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Dufour effect on free convection MHD flow past a moving vertical plate through porous medium with variable temperature and constant mass diffusion

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ABSTRACT

Dufour effect on unsteady free convection MHD flow past a moving vertical plate through porous medium with variable temperature and constant mass diffusion is studied here. The Laplace transform technique has been used to find the solutions for the velocity, temperature and concentration. The results obtained are discussed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof number, Prandtl number, permeability parameter, the Hartmann number, Schmidt number, Dufour number, time and inclination of magnetic field. The numerical values of Skin-friction have been tabulated.

Introduction

The Study of flows through porous medium has become of great interest in many scientific and engineering applications, such as, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, water and oil through the oil reservoirs. Bejan and Khair [1] have studied heat and mass transfer by natural convection in a porous medium. Nelson and Wood [2] have studied combined heat and mass transfer natural convection between vertical parallel plates with uniform heat flux boundary conditions. MHD flow between two parallel plates with heat transfer have been studied by Attia and Katb [4]. Rajput and Sahu [12] have studied combined effects of chemical reactions and heat generation/absorption on unsteady transient free convection MHD flow between two long vertical parallel plates through a porous medium with constant temperature and mass diffusion. Hossain and Shayo [3] have discussed the skin-friction in the unsteady free convection flow past an exponentially accelerated plate. Das and Jana [10] have studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [11] has studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Unsteady MHD Poiseuille flow between two infinite parallel plates through porous medium in an inclined magnetic field with heat and mass transfer have been discussed by Rajput and Kumar [15]. Sandeep and Sugunamma [14] have discussed effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Postelnicu [5] has studied influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam et al. [6] have studied Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Reddy [9] have discussed Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Dipak et al. [13] have studied Soret and Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate. Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects has studied by Postelnicu [7]. Ibrahim [8] has discussed analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect.

In this paper we are analyzing Dufour effect on unsteady free convection MHD flow past a moving vertical plate through porous medium with variable temperature and constant mass diffusion in an inclined magnetic field.

Mathematical Analysis

In this paper we have considered the flow of unsteady viscous incompressible fluid. The x- axis is taken along the plate in the upward direction and y- axis is taken normal to plate. The plate considered is electrically non-conducting, and its initial velocity is taken as u_0. A uniform inclined magnetic field B_0 is applied on the plate with angle α from vertical. Initially the fluid and plate are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time $t > 0$, temperature of the plate is raised to T_w and the concentration of the fluid is raised to C_w.

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The governing equations under the usual Boussinesq's approximations are as follows:

$$\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho}\sin^2(\alpha)u-\frac{\upsilon}{K}u, \tag{1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}+\frac{D_m K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}, \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}. \tag{3}$$

The initial and boundary conditions are given as:

$$\left.\begin{array}{l} t\le 0; u=0, T=T_\infty, C=C_\infty \text{ for each value of } y,\\ t>0; u=u_0, T=T_\infty+(T_w-T_\infty)\dfrac{u_0^2 t}{\upsilon}, C=C_w \; at \; y=0,\\ u\to 0, T\to T_\infty, C\to C_\infty \text{ as } y\to\infty. \end{array}\right\} \tag{4}$$

Here u is the velocity of the fluid, $g-$the acceleration due to gravity, $\beta-$volumetric coefficient of thermal expansion, β^*- volumetric coefficient of concentration expansion, t – time, $T-$the temperature of the fluid, $T_\infty-$the temperature of the plate at $y\to\infty, C-$species concentration in the fluid, $C_\infty-$ species concentration at $y\to\infty, \upsilon-$ the kinematic viscosity, $\rho-$the density, $C_P-$the specific heat at constant pressure, k – thermal conductivity of the fluid, K_T-thermal diffusion ratio, $D-$ the mass diffusion constant, $D_m-$the effective mass diffusivity rate, $T_w-$the temperature of the plate at $y=0, C_w-$species concentration at the plate at $y=0$, C_S- Concentration susceptibility B_0- the uniform magnetic field, $\sigma-$electrical conductivity, $K-$permeability of the porous medium and $\alpha-$angle of inclination from vertical.

By using the following dimensionless quantities, the above equations (1), (2), and (3) can be transformed into dimensionless form.

$$\left.\begin{array}{l} \bar{y}=\dfrac{yu_0}{\upsilon}, \bar{t}=\dfrac{tu_0^2}{\upsilon}, \bar{u}=\dfrac{u}{u_0}, \theta=\dfrac{T-T_\infty}{T_w-T_\infty}, \bar{K}=\dfrac{Ku_0^2}{\upsilon^2}\\ \bar{C}=\dfrac{C-C_\infty}{C_w-C_\infty}, Ha^2=\dfrac{\sigma B_0^2\upsilon}{\rho u_0^2}, D_f=\dfrac{D_m K_T(C_w-C_\infty)}{\upsilon C_S C_P(T_w-T_\infty)},\\ Gr=\dfrac{g\beta\nu(T_w-T_\infty)}{u_0^3}, \Pr=\dfrac{\mu C_p}{k}, M=Ha^2*\sin^2(\alpha),\\ \mu=\upsilon\rho, Sc=\dfrac{\upsilon}{D}, Gm=\dfrac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3}. \end{array}\right\} \tag{5}$$

Here $\bar{u}$ is dimensionless velocity, $\bar{t}-$ dimensionless time, *Pr*- Prandtl number, *Sc*- Schmidt number, *Gr*- thermal Grashof number, *Gm*- mass Grashof number, θ- dimensionless temperature, $\bar{C}$- dimensionless concentration, *Ha*- the Hartmann number, μ- the coefficient of viscosity, $\bar{K}-$permeability parameter and D_f- Dufour number. Then model is transformed into the following non dimensional form of equations:

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+Gr\theta+Gm\bar{C}-A\bar{u}, \tag{6}$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\bar{y}^2}+D_f\frac{\partial^2\bar{C}}{\partial\bar{y}^2}, \tag{7}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{Sc}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}. \tag{8}$$

Here

$$A=M+\frac{1}{\bar{K}}.$$

The initial and boundary condition become:

$$\left.\begin{array}{l} \bar{t}\le 0; \bar{u}=0, \theta=0, \bar{C}=0 \text{ for each value of } \bar{y},\\ \bar{t}>0; \bar{u}=1, \theta=\bar{t}, \bar{C}=1 \text{ at } \bar{y}=0,\\ \bar{u}\to 0, \theta\to 0, \bar{C}\to 0 \text{ as } \bar{y}\to\infty. \end{array}\right\} \tag{9}$$

Dropping bars in the above equations, we get:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC-Au, \tag{10}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+D_f\frac{\partial^2 C}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \tag{12}$$

Here $A=M+\dfrac{1}{K}.$

The initial and boundary condition become:

$$\left.\begin{array}{l} t\le 0; u=0, \theta=0, C=0 \text{ for each value of } y,\\ t>0; u=1, \theta=t, C=1 \text{ at } y=0,\\ u\to 0, \theta\to 0, C\to 0 \text{ as } y\to\infty. \end{array}\right\} \tag{13}$$

Now the solution of equation (10), (11) and (12) under the boundary conditions (13) are obtained by the Laplace - transform technique. The exact solutions for species concentration *C*, fluid temperature θ and fluid velocity *u* are respectively:

$$C=Erfc\left[\frac{\sqrt{Sc}\,y}{2\sqrt{t}}\right] \tag{14}$$

$$\theta=-\frac{e^{\frac{-\Pr y^2}{4t}}\sqrt{\Pr t}\,y}{\sqrt{\pi}}-\frac{A_{25}}{2}\left(t+\frac{\Pr y^2}{2}+\frac{\Pr ScD_f}{Sc-\Pr}\right)+\frac{\Pr ScD_f}{2(Sc-\Pr)}A_{27} \tag{15}$$

$$u=\frac{1}{2}A_9B_{19}+\frac{1}{4A^2}Gr\left[\begin{array}{l}2A_9B_{16}B_{13}+y\sqrt{A}A_9B_{17}\\ +A_{13}B_{18}(1-\Pr)\\ +\dfrac{A\Pr ScD_f}{Sc-\Pr}(-2A_9B_{19}+A_{13}B_{20})\end{array}\right]+ \frac{(A_{18}B_{11}-A_9B_{12})}{2A}\left[\frac{Gr\Pr ScD_f}{Sc-\Pr}+Gm\right] -Gr\left[\begin{array}{l}\dfrac{1}{2A^2}\{-2A_{22}B_{13}+\dfrac{A\sqrt{\Pr}y}{\sqrt{\pi}}(2e^{\frac{-\Pr y^2}{4t}}\sqrt{t}+\sqrt{\pi}\Pr yA_{22})+\\ \dfrac{1}{2}A_{13}B_{21}(\Pr-1)\}+\dfrac{\Pr Sc}{2A(Sc-\Pr)}\\ (A_{25}+\dfrac{A_{13}}{2}(1+A_{23}+A_{14}A_{26}))D_f\end{array}\right] -\frac{Gr\Pr ScD_fB_{14}}{2A(Sc-\Pr)}-\frac{1}{2A}GmB_{15} \quad (16)$$

Skin- Friction

We calculate the non-dimensional form of skin friction (τ) from the velocity field as:

$$\tau=\left(-\frac{\partial u}{\partial y}\right)_{y=0}$$

And numerical values of τ are given in table-1 for different parameters.

Table 1. Skin-Friction for different values of parameters.

α (In degree)	*Ha*	*Pr*	D_f	*t*	*K*	*Gm*	*Gr*	*Sc*	τ
15	4	07	0.70	0.20	0.2	50	05	2.01	-3.5343
30	4	07	0.70	0.20	0.2	50	05	2.01	-2.6773
60	4	07	0.70	0.20	0.2	50	05	2.01	-0.8381
30	2	07	0.70	0.20	0.2	50	05	2.01	-3.5570
30	6	07	0.70	0.20	0.2	50	05	2.01	-1.4596
30	4	03	0.70	0.20	0.2	50	05	2.01	+1.3322
30	4	10	0.70	0.20	0.2	50	05	2.01	-3.0926
30	4	07	0.15	0.20	0.2	50	05	2.01	-4.9295
30	4	07	0.23	0.20	0.2	50	05	2.01	-4.6019
30	4	07	0.50	0.20	0.2	50	05	2.01	-3.4963
30	4	07	0.70	0.15	0.2	50	05	2.01	-2.0910
30	4	07	0.70	0.18	0.2	50	05	2.01	-2.4643
30	4	07	0.70	0.20	0.4	50	05	2.01	-3.4008
30	4	07	0.70	0.20	0.6	50	05	2.01	-3.6636
30	4	07	0.70	0.20	0.2	60	05	2.01	-4.3762
30	4	07	0.70	0.20	0.2	70	05	2.01	-6.0751
30	4	07	0.70	0.20	0.2	50	10	2.01	+0.1043
30	4	07	0.70	0.20	0.2	50	15	2.01	+2.8861
30	4	07	0.70	0.20	0.2	50	05	2.10	-2.4249
30	4	07	0.70	0.20	0.2	50	05	2.20	-2.1423

Results and Discussion

The numerical values of velocity, concentration, temperature and skin-friction are computed for different parameters like thermal Grashof number *Gr*, mass Grashof number *Gm*, Hartmann number *Ha*, Prandtl number *Pr*, Schmidt number *Sc*, inclination α, permeability parameter *K*, Dufour number D_f and time *t*. The values of the parameters considered are *Gr* = 5, 10, 15, *Ha* = 2, 4, 6, *Gm* = 50, 60, 70, $\alpha=15°,30°,60°$, *Pr* = 7, 10, *Sc* = 2.01, 2.10, 2.20, *K* = 0.2, 0.4, 0.6, D_f = 0.15, 0.23, 0.5 and *t* = 0.15, 0.18, 0.2. Figures 3, 6, 7 and 9 show that velocity increases when *Gm, K, Pr* and *t* are increased. Figures 1, 2, 4, 5 and 8 show that velocity decreases when α, D_f, *Gr, Ha,* and *Sc* are increased. Figures 10, 11, 12 and 13 show that temperature of the fluid increases with increasing the values of D_f, *Pr* , *Sc* and *t* respectively.

Skin- friction is given in table-1. The value of skin-friction increases with increasing the values of *Ha, Sc, Gr,* α and D_f and decreases with increasing the values of *Gm, t , Pr,* and *K*.

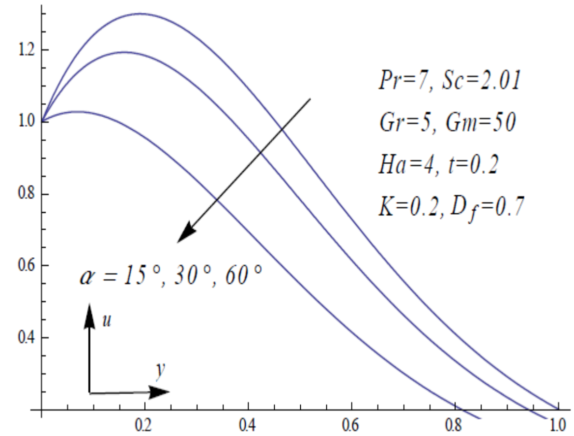

Figure 1. Velocity profile for different values of α .

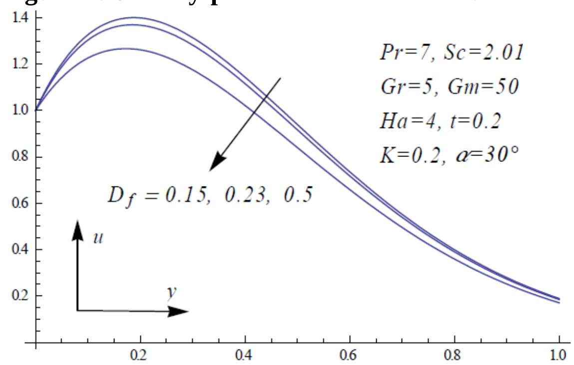

Figure 2. Velocity profile for different values of D_f .

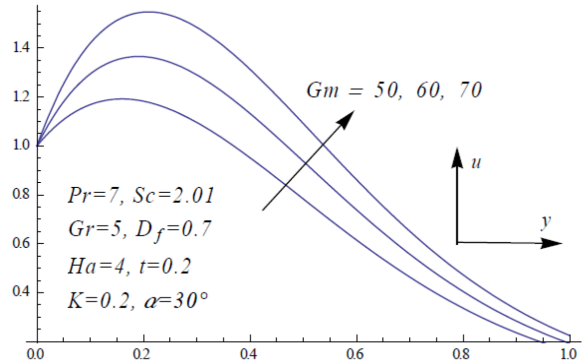

Figure 3. Velocity profile for different values of *Gm*.

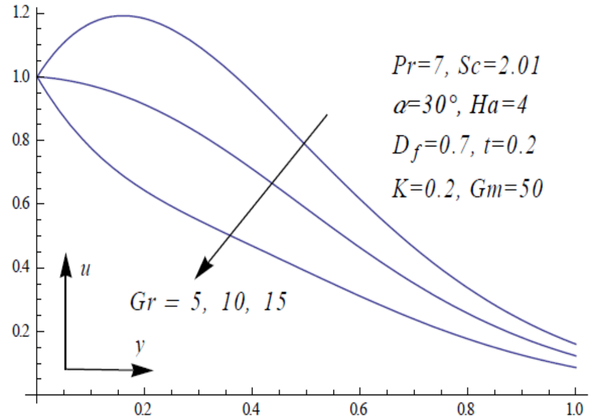

Figure 4. Velocity profile for different values of *Gr*

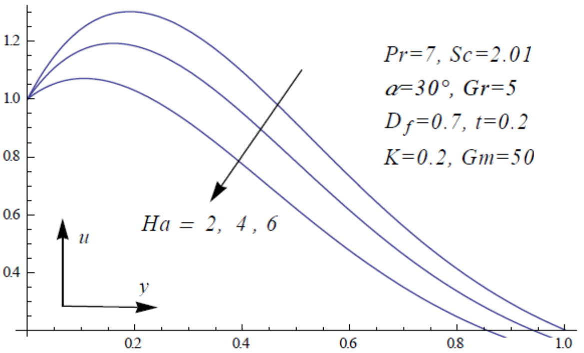

Figure 5. Velocity profile for different values of *Ha*.

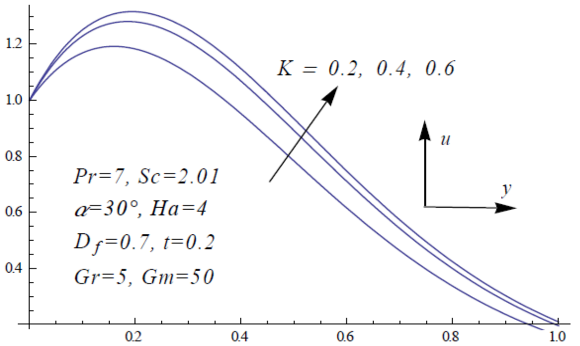

Figure 6. Velocity profile for different values of *K*.

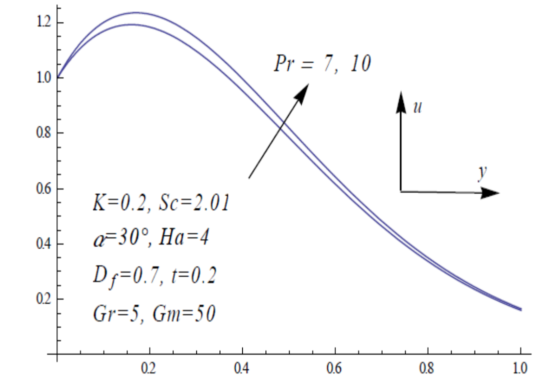

Figure 7. Velocity profile for different values of *Pr*

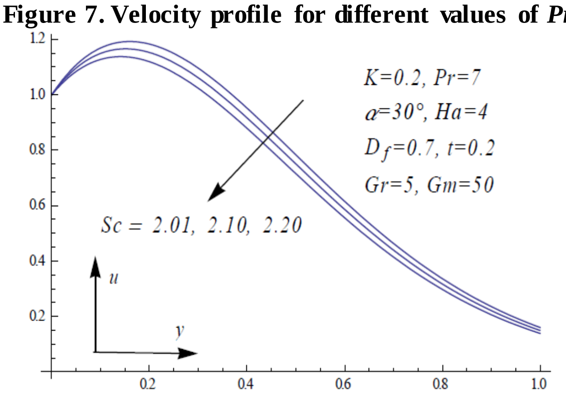

Figure 8. Velocity profile for different values of *Sc*.

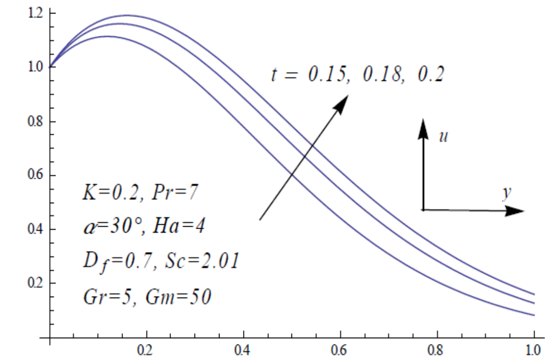

Figure 9. Velocity profile for different values of *t*.

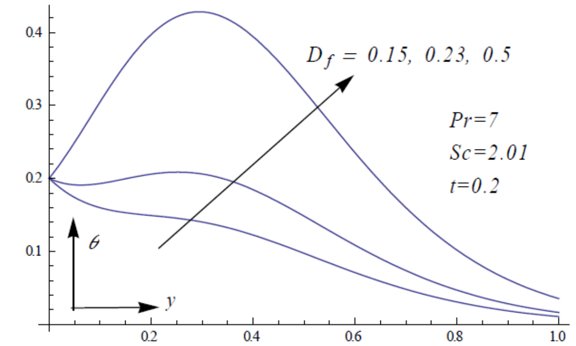

Figure-10: Temperature profile for different values of D_f

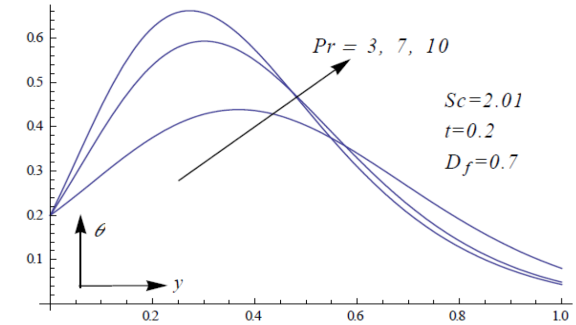

Figure 11. Temperature profile for different values of *Pr*.

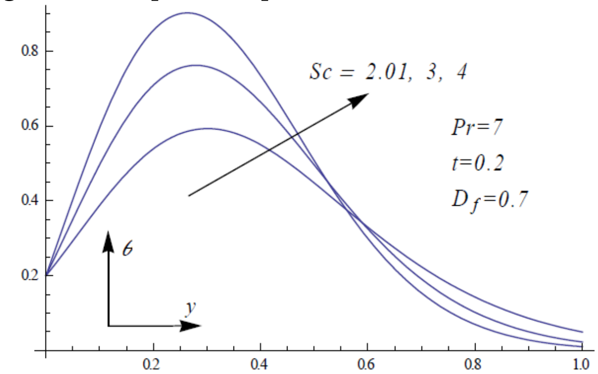

Figure 12. Temperature profile for different values of *Sc*.

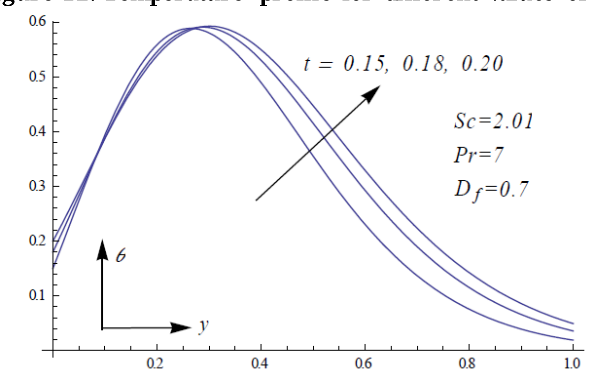

Figure 13. Temperature profile for different values of *t*.

Conclusion

Some conclusions of study are as below:

- The velocity of the fluid increases with increasing the values of *K*, *Gm*, *t* and *Pr*.
- The velocity of the fluid decreases with increasing the values of *Ha*, *Gr*, *Sc*, α and D_f.
- The temperature of the fluid increases with increasing *Pr*, *Sc*, *t* and D_f.
- The skin-Friction of the fluid increases with increasing the values of *Ha*, *Sc*, *Gr*, α and D_f.
- The skin-Friction of the fluid decreases with increasing the values of *Gm*, *t* , *Pr*, and *K*.

Appendix

$$A_9 = e^{-\sqrt{A}y},$$

$$A_{10} = Erfc\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right],\ A_{11} = Erf\left[\frac{2\sqrt{A}t - y}{2\sqrt{t}}\right],$$

$$A_{12} = Erf\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right]$$

$$A_{13} = 2e^{\frac{At}{\Pr-1}-\sqrt{\frac{A\Pr}{\Pr-1}}y},$$

$$A_{14} = e^{2\sqrt{\frac{A\Pr}{\Pr-1}}y}$$

$$A_{17} = Erfc\left[\frac{2\sqrt{\frac{A\Pr}{\Pr-1}}t + y}{2\sqrt{t}}\right],$$

$$A_{18} = e^{\frac{At}{Sc-1}-\sqrt{\frac{ASc}{Sc-1}}y},$$

$$A_{19} = e^{2\sqrt{\frac{ASc}{Sc-1}}y},$$

$$A_{20} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t - y}{2\sqrt{t}}\right]$$

$$A_{21} = Erf\left[\frac{2\sqrt{\frac{ASc}{Sc-1}}t + y}{2\sqrt{t}}\right],\quad A_{22} = -1 + Erf\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{23} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t - \sqrt{\Pr}y}{2\sqrt{t}}\right]$$

$$A_{24} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{25} = -2Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$A_{26} = Erfc\left[\frac{2\sqrt{\frac{A}{\Pr-1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right]$$

$$A_{27} = -2Erfc\left[\frac{\sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$A_{28} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t - \sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$A_{29} = Erfc\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right]$$

$$A_{30} = Erf\left[\frac{2\sqrt{\frac{A}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right],$$

$$B_{11} = 1 + A_{19} + A_{20} - A_{19}A_{21},\ B_{12} = 1 + A_{11} + A_9^{-2}A_{10}$$

$$B_{13} = 1 - \Pr - At,\ B_{14} = A_{27} + A_{18}(1 + A_{28} + A_{19}A_{29}),$$

$$B_{15} = A_{18}(1 + A_{19} + A_{28} - A_{19}A_{30}) + A_{27}$$

$$B_{16} = 1 + A_9^{-2} + A_{11} - A_9^{-2}A_{12},$$

$$B_{17} = 1 - A_9^{-2} + A_{11} + A_9^{-2}A_{12},$$

$$B_{18} = -1 - A_{14} - A_{15} + A_{14}A_{17}$$

$$B_{19} = 1 + A_{11} + A_9^{-2}A_{10},\qquad B_{20} = 1 + A_{15} + A_{14}A_{17},$$

$$B_{21} = 1 + A_{14} + A_{23} - A_{14}A_{24}$$

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