{e_j} = A\tilde{v}_{e_j}\}$

$\Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, q\tilde{t}\right) \tilde{\geq} r\left[\Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right] \tilde{>} \Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)$

$\Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) \tilde{>} \Delta\left(A\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)$

This implies

$A\tilde{v}_{e_j} = \tilde{v}_{e_j}$

Hence $\tilde{v}_{e_j}$ is the common fixed point of A and S .

Now to show that the uniqueness

Suppose there is another fixed point $\tilde{z}_{e_k} \neq \tilde{v}_{e_j}$. Then

$\Delta\left(A\tilde{v}_{e_j}, A\tilde{z}_{e_k}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(S\tilde{v}_{e_j}, A\tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(S\tilde{v}_{e_j}, A\tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(A\tilde{v}_{e_j}, A\tilde{z}_{e_k}, \tilde{t}\right)\right\}\right]$

$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)\right\}\right]$

$= r\left[\min\left\{\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right), 1, \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)\right\}\right]$

$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) = r\left[\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)\right]$

$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) \tilde{>} \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)$

$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) \tilde{>} \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)$

And so $\tilde{v}_{e_j} = \tilde{z}_{e_k}$ which is a contradiction to our assumption.

Therefore $\tilde{v}_{e_j} = \tilde{z}_{e_k}$.

Hence, A and S have unique common fixed point in $SSP\left(\tilde{X}\right)$.

Theorem 4.5

Let $\left(\tilde{X}, \Delta, *\right)$ be a complete fuzzy soft metric space and let A be a self mapping of $SSP\left(\tilde{X}\right)$ also let S be a continuous self mapping of $SSP\left(\tilde{X}\right)$. Let the pair $\{A, S\}$ be R–weakly commuting and

1) $S\left(\tilde{x}_e\right) \subseteq A\left(\tilde{x}_e\right)$

2) $\Delta\left(A\tilde{x}_e, A\tilde{y}_{e'}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(S\tilde{x}_e, A\tilde{y}_{e'}, \tilde{t}\right), \Delta\left(S\tilde{x}_e, A\tilde{x}_e, \tilde{t}\right), \Delta\left(A\tilde{x}_e, A\tilde{y}_{e'}, \tilde{t}\right)\right\}\right]$

for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \mathrm{SSP}(\tilde{X})$ and for all $\tilde{t} > 0$ where $r:[0,1]\to[0,1]$ is a continuous function such that $r(\tilde{t}) > \tilde{t}$ for each $0 \le \tilde{t} \le 1$ and $r(\tilde{t}) = 1$ for $\tilde{t} = 1$. Then A and S have unique common fixed point in $\mathrm{SSP}(\tilde{X})$.

Proof

Define two sequences $\{\tilde{x}_{e_n}\}$ and $\{\tilde{y}_{e'_n}\}$ in $\mathrm{SSP}(\tilde{X})$ as $\mathrm{S}\tilde{x}_{e_{2n+1}} = \mathrm{A}\tilde{x}_{e_{2n}} = \tilde{y}_{e'_{2n}}$

Applying condition (2),

$$\Delta\left(\mathrm{A}\tilde{x}_{e_{2n+1}}, \mathrm{A}\tilde{x}_{e_{2n}}, \tilde{t}\right) \tilde{\ge} r\left[\min\left\{\Delta\left(\mathrm{S}\tilde{x}_{e_{2n+1}}, \mathrm{A}\tilde{x}_{e_{2n}}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{x}_{e_{2n+1}}, \mathrm{A}\tilde{x}_{e_{2n+1}}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{x}_{e_{2n+1}}, \mathrm{S}\tilde{x}_{e_{2n}}, \tilde{t}\right)\right\}\right]$$

$$\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{\ge} r\left[\min\left\{\Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right), \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n+1}}, \tilde{t}\right), \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)\right\}\right]$$

$$= r\left[\min\left\{1, \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n+1}}, \tilde{t}\right), \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)\right\}\right]$$

$$\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{\ge} \begin{cases} r\left[\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right], & \text{if } \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{<} \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right) \\ r\left[\Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)\right], & \text{if } \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{>} \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right) \end{cases}$$

If $\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{<} \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)$ then $\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{\ge} r\left[\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right]$.

$$\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{\ge} r\left[\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)\right] \tilde{>} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)$$

This implies $\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{>} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right)$ which is a contradiction.

Therefore $\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{\ge} r\left[\Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)\right] \tilde{>} \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)$ (1)

$\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{>} \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)$ which implies $\left\{\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right), n \ge 0\right\}$ is an increasing sequence of positive real numbers in $[0,1]$ and therefore tends to a limit $l \le 1$.

If $l < 1$ then on taking limit $n \to \infty$ in (1) $l \ge r(l) > l$, which is a contradiction.

Therefore $l = 1$.

For every $n \in \mathrm{N}$, using analogous arguments we can show that $\left\{\Delta\left(\tilde{y}_{e'_{2n+2}}, \tilde{y}_{e'_{2n+1}}, \tilde{t}\right), n \ge 0\right\}$ is a sequence of positive real numbers in $[0,1]$ which tends to a limit $l = 1$.

Therefore, for every $n \in \mathrm{N}$, $\Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{>} \Delta\left(\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)$ and

$\lim_{t\to\infty} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) = 1$ as $n \to \infty$.

Also for any integer m,

$$\Delta\left(\tilde{y}_{e'_n}, \tilde{y}_{e'_{n+m}}, \tilde{t}\right) \tilde{\ge} \Delta\left(\tilde{y}_{e'_n}, \tilde{y}_{e'_{n+1}}, \frac{\tilde{t}}{m}\right) * \Delta\left(\tilde{y}_{e'_n}, \tilde{y}_{e'_{n+2}}, \frac{\tilde{t}}{m}\right) * \ldots * \Delta\left(\tilde{y}_{e'_n}, \tilde{y}_{e'_{n+m}}, \frac{\tilde{t}}{m}\right)$$

As $n \to \infty$

$$\lim_{\tilde{t}\to\infty} \Delta\left(\tilde{y}_{e'_n}, \tilde{y}_{e'_{n+m}}, \tilde{t}\right) \tilde{\ge} 1*1*\ldots*1 = 1$$

$$\lim_{\tilde{t}\to\infty} \Delta\left(\tilde{y}_{e'_n}, \tilde{y}_{e'_{n+m}}, \tilde{t}\right) = 1$$

Hence $\{\tilde{y}_{e'_n}\}$ is a Cauchy sequence in $\mathrm{SSP}(\tilde{X})$ and by completeness of $\tilde{X}$, $\{\tilde{y}_{e'_n}\}$ converges to $\{\tilde{v}_{e_j}\}$ in $\mathrm{S}(\tilde{x}_e)$. Using condition (1) $\{\tilde{y}_{e'_n}\}$ also converges to $\{\tilde{v}_{e_j}\}$ in $\mathrm{A}(\tilde{x}_e)$.

Given $\{\mathrm{A},\mathrm{S}\}$ is R–weakly commuting

$$\Delta\left(\mathrm{AS}\tilde{x}_{e_{2n+1}}, \mathrm{SA}\tilde{x}_{e_{2n+1}}, \tilde{t}\right) \tilde{\ge} \Delta\left(\mathrm{A}\tilde{x}_{e_{2n+1}}, \mathrm{S}\tilde{x}_{e_{2n+1}}, \frac{\tilde{t}}{\mathrm{R}}\right)$$

Since $\mathrm{S}\tilde{x}_{e_{2n+1}} = \tilde{y}_{e'_{2n}}$, $\mathrm{A}\tilde{x}_{e_{2n+1}} = \tilde{y}_{e'_{2n+1}}$

$$\Delta\left(\mathrm{A}\tilde{y}_{e'_{2n}}, \mathrm{S}\tilde{y}_{e'_{2n+1}}, \tilde{t}\right) \tilde{\ge} \Delta\left(\tilde{y}_{e'_{2n+1}}, \tilde{y}_{e'_{2n}}, \frac{\tilde{t}}{\mathrm{R}}\right)$$

On taking limit $n \to \infty$,

$$\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \mathrm{S}\tilde{v}_{e_j}, \tilde{t}\right) \tilde{\ge} \Delta\left(\tilde{v}_{e_j}, \tilde{v}_{e_j}, \frac{\tilde{t}}{\mathrm{R}}\right) = 1$$

$$\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \mathrm{S}\tilde{v}_{e_j}, \tilde{t}\right) = 1$$

$$\mathrm{A}\tilde{v}_{e_j} = \mathrm{S}\tilde{v}_{e_j}$$

To show that

$\mathrm{A}\tilde{v}_{e_j} = \mathrm{S}\tilde{v}_{e_j} = \tilde{v}_{e_j}$

Suppose $\mathrm{A}\tilde{v}_{e_j} \neq \tilde{v}_{e_j}$, then there exists $\tilde{t} > 0$ such that $\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) \tilde{<} 1$

$$\Delta\left(\mathrm{AS}\tilde{x}_{e_{2n+1}}, \mathrm{A}\tilde{x}_{e_{2n}}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\mathrm{S}^2\tilde{x}_{e_{2n+1}}, \mathrm{A}\tilde{x}_{e_{2n}}, \tilde{t}\right), \Delta\left(\mathrm{S}^2\tilde{x}_{e_{2n+1}}, \mathrm{AS}\tilde{x}_{e_{2n+1}}, \tilde{t}\right), \Delta\left(\mathrm{S}^2\tilde{x}_{e_{2n+1}}, \mathrm{S}\tilde{x}_{e_{2n}}, \tilde{t}\right)\right\}\right]$$

$$\Delta\left(\mathrm{A}\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\mathrm{S}\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n}}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{y}_{e'_{2n}}, \mathrm{A}\tilde{y}_{e'_{2n}}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{y}_{e'_{2n}}, \tilde{y}_{e'_{2n-1}}, \tilde{t}\right)\right\}\right]$$

As $n \to \infty$

$$\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\mathrm{S}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{v}_{e_j}, \mathrm{A}\tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right\}\right]$$

Since

$\mathrm{S}\tilde{v}_{e_j} = \mathrm{A}\tilde{v}_{e_j}$

$$\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\mathrm{A}\tilde{v}_{e_j}, \mathrm{A}\tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right\}\right]$$

$$= r\left[\min\left\{\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), 1, \Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right\}\right]$$

$$\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) \tilde{\geq} r\left[\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)\right] \tilde{>} \Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)$$

This implies, $\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) \tilde{>} \Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right)$ which is a contradiction.

Therefore, $\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right) = 1$ and this implies $\mathrm{A}\tilde{v}_{e_j} = \tilde{v}_{e_j}$.

Hence $\tilde{v}_{e_j}$ is the common fixed point of A and S.

To show that the uniqueness

Suppose there is another fixed point $\tilde{z}_{e_k} \neq \tilde{v}_{e_j}$. Then

$$\Delta\left(\mathrm{A}\tilde{v}_{e_j}, \mathrm{A}\tilde{z}_{e_k}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\mathrm{S}\tilde{v}_{e_j}, \mathrm{A}\tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{v}_{e_j}, \mathrm{A}\tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\mathrm{S}\tilde{v}_{e_j}, \mathrm{S}\tilde{z}_{e_k}, \tilde{t}\right)\right\}\right]$$

$$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) \tilde{\geq} r\left[\min\left\{\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(\tilde{v}_{e_j}, \tilde{v}_{e_j}, \tilde{t}\right), \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)\right\}\right]$$

$$= r\left[\min\left\{\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right), 1, \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)\right\}\right]$$

$$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) \tilde{>} r\left[\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)\right]$$

$$\tilde{>} \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)$$

$$\Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) \tilde{>} \Delta\left(\tilde{v}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right)$$

And so $\tilde{v}_{e_j} = \tilde{z}_{e_k}$ which is a contradiction to our assumption.

Therefore $\tilde{v}_{e_j} = \tilde{z}_{e_k}$.

Hence A and S have unique common fixed point in $\mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$.

Theorem 4.6

Let $\left(\tilde{\mathrm{X}}, \Delta, *\right)$ be a complete fuzzy soft metric space and let $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}$ be self mappings of $\mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$. Let the pairs $\{\mathrm{A}, \mathrm{S}\}$ and $\{\mathrm{B}, \mathrm{T}\}$ be owc satisfying the condition if there exists $q \in (0,1)$ and $\alpha, \beta > 0$, $\alpha + \beta > 1$ such that for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$ and $\tilde{t} > 0$, $\Delta\left(\mathrm{A}\tilde{x}_e, \mathrm{B}\tilde{y}_{e'}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(\mathrm{S}\tilde{x}_e, \mathrm{T}\tilde{y}_{e'}, \tilde{t}\right) + \beta\min\left\{\Delta\left(\mathrm{S}\tilde{x}_e, \mathrm{A}\tilde{x}_e, \tilde{t}\right), \Delta\left(\mathrm{B}\tilde{y}_{e'}, \mathrm{T}\tilde{y}_{e'}, \tilde{t}\right), \Delta\left(\mathrm{B}\tilde{y}_{e'}, \mathrm{S}\tilde{x}_e, \tilde{t}\right)\right\}$.

Then $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}$ have unique common fixed point in $\mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$.

Proof

Given the pairs $\{\mathrm{A}, \mathrm{S}\}$ and $\{\mathrm{B}, \mathrm{T}\}$ be owc. Therefore for all $\tilde{x}_e, \tilde{y}_{e'} \tilde{\in} \mathrm{SSP}\left(\tilde{\mathrm{X}}\right)$

$\mathrm{A}\tilde{x}_e = \mathrm{S}\tilde{x}_e$ and $\mathrm{B}\tilde{y}_{e'} = \mathrm{T}\tilde{y}_{e'}$ (1)

Therefore the given condition

$$\Delta\left(\mathrm{A}\tilde{x}_e, \mathrm{B}\tilde{y}_{e'}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(\mathrm{S}\tilde{x}_e, \mathrm{T}\tilde{y}_{e'}, \tilde{t}\right) + \beta\min\left\{\Delta\left(\mathrm{S}\tilde{x}_e, \mathrm{A}\tilde{x}_e, \tilde{t}\right), \Delta\left(\mathrm{B}\tilde{y}_{e'}, \mathrm{T}\tilde{y}_{e'}, \tilde{t}\right), \Delta\left(\mathrm{B}\tilde{y}_{e'}, \mathrm{S}\tilde{x}_e, \tilde{t}\right)\right\}$$

becomes

$$\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right) + \beta \min\left\{\Delta\left(A\tilde{x}_e, A\tilde{x}_e, \tilde{t}\right), \Delta\left(B\tilde{y}_{e'}, B\tilde{y}_{e'}, \tilde{t}\right), \Delta\left(B\tilde{y}_{e'}, A\tilde{x}_e, \tilde{t}\right)\right\}$$

$$\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right) + \beta \min\left\{1, 1, \Delta\left(B\tilde{y}_{e'}, A\tilde{x}_e, \tilde{t}\right)\right\}$$

$$\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right) + \beta\Delta\left(B\tilde{y}_{e'}, A\tilde{x}_e, \tilde{t}\right)$$

$$= (\alpha+\beta)\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right)$$

Since $\alpha + \beta > 1$

$$\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, q\tilde{t}\right) \tilde{\geq} (\alpha+\beta)\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right) \tilde{>} \Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right)$$

$$\Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, q\tilde{t}\right) \tilde{>} \Delta\left(A\tilde{x}_e, B\tilde{y}_{e'}, \tilde{t}\right)$$

This implies $A\tilde{x}_e = B\tilde{y}_{e'}$

By (1), $A\tilde{x}_e = S\tilde{x}_e = B\tilde{y}_{e'} = T\tilde{y}_{e'}$

Suppose $\tilde{z}_{e_j}$ is the common fixed point of A and S and $\tilde{z}_{e_k}$ is the common fixed point of B and T_{up}. Then $A\tilde{z}_{e_j} = S\tilde{z}_{e_j} = \tilde{z}_{e_j}$ and $B\tilde{z}_{e_k} = T\tilde{z}_{e_k} = \tilde{z}_{e_k}$.

$$\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) = \Delta\left(A\tilde{z}_{e_j}, B\tilde{z}_{e_k}, q\tilde{t}\right)$$

$$\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(A\tilde{z}_{e_j}, B\tilde{z}_{e_k}, \tilde{t}\right) + \beta \min\left\{\Delta\left(A\tilde{z}_{e_j}, A\tilde{z}_{e_j}, \tilde{t}\right), \Delta\left(B\tilde{z}_{e_k}, B\tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(B\tilde{z}_{e_k}, A\tilde{z}_{e_j}, \tilde{t}\right)\right\}$$

$$\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) + \beta \min\left\{\Delta\left(\tilde{z}_{e_j}, A\tilde{z}_{e_j}, \tilde{t}\right), \Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right)\right\}$$

$$\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, q\tilde{t}\right) \tilde{\geq} \alpha\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) + \beta \min\left\{\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_j}, \tilde{t}\right), \Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_k}, \tilde{t}\right), \Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right)\right\}$$

$$= \alpha\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) + \beta \min\left\{1, 1, \Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right)\right\}$$

$$= \alpha\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) + \beta\Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right)$$

$$= (\alpha+\beta)\Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right)$$

Since $\alpha + \beta > 1$

$$\Delta\left(\tilde{z}_{e_j}, \tilde{z}_{e_k}, \tilde{t}\right) \tilde{\geq} (\alpha+\beta)\Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right) \tilde{>} \Delta\left(\tilde{z}_{e_k}, \tilde{z}_{e_j}, \tilde{t}\right)$$

This implies $\tilde{z}_{e_j} = \tilde{z}_{e_k}$

Therefore A, B, S, T have unique common fixed point in $SSP\left(\tilde{X}\right)$.

5. References

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