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On Taxicab Geometry

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ABSTRACT

In this paper, we explain a new type of geometry based on a different way of measuring distance between points, as we explained on different planes like Minkowski and Galilean. Now, we'll give a little background on this unfamiliar geometry. The purpose of this paper is to introduce high school students to taxicab geometry, one type of non-Euclidean geometry in which a new metric to measure distance replaces the usual metric of Euclidean geometry.

Introduction

A long time ago, most of the people thought that the only sensible way to do Geometry was to do it the way Euclid did in the 300s B.C. But starting in the 19th century, mathematicians began exploring versions of Geometry that looked a whole lot different. Not only mathematicians but also physicists were beginning to explore ways of creating non-Euclidean Geometries. Hermann Minkowski is usually credited with introducing Taxicab Geometry, along with a whole family of different geometries, based on different ways of measuring distance between points.

The ordinary way to describe a (plane) geometry is to tell what its points are, what its lines are, how distance is measured, and how angle measure is determined [5]. Taxicab geometry is a non-Euclidean geometry that is accessible in a particular form and is only one axiom away from being Euclidean in its basic structure. The points are the same, the lines are the same, and angles are measured the same way. Only the distance function is different, and the minimum distance between two points is a straight line in Euclidean geometry. Whereas, in taxicab geometry there may be many paths, all equally minimal, that join two points. For instance, taxicab distance between two points A and B is the length of a shortest path from A to B composed of line segments parallel and perpendicular to the x- axis (Figure1).

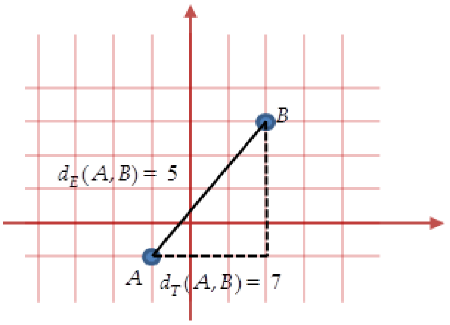

Figure 1. Distance between two points.

In taxicab geometry, we can consider the grid as a net of streets, which a taxi driver navigates through. The crossings are the places, where he can stop. It is remarkable that the taxi driver starting at point A can take different ways, which have the same length, if he is continuously approaching the destination point B (Figure 2).

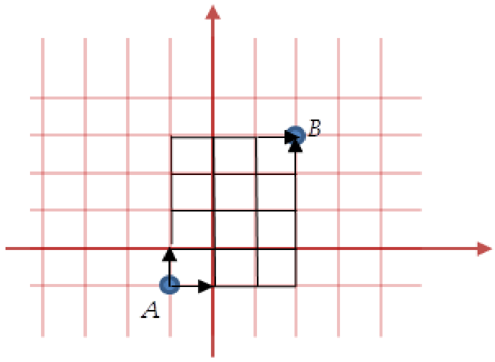

Figure 2. Taxicab distance between two points.

The general aim of geometry education can be defined as having students learn about the environment, and use geometry in the problem solving process. Euclidean geometry, with its theoretical knowledge, fails to make connections and integrations in real life. And also, Euclidean geometry does not always help students to correctly visualize a realistic environment. For instance, considering that roads are generally horizontal and vertical, a taxi driver may not always use Euclidean distance in real life.

Taxicab allows students to do math in a way that suggests a more realistic idea of what they are trying to conclude. Therefore, it could be suggested that taxicab geometry provides students with more opportunities than Euclidean geometry does in terms of making meaning out of the real world[1]. As it is well known, one of the most important agreements in mathematics education is to provide opportunites to students in mathematics lessons for developing their problem posing skills[2].

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Distance Formulas

Euclidean geometry is based on the Euclidean metric, which is a function that takes any two points as input and gives us the distance between them. In two dimensions, this is just the familiar distance formula between points in the plane as follows $d_E = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

In Galilean geometry, the distance between the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $d_1 = |x_2 - x_1|$, if $d_1 = 0$ then $d_2 = |y_2 - y_1|$. Obviously, it can be seen that we calculate the distance as projection (Figure 3). If both, $d_1 = d_2 = 0$ then we say that these points are coincide, $A = B$ [4].

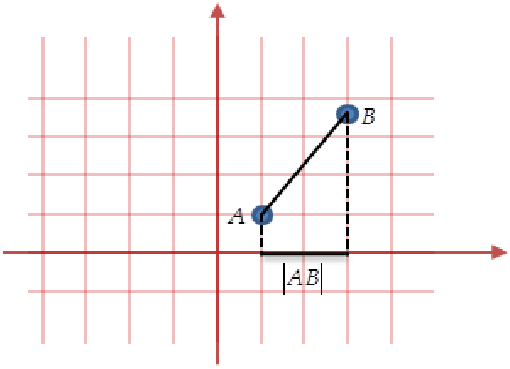

Figure 3. Galilean distance between two points.

Taxicab geometry is a form of geometry, where the distance between two points is not the length of the line segment as in the Euclidean geometry, but the sum of the absolute differences of their coordinates. In other words, the of the distances d_1 and d_2 in Galilean plane. So, taxicab distance formula between two points is $d_T = |x_2 - x_1| + |y_2 - y_1|$.

By the way, we can show that taxicab distance formula satisfies metric conditions. Let $P(x_p, y_p)$, $Q(x_q, y_q)$ and $R(x_r, y_r) \in \mathbb{R}^2$.

M. Axiom 1: $d(P,Q) \geq 0$ and $d(P,Q) = 0$ if and only if $P = Q$

If $P = Q$ then $d(P,Q) = d(P,P)$

$= |x_p - x_p| + |y_p - y_p|$

$= |0| + |0|$

$= 0$

If $P \neq Q$ then either $x_p \neq x_q$ or $y_p \neq y_q$ and $|x_p - x_q| > 0$ or $|y_p - y_q| > 0$. Suppose that $x_p \neq x_q$, then $|x_p - x_q| + |y_p - y_q| \geq |x_p - x_q| > 0$. Therefore, $|x_p - x_q| + |y_p - y_q| \geq 0$.

M. Axiom 2: $d(P,Q) = d(Q,P)$

$d(P,Q) = |x_p - x_q| + |y_p - y_q|$

$d(P,Q) = |(-1)(-x_p + x_q)| + |(-1)(-y_p + y_q)|$

$d(P,Q) = |(-1)(x_q - x_p)| + |(-1)(y_q - y_p)|$

$d(P,Q) = |(-1)||(x_q - x_p)| + |(-1)||(y_q - y_p)|$

$d(P,Q) = |x_q - x_p| + |y_q - y_p|$

$d(P,Q) = d(Q,P)$

M. Axiom 3: $d(P,Q) + d(Q,R) \geq d(P,R)$

Let P, Q and R be non-collinear, thus forming a triangle.

$d(P,Q) + d(Q,R) = |x_p - x_q| + |y_p - y_q| + |x_q - x_r| + |y_q - y_r|$

$d(P,Q) + d(Q,R) = (|x_p - x_q| + |x_q - x_r|) + (|y_p - y_q| + |y_q - y_r|)$

$d(P,Q) + d(Q,R) \geq (|x_p - x_q + x_q - x_r|) + (|y_q - y_q + y_q - y_r|)$

$d(P,Q) + d(Q,R) = |x_p - x_r| + |y_p - y_r|$

$d(P,Q) + d(Q,R) = d(P,R)$

hence, $d(P,Q) + d(Q,R) \geq d(P,R)$ [6].

T-Line Segment

There is one line segment to one length in Euclidean geometry, but several line segments to one length in taxicab geometry. This is shown below by (Figure 4). Both lines have the length 9, and we call this new kind of line as *taxicab-line* or $t-$ *line*.

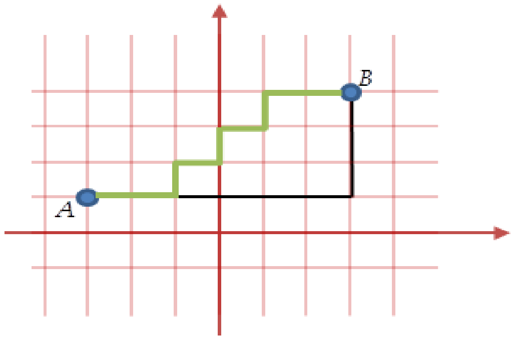

Figure. 4

Example: How many $t-$ *line* segments are there (Figure 5)between the points A and B?

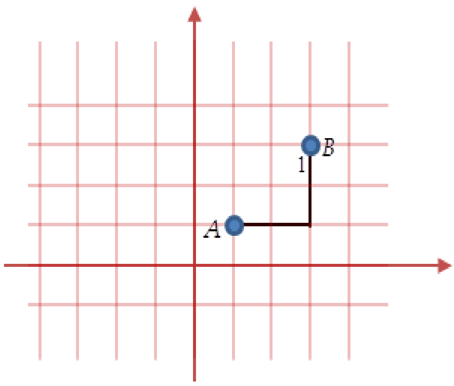

Figure. 5

We can count them simply and get 6 $t-$ *line* segments. If we examine the number of $t-$ *lines* systematically, we may discover a rule. The number of $t-$ *line* segments on a crossing is always the sum of the segments at the foregoing crossings.

.1

.1 .4

.1 .3 $_B$.6 .6 + 4

.1 .2 .3 .4

$_A$. .1 .1 .1 .1

This leads to Pascal's triangle, whose rows are diagonal here. Point B is on the ninth diagonal and thus on the ninth row of Pascal's triangle, related to A, and on the fourth place.

There is the number $28 + 56 = 84$ or "9 choose 3" which is $\binom{9}{3} = \frac{9!}{(9-3)!3!} = \frac{9.8.7.6!}{6!3.2.1} = 84$, and these 84 $t-$ *lines* for the case "6 to the right, 3 to the top" to go from A to B (Figure 6).

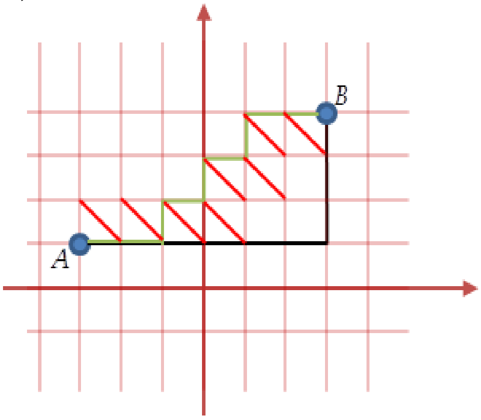

Figure. 6

Generally the number is "$n + m$ choose m" for the case "m to the right, n to the top" [3].

In summary

Taxicab geometry takes a more realistic approach in geometry. In Euclidean geometry, the shortest distance between two points is a straight line. In theory, this method works perfectly. However, in real life applications it is not as expected. This is where taxicab geometry comes to play in math. The basic premise of taxicab geometry is that the shortest distance between two points is not always a straight line. We can see the taxicab geometry is a very useful model of urban geography. Only a pigeon would benefit from the knowledge that the distance between two buildings on opposite ends of a city is a straight line. For people, taxicab distance is the "real" distance, and taxicab geometry has many applications also it is relatively easy to explore.

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