Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 95 (2016) 40767-40771

ISSN: 2229-712X

Unsteady MHD flow past an impulsively started vertical plate with constant wall temperature and variable mass diffusion in the presence of Hall current

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ARTICLE INFO

Article history:

Received: 25 April 2016;

Received in revised form: 28 May 2016;

Accepted: 3 June 2016;

Keywords

MHD,

Hall current,

Skin friction,

constant wall temperature,

variable mass diffusion.

ABSTRACT

In the present paper, unsteady MHD flow past an impulsively started vertical plate with constant wall temperature and variable mass diffusion in the presence of Hall current is studied. The fluid considered is an electrically conducting, absorbing-emitting radiation but a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profile and skin friction. The velocity profile and skin friction have been studied for different parameters like Schmidt number, Hall parameter, magnetic parameter, mass Grashof number, thermal Grashof number, Prandtl number, and time. The effect of parameters are shown graphically and the value of the skin-friction for different parameters has been tabulated.

Introduction

The study of MHD flow with Hall effect plays an important role in engineering and astrophysics. In engineering, it finds its application in MHD generators, ion propulsion, MHD bearings, the three- dimensional free convective channel flow MHD pumps, MHD boundary layer control of re-entry vehicles etc. In astrophysics, it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms etc. MHD flow models with Hall effects have been studied by a number of researchers, some of which are mentioned here. Soundalgekar and Uplekar [1] have studied Hall effects in MHD Couette flow with heat transfer. Watanabe et al. [2] have studies Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate. Katagiri [3] have studied the effect of Hall current on the magnetohydrodynamic boundary layer flow past a semi-infinite plate.

Attia [4] have studied Hall effect on Couette flow with heat transfer of a dusty conducting fluid in the presence of uniform suction and injection. Attia [5] has studied the effect of variable properties on the unsteady Hartmann flow with heat transfer considering the Hall effect. Attia [6] have studied unsteady Hartmann flow of a visco elastic fluid considering the Hall effect. Pop [7] has studied the effect of Hall currents on hydromagnetic flow near an accelerated plate. Pandurangan [8] have studies combined effects of radiation and Hall current on MHD flow past an exponentially accelerated vertical plate in the presence of rotation. We would mention some of the research articles related to my work. Hossain and Takhar [9] have studied radiation effect on mixed convection along a vertical plate with uniform surface temperature heat and mass transfer. Reddy et al. [10] have studied radiation effects on MHD combined con-vection and mass transfer flow past a vertical porous plate embedded in a porous medium with heat generation. Debnath[11] has studied an unsteady magnetohydrodynamic boundary layers in a rotating flow. Rajput and Kumar [12] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Reddy et al. [13] have studied unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical porous plate with variable viscosity and thermal con-ductility. Rajput and Sahu [14] have studied combined effects of MHD and radiation on unsteady transient free convection flow between two long vertical parallel plates with constant temperature and mass diffusion.

We are considering unsteady MHD flow past an impulsively started vertical plate with constant wall temperature and variable mass diffusion in the presence of Hall current. The effect of Hall current on the velocity have been observed with the help of graphs, and the skin friction has been tabulated.

Mathematical Analysis

An unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate is considered here. The plate is electrically non-conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially, at time $t \leq 0$ the temperature of the fluid and the plates are same as T_∞ and the concentration of the fluid is C_∞. At time t > 0, temperature of the plate is raised to T_w and the concentration of the plate is raised to C_w. Using the relation $\nabla \cdot B = 0$, for the magnetic field $\overline{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. Due to Hall effect, to there will be two components of the momentum equation, which are as under. The usual assumptions have been taken in to consideration. The fluid model is as under:

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$$\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^{\,2}}{\rho(1+m^2)}(u+mw), \tag{1}$$

$$\frac{\partial w}{\partial t}=\upsilon\frac{\partial^2 w}{\partial y^2}-\frac{\sigma B_0^{\,2}}{\rho(1+m^2)}(w-mu), \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The following boundary conditions have been assumed:

$$\left.\begin{aligned}&t\le 0: u=0, w=0, C=C_\infty, T=T_\infty, \text{ for all the values of } y,\\ &t>0: u=u_0, w=0, T=T_w, C=C_\infty+(C_w-C_\infty)\frac{u_0^{\,2}t}{\upsilon} \text{ at } y=0,\\ &u\to 0, w\to 0, C\to C_\infty, T\to T_\infty \quad \text{as } y\to\infty.\end{aligned}\right\} \tag{5}$$

Here u is the velocity of the fluid in x- direction, w - the velocity of the fluid in z - direction, m - the Hall parameter, g - acceleration due to gravity, β - volumetric coefficient of thermal expansion, β^* - volumetric coefficient of concentration expansion, t - time, C_∞ - the concentration in the fluid far away from the plate, C - species concentration in the fluid , C_w - species concentration at the plate, D - mass diffusion, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate, T - the temperature of the fluid , k - the thermal conductivity, υ - the kinematic viscosity, ρ - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, K - permeability of the medium, and C_P is specific heat at constant pressure. Here $m=\omega_e\tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left.\begin{aligned}&\bar{u}=\frac{u}{u_0}, \bar{w}=\frac{w}{u_0}, \bar{y}=\frac{yu_0}{\upsilon}, Sc=\frac{\upsilon}{D}, \Pr=\frac{\mu C_P}{k},\\ &M=\frac{\sigma B_0^2\upsilon}{\rho u_0^2}, \bar{t}=\frac{tu_0^{\,2}}{\upsilon}, G_r=\frac{g\beta\upsilon(T_w-T_\infty)}{u_0^{\,3}},\\ &Gm=\frac{g\beta\upsilon(C_w-C_\infty)}{u_0^{\,3}}, \bar{C}=\frac{C-C_\infty}{C_w-C_\infty}, \theta=\frac{(T-T_\infty)}{(T_w-T_\infty)}.\end{aligned}\right\} \tag{6}$$

Here the symbols used are:

$\bar{u}$ - the dimensionless velocity, $\bar{w}$ - dimensionless velocity, θ - the dimensionless temperature, $\bar{C}$ - the dimensionless concentration, G_r - thermal Grashof number, G_m - mass Grashof number, μ - the coefficient of viscosity, Pr - the Prandtl number, Sc - the Schmidt number, M - the magnetic parameter.

The dimensionless forms of equation (1), (2), (3) and (4) are as follows

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+G_r\theta+G_m\bar{C}-\frac{M(\bar{u}+m\bar{w})}{(1+m^2)}, \tag{7}$$

$$\frac{\partial\bar{w}}{\partial\bar{t}}=\frac{\partial^2\bar{w}}{\partial\bar{y}^2}-\frac{M(\bar{w}-m\bar{u})}{(1+m^2)}, \tag{8}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{S_C}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}, \tag{9}$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial\bar{y}^2}, \tag{10}$$

with the corresponding boundary conditions:

$$\left.\begin{aligned}&\bar{t}\le 0, \bar{u}=0, \bar{C}=0, \theta=0, \bar{w}=0, \text{ for all value of } \bar{y},\\ &\bar{t}>0, \bar{u}=1, \bar{w}=0, \theta=1, \bar{C}=\bar{t} \text{ at } \bar{y}=0,\\ &\bar{u}\to 0, \bar{C}\to 0, \theta\to 0, \bar{w}\to 0 \text{ as } \bar{y}\to\infty.\end{aligned}\right\} \tag{11}$$

Dropping the bars and combining the Equations (7) and (8), we get

$$\frac{\partial q}{\partial t}=\frac{\partial^2 q}{\partial y^2}+Gr\theta+GmC-\left(\frac{M}{1+m^2}(1-mi)\right)q \tag{12}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{13}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}, \tag{14}$$

where $q=u+iw$, with corresponding boundary conditions

$$\left.\begin{aligned}&t\le 0: q=0, \theta=0, C=0, \text{ for all value of } y,\\ &t>0: q=1, w=0, \theta=1, C=t, \text{ at } y=0,\\ &q\to 0, C\to 0, \theta\to 0, \text{ as } y\to\infty.\end{aligned}\right\} \tag{15}$$

Equation (12), (13) and (14), are subject to boundary condition (15), solved by Laplace transform technique.

The solution obtained is as under-

$$\begin{aligned}q=[&\frac{1}{2}e^{-\sqrt{a}y}(A_0)+\frac{1}{2a}Gr\{-e^{-\sqrt{a}y}(A_0)+e^{\frac{at}{-1+\Pr}\sqrt{\frac{at}{-1+\Pr}}y}(1+A_{10}+A_{15}A_{11})\}+\\ &\frac{1}{4a^2}Gmy\{\frac{1}{y}A(1-Sc-e^{-\sqrt{a}y}t)+\\ &\sqrt{a}e^{-\sqrt{a}y}(A_3)-\frac{1}{y}2e^{\frac{at}{-1+Sc}\sqrt{\frac{at}{-1+Sc}}y}B(1-Sc)\}-\\ &\frac{1}{2a}Gr\{-2A_{12}+e^{\frac{at}{-1+\Pr}\sqrt{\frac{at}{-1+\Pr}}y}(1+A_{13}+A_{15}A_{14})\}\\ &-\frac{1}{2a^2\sqrt{\pi}}Gm\sqrt{Sc}y\{\frac{-2\sqrt{\pi}B_{13}}{\sqrt{Sc}y}(1-Sc-t)+\\ &a\{2e^{-\frac{Scy^2}{4t}}\sqrt{t}-\sqrt{\pi}\sqrt{Sc}y(1-B_{14})\\ &+\frac{1}{y}e^{\frac{at}{-1+Sc}\sqrt{\frac{at}{-1+Sc}}\sqrt{Sc}y}\sqrt{\pi}B_0(\sqrt{Sc}-\frac{1}{\sqrt{Sc}})\}\}]\end{aligned}$$

$$\theta=Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$C=\left[\left(t+\frac{Scy^2}{2}\right)Erf\left(\frac{\sqrt{Sc}y}{2\sqrt{t}}\right)-e^{-\frac{Scy^2}{4t}}\frac{\sqrt{Sct}\,y}{\sqrt{\pi}}\right].$$

Skin Friction

The dimensionless skin friction at the plate $y=0$ is computed by

$$\left(\frac{dq}{dy}\right)_{y=0}=\tau_x+i\tau_z$$

Separating real and imaginary parts in $\left(\frac{dq}{dy}\right)_{y=0}$, the dimensionless skin– friction components:

$$\tau_x=\left(\frac{du}{dy}\right)_{y=0}$$

and

$$\tau_z=\left(\frac{dw}{dy}\right)_{y=0}$$

can be computed.

Result and Discussions

The numerical values of velocity and skin friction are computed for different parameters like Hall parameter m, mass Grashof number Gm, Schmidt number Sc, time t, thermal Grashof number Gr, magnetic field parameter M, and Prandtl number Pr,. The values of the main parameters considered are-

$m = 1, 1.5, 2; Sc = 2.01, 3, 4; Pr = 2, 5, 7; M = 1, 3, 5;$
$Gr = 10, 15, 20; Gm = 10, 20, 30; t = 0.1, 0.12, 0.13;$

Figures 2, 3, and 7 show that primary velocity (u) increases when Gr, m, and t are increased. Figures 1, 4, 5 and 6 show that primary velocity (u) decreases when Gm, M, Pr, and Sc, are increased. And figures 8, 9, 11, and 14 show that the secondary velocity (w) increases when Gm, Gr, M, and t are increased. Figures 10, 12 and 13 show that secondary velocity (w) decreases when m, Pr, and Sc are increased.

From table – 1 is observed that τ_x decreases with increase in Pr, M, Sc and it increases with increase in m, Gm, Gr, and t. τ_z increases with increase in Gr, t, Gm, and M and it decreases when Sc, Pr, and m are increased.

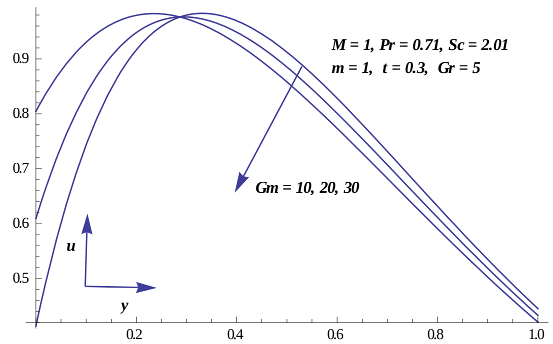

Figure 1. velocity profiles u for different values of Gm.

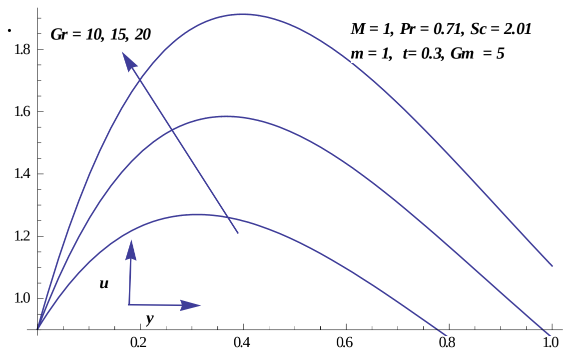

Figure 2. velocity profiles u for different values of Gr.

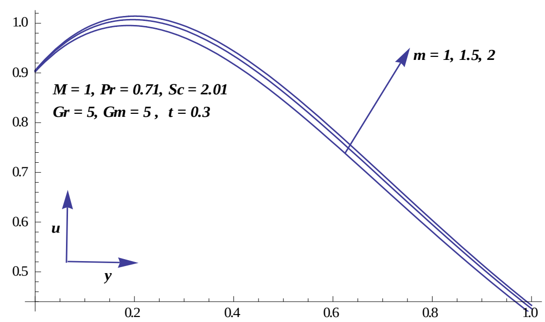

Figure 3. velocity profiles u for different values of m.

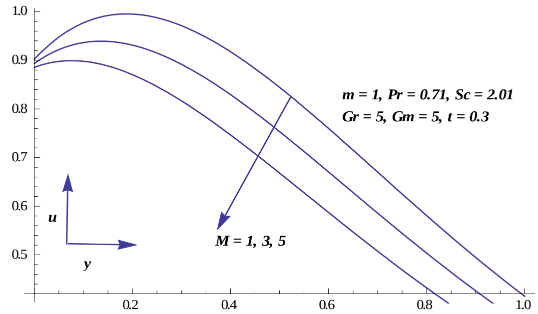

Figure 4. velocity profiles u for different values of M.

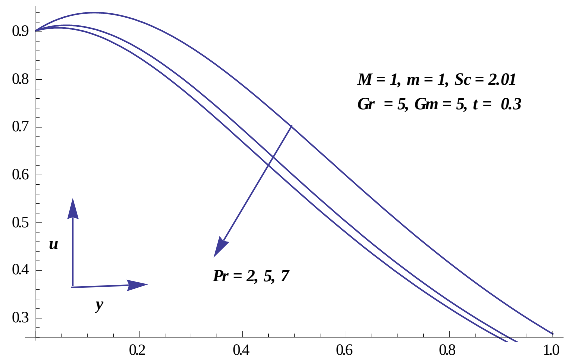

Figure 5. velocity profiles u for different values of Pr.

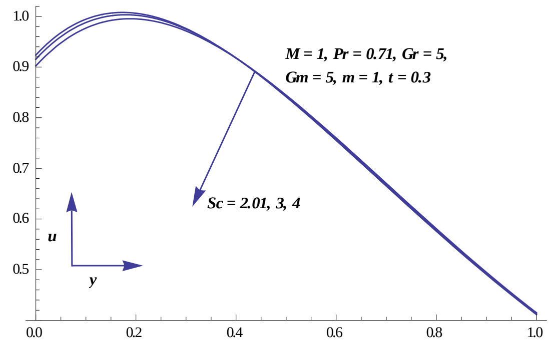

Figure 6. velocity profile u for different values of Sc.

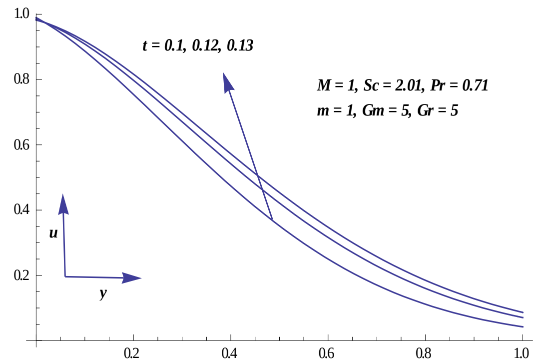

Figure 7. velocity Profile u for different values for t.

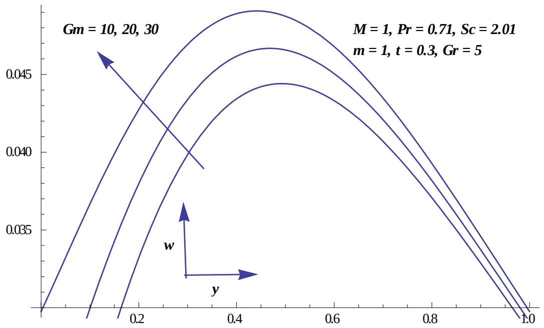

Figure 8. velocity profile w for different values of Gm.

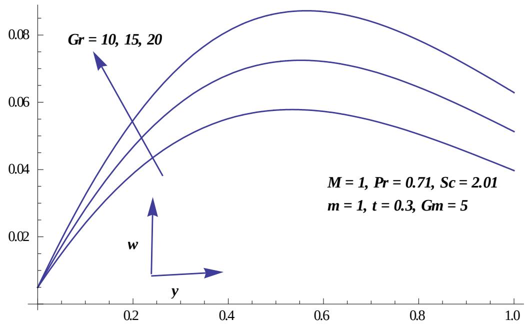

Figure 9. velocity profile w for different values of Gr.

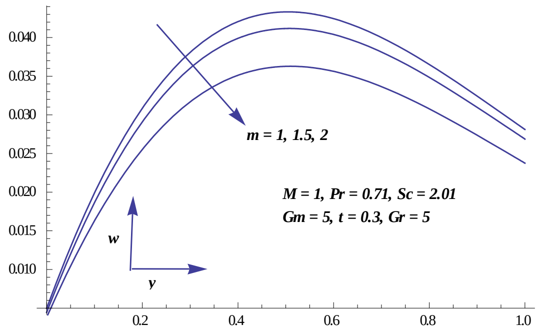

Figure 10. velocity profile w for different values of m.

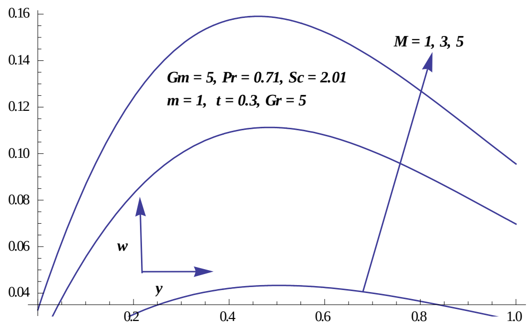

Figure 11. velocity profile w for different values of M.

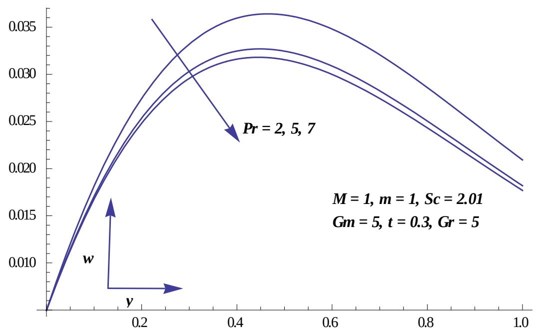

Figure 12. velocity profile w for different values of Pr.

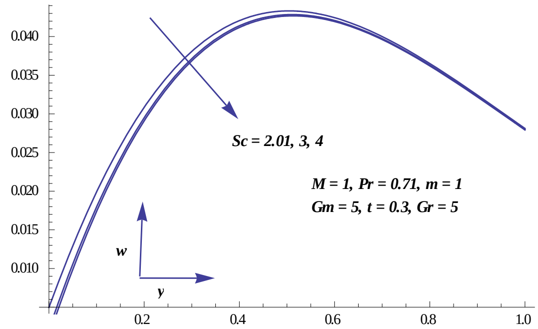

Figure 13. velocity profile w for different values of Sc.

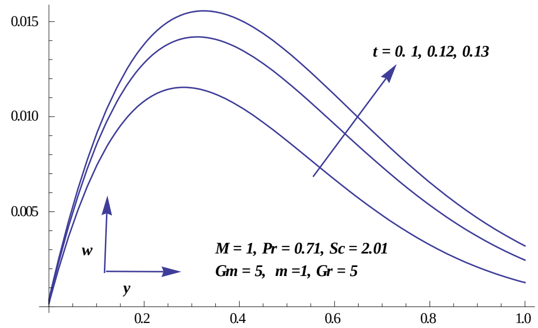

Figure 14. velocity profile w for different values of t.

Conclusion

Some conclusions of the study are as under:

- Primary velocity increases with the increase in Hall parameter, thermal Grashof number, and time. However, it decreases with the increase in mass Grashof number, magnetic field parameter and Prandtl number, and, Schmidt number.
- Secondary velocity increases with increase in thermal Grashof number, mass Grashof number, time, and magnetic field parameter. However, it decreases with the increase, Hall parameter, Prandtl number, Schmidt number.
- Skin fraction τ_x decreases with increase in Prandtl number, magnetic field parameter, Schmidt number, and it increases with increase in Hall parameter, mass Grashof number, thermal Grashof number, time are increased. τ_z increases with increase in thermal Grashof number, mass Grashof number, time, and magnetic field parameter, and it decreases when Prandtl number, Hall parameter, Schmidt number are increased.

Appendix

$A = 2e^{-\sqrt{a}y}A_3,\ A_0 = (1 + A_1 + e^{2\sqrt{a}y}A_2),\ A_1 = Erf[\frac{1}{2\sqrt{t}}2\sqrt{a}t - y],$

$A_2 = Erf[\frac{1}{2\sqrt{t}}2\sqrt{a}t + y],\ A_3 = \left(1 + e^{2\sqrt{a}y} + A_1 - e^{2\sqrt{a}y}A_2\right),$

$A_{10} = Erf[\frac{2\sqrt{\frac{a\Pr}{-1+\Pr}}t - y}{2\sqrt{t}}],\ A_{11} = Erf[\frac{2\sqrt{\frac{a\Pr}{-1+\Pr}}t + y}{2\sqrt{t}}],\ A_{12} = Erfc[\frac{\sqrt{\Pr}y}{2\sqrt{t}}],$

$A_{13} = Erf[\frac{2\sqrt{\frac{a}{-1+\Pr}}t - \sqrt{\Pr}y}{2\sqrt{t}}],\ A_{14} = Erf[\frac{2\sqrt{\frac{a}{-1+\Pr}}t + \sqrt{\Pr}y}{2\sqrt{t}}],$

$A_{15} = e^{2\sqrt{\frac{a\Pr}{-1+\Pr}}y},\ B = 1 + B_{17} + B_{11} - B_{17}B_{12},$

$B_0 = 1 + B_{17} + B_{15} - B_{17}B_{16},$

$B_{11} = Erf[\frac{2\sqrt{\frac{aSc}{-1+Sc}}t - y}{2\sqrt{t}}],\ B_{12} = Erf[\frac{2\sqrt{\frac{aSc}{-1+Sc}}t + y}{2\sqrt{t}}],\ B_{13} = [-1 + Erf\frac{\sqrt{Sc\,y}}{2\sqrt{t}}],$

$$B_{14} = Erf\,\frac{\sqrt{Sc}\,y}{2\sqrt{t}},\ B_{15} = Erf\left[\frac{2\sqrt{\frac{a}{-1+Sc}}\,t - \sqrt{Sc}\,y}{2\sqrt{t}}\right],\ B_{16} = Erf\left[\frac{2\sqrt{\frac{a}{-1+Sc}}\,t + \sqrt{Sc}\,y}{2\sqrt{t}}\right],$$

$$B_{17} = e^{2\sqrt{\frac{aSc}{-1+Sc}}\,y},\ a = \frac{M}{1+m^2}(1-im),$$

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Table for the skin friction

m	***Gr***	***Gm***	***M***	***Sc***	***Pr***	***t***	τ_x	τ_z
1.0	5.0	10	1	2.01	0.71	0.3	0.9521	0.1958
1.0	5.0	20	1	2.01	0.71	0.3	1.4570	0.2019
1.0	5.0	30	1	2.01	0.71	0.3	1.9618	0.2080
1.0	10	5.0	1	2.01	0.71	0.3	2.3312	0.2355
1.0	15	5.0	1	2.01	0.71	0.3	3.9628	0.2782
1.0	20	5.0	1	2.01	0.71	0.3	5.5944	0.3210
1.0	5.0	5.0	1	2.01	0.71	0.3	0.6996	0.1928
1.5	5.0	5.0	1	2.01	0.71	0.3	0.7752	0.1815
2.0	5.0	5.0	1	2.01	0.71	0.3	0.8188	0.1591
1.0	5.0	5.0	1	2.01	0.71	0.3	0.6996	0.1928
1.0	5.0	5.0	3	2.01	0.71	0.3	0.3002	0.5216
1.0	5.0	5.0	5	2.01	0.71	0.3	-0.0833	0.7866
1.0	5.0	5.0	1	2.01	2.00	0.3	0.3215	0.1751
1.0	5.0	5.0	1	2.01	5.00	0.3	0.0082	0.1641
1.0	5.0	5.0	1	2.01	7.00	0.3	-0.0959	0.1611
1.0	5.0	5.0	1	3.00	0.71	0.3	0.6709	0.1921
1.0	5.0	5.0	1	4.00	0.71	0.3	0.6511	0.1917
1.0	5.0	5.0	1	2.01	0.71	0.1	-0.8648	0.0965
1.0	5.0	5.0	1	2.01	0.71	0.12	-0.6129	0.1073
1.0	5.0	5.0	1	2.01	0.71	0.13	-0.5029	0.1125