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Estimation of Temperature Distribution and Thermal Stress Analysis of Composite Circular Rod by Finite Element Method

R. B. Thete and Dr. K. P. Ghadle

Department of Mathematics, Dr. B.A.M. University Aurangabad,Maharashtra, India.

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ABSTRACT

In this paper the temperature distribution and thermal stress analysis of multilayered composite circular rod is discussed by finite element method and the result temperature and thermal stresses has been computed numerically and illustrated graphically.

Introduction

The temperature distribution and thermal stress analysis of multilayered composite solid has a variety of engineering application. The finite element method has become a powerful tool for the numerical solution of a wide range of engineering problems involves stress analysis, heat transfer, electromagnetism and fluid flow. Dhawan Sharanjeet [6] had investigated approximation of temperature distribution in a solid body using semi- finite element techniques. Dechaumphai et al. [8] used finite element analysis for predicting temperatures and thermal stresses of heated products.

In present paper an attempt has been made to determine the temperature distribution and thermal stress analysis of a composite rod with four different material aluminium, platinum, steel and titanium and these material are usually use in construction of aero plane. Transient temperature and stresses can be obtained by using numerical technique (finite difference method, finite element method) and numerical calculations are obtained by using MATLAB.

Statement of problem:

Consider a one-dimensional composite rod $0 \le x \le l$, the time dependent heat conduction differential equation is

$$k\frac{\partial^2 T}{\partial x^2} + Q = C\rho\frac{\partial T}{\partial t} \qquad (1)$$

where C and ρ denotes material specific heat and density respectively and t-time , T-temperature, k- thermal conductivity, Q-heat generation.

With boundary condition

at $x = 0$, $T = T_i$ (2)

and

at $x = l$, $k\frac{\partial T}{\partial t} = -h(T - T_a)$ (3)

and $T(x,0) = T_i$ at $t = 0$, $0 \le x \le l$.

Solution of problem by finite element method:

Discretization of circular rod by into finite element-

The circular rod of length 1m. is discretized into 5 number of node and 4 element

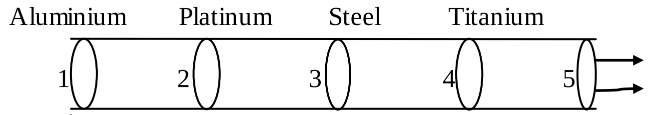


Figure.1.Geometry of the problem.

By finite element approach for solution of equation [1] by dividing the problem domain into finite length, one-dimensional and discretizing the temperature distribution within each element as

$$T(x,t) = N_1(x)T_1(t) + N_2(x)T_2(t) = [N(x)]^T\{T(t)\} \qquad (4)$$

Applying Galerkin finite element method

$$\int\left[k\frac{\partial^2 T}{\partial x^2} + Q - C\rho\frac{\partial T}{\partial t}\right] N_i(x)A\,dx \,,\; i = 1,2,3\ \ldots \qquad (5)$$

Integrating first term by parts and rearranging the above equation reduces to

$$kA\int\frac{dN_i}{dx}\frac{dT}{dx}dx + c\rho A\int N_i\frac{dT}{dt}dt = QA\int N_i\,dx + \left[kAN_i\frac{dT}{dx}\right]_{x=0}^{x=l} \qquad (6)$$

Applying boundary condition

$$kA\int\frac{dN_i}{dX}\frac{dT}{dx}dx + c\rho A\int N_i\frac{dT}{dt}dx = QA\int N_i\,dx - AN_i(L)\,h(T - T_a) - kAN_i(0)T_i \qquad (7)$$

Substitute the temperature distribution two node linear elements.

$$T(x,t) = N_1(x)T_1(t) + N_2(x)T_2(t) = [N(x)]^T\{T(t)\} \text{ and } \frac{\partial T}{\partial t} = \{\dot{T}\} \qquad (8)$$

The equation(7) reduces to

$$kA\int\frac{d[N]}{dx}\frac{d[N]^T}{dx}\{T\}dx + c\rho A\int[N][N]^T\{\dot{T}\}\,dx = QA\int[N]\,dx - AN_i(L)h(T - T_a) - kAN_i(0)T_i \qquad (9)$$

Tele:
E-mail address: rbthete1979@gmail.com

And is of the form

$$[K][T] + [C]\{\dot{T}\} = \{f_g\} \quad (10)$$

Where the conductance matrix

$$[K] = kA\int \frac{d[N]}{dx}\frac{d[N]^T}{dx}\,dx \quad (11)$$

Capacitance matrix $[C]$ is given by

$$[C] = c\rho A\int [N][N]^T\,dx \quad (12)$$

Internal heat generation matrix $\{f_Q\} = QA\int [N]\,dx$ (13)

Gradient matrix

$$\{f_g\} = AN_i(L)\,h(T_a - T) - kAN_i(0)T_i \quad (14)$$

Numerical value for calculation

Numerical calculation has been carried out for composite circular rod with the following parameter.

Dimension

Length $(L) = 1m$

Diameter $(d) = 0.12m$.

Initial conditions as

Convection coefficient of air $(h) = 15(w\backslash m^2 - k)$

Source temperature $(T_0) = 273k$

Ambient temperature = $(T_a) = 150k$

Internal heat generation $(Q) = 0.$

Point heat source $(x_0) = 0.$

Structural properties are

Number of node $(M) = 5.$

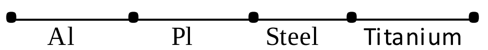
Number of element $(NE) = 4.$

Table 1.Material properties.

Property	Symbol	Aluminium	Platinum	Steel	Titanium
Thermal conductivity (w/m-k)	K	204.2	71.6	53.6	21.9
Density (kg/m3)	ρ	2707	21450	7833	4506
Specific heat(j/kg-k)	C	896	133	465	520
Young modulus of elasticity (Gpa)	E	69	168	200	116
Coefficient of linear thermal expansion(1/k)	α	$22.2*10^{-6}$	$8.8*10^{-6}$	$13*10^{-6}$	$8.6*10^{-6}$

Result Analysis:

The circular rod is discretized into 5 numbers of node and 4 elements



By Hutton David [2] , All the 4 element are assembled and we get equation

$$[K]\{T\} + [C]\{\dot{T}\} = \{f_q\} + \{f_g\} \quad (15)$$

Applying finite difference method and substituting

$$\{\dot{T}\} = \{\frac{T(t+\Delta t) - T(t)}{\Delta t}\} \quad (16)$$

The above equation reduces to

$$[K]\{T\} + [C]\{\frac{T(t+\Delta t) - T(t)}{\Delta t}\} = \{f_q\} + \{f_g\} \quad (17)$$

After simplifying we get,

$$T(t+\Delta t) = [C]^{-1}\{f_q\}\Delta t + [C]^{-1}\{f_g\}\Delta t - [C]^{-1}[K][T]\Delta t + \{T\} \quad (18)$$

Solving above equation by using MATLAB programming obtain the temperature at all nodes at different time

Table 2.Nodal temperature at different time

Node/time	5 sec	10sec	15sec	20sec	25sec
1	421.5862	420.2667	419.0383	417.8979	416.8424
2	426.8933	429.9840	433.0114	435.9775	438.8843
3	425.0507	424.2879	423.5502	422.8368	422.1469
4	417.1377	417.4116	417.6750	417.9282	418.1715
5	423.1318	422.9955	422.8650	422.7403	422.6210

Graph of node verses temperature at different time.

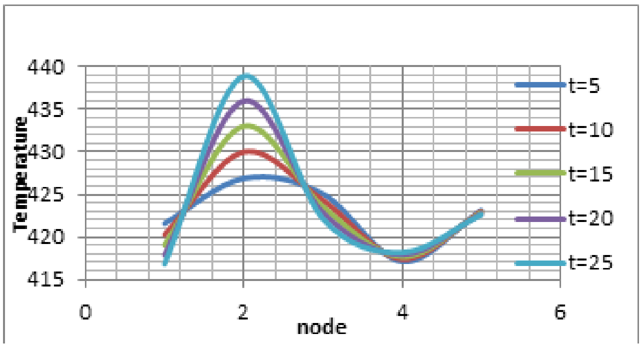


Figure.2.Temperature distribution at nodal point at different time.

By Cook R.D [1] the stress relation is

$$\epsilon = B\Delta t \quad (19)$$

stress strain relation is

$$\sigma = E\,\epsilon = EB\Delta t \quad (20)$$

The thermal stresses along x-axis at $t = 5, 10, 15, 20, 25sec$ are obtained using the temperature at the node which is in above table.

Table 3.Thermal stresses in element at different time.

Stress/X-axis	0 - 0.25	0.25 – 0.50	0.50 – 0.75	0.75 – 1
t = 5 sec	0.0813	-0.0272	-0.2057	0.0598
t =10 sec	0.1488	-0.0842	-0.1788	0.0557
t =15 sec	0.2140	-0.1399	-0.1528	0.0518
t =20 sec	0.2770	-0.1943	-0.1261	0.0474
t=25 sec	0.3376	-0.2474	-0.1041	0.0447

Graph of element verses stresses at different time

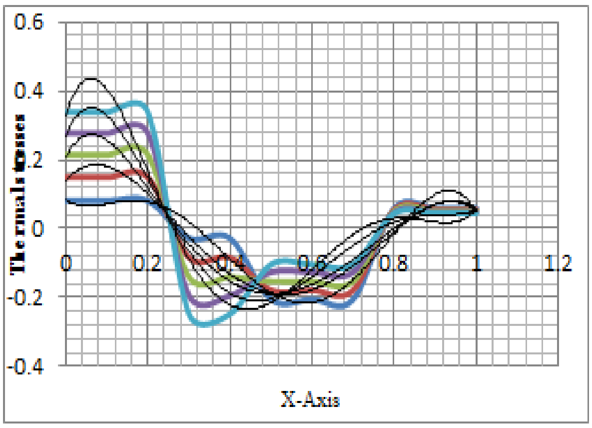


Figure 3.Thermal stresses in all four Element at different time.

Conclusion

In this paper numerical solution of transient heat conduction and stress analysis of composite circular rod are computed by using finite element method. Graph of element verses thermal stresses at different time shows that maximum thermal stress generated in aluminium and titanium material as compared to platinum and steel material.

Also variation of temperature verses node shown graphically. Temperature is high in platinum material and least in titanium material. Thermal stresses are extreme in aluminium at point heat source and least in platinum and steel. Composite material has high strength to its weight ratio. So that they are use in aerospace application. This study of composite material of different layer solid are useful in construction of aircraft, refrigeration and variety of industrial engineering application.

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