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Magnetohydrodynamic Peristaltic Transport with Porous Medium through a Coaxial Asymmetric Vertical Tapered Channel and Joule Heating with Radiation

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ABSTRACT

The main objective of present investigation is to introduce the magnetohydrodynamic peristaltic transport with porous medium through a coaxial asymmetric vertical tapered channel and Joule heating with radiation. Effects of sundry parameters on the temperature and heat transfer coefficient at the wall $y = h_1$ are studied through graphs. It is noted that the temperature increases when increase in Radiation parameter (N), Prandtl number (Pr), heat source/sink parameter (γ), Brinkman number (Br), Hartmann number (M), non-uniform parameter (K_1) and non-dimensional amplitude (ε) in entire tapered channel. Further, we observe that the heat transfer coefficient decreases when non-uniform parameter (K_1) is assigned higher values.

Introduction

Past five decades researchers have extensively paying attention on the peristaltic pumping of Newtonian and non-Newtonian fluids. In particular, the study of peristaltic flow has generated a lot of interest and hence good literature is currently available on the subject. A thorough understanding of peristalsis is of great interest, due to its natural property of many biological systems having smooth muscle tubes which transports biofluids through its propulsive movements. It is found in the movement of food bolus through oesophagus, transport of urine from kidney to the bladder through the urethra, the movement of spermatozoa in the ducts afferents of the male reproductive tract, circulation of blood in the small blood vessels, the movement of chyme in the gastro-intestinal tract, intra-uterine fluid motion, movements of ovum in the female fallopian tube are only some examples of peristaltic fluid flow.

LATHAM [1] made initial effort regarding peristaltic mechanism of viscous fluids. The primary mathematical models of peristalsis obtained by a train of sinusoidal waves in an infinitely long symmetric channel or tube were introduced by Fung and Yih [2] and Shapiro et al. [3]. After these studies, numerous numerical, analytical and experimental attempts have been made to understand peristaltic action in different situations for non-Newtonian/Newtonian fluid flows. Afterward, few relevant interested discussions can be seen via attempts such as Brown and Hung [4] and Hayat et al.[5 & 6], Takabatake and Ayukawa [7& 8], Srivastava and Srivastava [9, 10&11], Siddiqui and Schwarz [12], Ramachandra and Usha[13],Elshehawey and Sobh [14], Sobh [15], Abd El Naby et al.[16], J.B. Shukla et al.[17], T. Hayat et al.[18], M.H. Haroun [19], T. Hayat et al.[20], Ravikumar et al. [21, 22 & 23].

Heat transfer and mass transfer are natural processes which occur quite often in the field of power engineering, refrigeration and air conditioning, chemical engineering, metallurgical engineering etc. They are also widely used in porous industries. Heat transfer is the transition of thermal energy from a region of higher temperature to a region of lower temperature. The transfer of thermal energy continues until the object and its surroundings reach the state of thermal equilibrium. The energy transfer by heat flow cannot be measured directly. But the concept has physical meaning because it is related to the measurable quantity called temperature. Vajravelu *et al*. [24] have been investigated on heat transfer characteristics on peristaltic flow in a porous annulus. In another research paper, Nadeem et al. [25] examined an influence of heat transfer in peristalsis with variable viscosity. Very recently, Sk Abzal [26] examined on an influence of heat transfer on magnetohydrodynamic peristaltic blood flow with porous medium through a coaxial vertical asymmetric tapered channel - an analysis of blood flow study. An interested investigation discussed by Abbasi Fahad Munir et al. [27] on Peristaltic flow in an asymmetric channel with convective boundary conditions and Joule heating. In another attempt, K. Venugopal Reddy et al. [28] gave on Velocity slip and joule heating effects on MHD peristaltic flow in a porous medium. Influence of convective conditions in radiative peristaltic flow of pseudo plastic nanofluid in a tapered asymmetric channel by T. Hayat et al. [29]. Shehzad SA et al. [30] discussed on MHD mixed convective peristaltic motion of nanofluid with Joule heating and thermophoresis effects.

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2. Formulation of the problem

Consider the peristaltic transport of a viscous fluid through an asymmetric vertical tapered channel through the porous medium. Asymmetry in the flow is due to the propagation of peristaltic waves of different amplitudes and phase on the channel walls. We assume that the fluid is subject to a constant transverse magnetic field B_0. The flow is generated by sinusoidal wave trains propagating with steady speed c along the tapered asymmetric channel walls.

The geometry of the wall surface is defined as

$$Y = H_2 = b + m'X + d\sin\left[\frac{2\pi}{\lambda}(X - ct)\right] \tag{1}$$

$$Y = H_1 = -b - m'X - d\sin\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right] \tag{2}$$

Where b is the half-width of the channel, d is the wave amplitude, c is the phase speed of the wave and m' $(m' << 1)$ is the non-uniform parameter, λ is the wavelength, t is the time and X is the direction of wave propagation. The phase difference ϕ varies in the range $0 \le \phi \le \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and further b, d and ϕ satisfy the following conditions for the divergent channel at the inlet $d\cos(\frac{\phi}{2}) \le b$

It is assumed that the left wall of the channel is maintained at temperature T_0, while the right wall has temperature T_1.

The equations governing the motion for the present problem prescribed by\

$$u_x + v_y = 0 \tag{3}$$

$$\rho(uu_x + vu_y) = -p_x + \mu(u_{xx} + u_{yy}) - (\sigma B_0^2)(u + c) - \left(\frac{\mu}{k_1}\right)(u + c) \tag{4}$$

$$\rho(uv_x + vv_y) = -p_y + \mu(v_{xx} + v_{yy}) - (\sigma B_0^2)v - \left(\frac{\mu}{k_1}\right)v \tag{5}$$

$$\rho C_p(uT_x + T_y) = k(T_{xx} + T_{yy}) + Q_0 + \sigma B_0^2 u^2 - q_y \tag{6}$$

u and v are the velocity components in the corresponding coordinates, k_1 is the permeability of the porous medium, ρ is the density of the fluid, p is the fluid pressure, k is the thermal conductivity, μ is the coefficient of the viscosity, Q_0 is the constant heat addition/absorption, C_p is the specific heat at constant pressure, σ is the electrical conductivity and T is the temperature of the fluid.

The relative boundary conditions are

$$\bar{U} = 0, \bar{T} = T_0, \ \bar{C} = C_0 \ \text{at} \ \ \bar{Y} = \bar{H}_1$$

$$\bar{U} = 0, \bar{T} = T_1, \ \bar{C} = C_1 \ \text{at} \ \ \bar{Y} = \bar{H}_2$$

The radioactive heat flux (Cogley et al. [31]) is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2(T_0 - T_1) \tag{7}$$

here α is the mean radiation absorption coefficient.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$x = X - ct$, $y = Y$, $u = U - c$, $v = V$ and $p(x) = P(X, t)$ (8)

Introducing the following non-dimensional quantities:

$$\bar{x} = \frac{x}{\lambda} \quad \bar{y} = \frac{y}{b} \quad \bar{t} = \frac{ct}{\lambda} \quad \bar{u} = \frac{u}{c} \quad \bar{v} = \frac{v}{c\delta} \quad h_1 = \frac{H_1}{b} \quad h_2 = \frac{H_2}{b} \quad p = \frac{b^2 p}{c\lambda\mu} \quad \theta = \frac{T - T_0}{T_1 - T_0} \quad \delta = \frac{b}{\lambda}$$

$$\mathrm{Re} = \frac{\rho c b}{\mu} \ M = B_0 b\sqrt{\frac{\sigma}{\mu}} \ \Pr = \frac{\mu C_p}{k} \ E_c = \frac{c^2}{C_p(T_1 - T_0)} \ \gamma = \frac{Q_0 b^2}{\mu C_p(T_1 - T_0)} \ N^2 = \frac{4\alpha^2 d_1^2}{k} \ \varepsilon = \frac{d}{b} \tag{9}$$

where $\varepsilon = \frac{d}{b}$ is the non-dimensional amplitude of channel, $\delta = \frac{b}{\lambda}$ is the wave number, $k_1 = \frac{\lambda m'}{b}$ is the non - uniform parameter, Re is the Reynolds number, M is the Hartman number, $K = \frac{k}{b^2}$ Permeability parameter,Pr is the Prandtl number, E_c is the Eckert number, γ is the heat source/sink parameter, B_r (= $E_c P_r$) is the Brinkman number, and N^2 is the radiation parameter.

3. Solution of the problem

In view of the above transformations (8) and non-dimensional variables (9), equations (3-6) are reduced to the following non-dimensional form after dropping the bars,

$$\mathrm{Re}\,\delta\left[u\,u_x + v u_y\right] = \left[-p_x + \delta^2 u_{xx} + u_{yy} - Au - A\right] \tag{10}$$

$$\mathrm{Re}\,\delta^3\left[u v_x + v v_y\right] = \left[-p_y + \delta^4 v_{xx} + \delta^2 v_{yy} - M^2\delta^2 v - \delta^2\frac{1}{Da}v\right] \tag{11}$$

$$\mathrm{Re}\,\delta\left[u\theta_x + v\theta_y\right] = \frac{1}{\mathrm{Pr}}\left[\delta^2\theta_{xx} + \theta_{yy}\right] + \gamma + M^2 E u^2 + \frac{N^2\theta}{P_r} \tag{12}$$

Where $A = \left(M^2 + \frac{1}{Da}\right)$

Applying long wave length approximation and neglecting the wave number along with low-Reynolds numbers. Equations (10-12) become

$$\frac{\partial^2 u}{\partial y^2} - Au = \frac{\partial p}{\partial x} + A \tag{13}$$

$$\frac{\partial p}{\partial y} = 0 \tag{14}$$

$$\frac{1}{\mathrm{Pr}}\left[\delta^2\frac{\partial^2\theta}{\partial x^2}\right] + \gamma + M^2 E u^2 + \frac{N^2\theta}{P_r} = 0 \tag{15}$$

The relative boundary conditions in dimensionless form are given by

u = -1, θ = 0 at $y = h_1 = 1 - k_1 x - \varepsilon\sin\left[2\pi(x-t) + \phi\right]$ (16)

u = -1, θ = 1 at $y = h_2 = 1 + k_1 x + \varepsilon\sin\left[2\pi(x-t)\right]$ (17)

The solutions of velocity and temperature with subject to boundary conditions (16) and (17) are given by

$$u = a_1 Sinh[\alpha_1 y] + a_2 Cosh[\alpha_1 y] - a \tag{18}$$

Where

$$a = 1 + \frac{p}{A} \qquad \alpha_1 = \sqrt{M^2 + \frac{1}{Da}}$$

$$a_1 = \left(\frac{-p}{A\,Sinh[\alpha_1 h_2]}\right)\left(\frac{Sinh[\alpha_1 h_2]\left(Cosh[\alpha_1 h_1] - Cosh[\alpha_1 h_2]\right)}{Sinh[\alpha_1 h_1]\,Cosh[\alpha_1 h_2] - Sinh[\alpha_1 h_2]Cosh[\alpha_1 h_1]}\right)$$

$$a_2 = \left(\frac{p}{A}\right)\left(\frac{Sinh[\alpha_1 h_1] - Sinh[\alpha_1 h_2]}{Sinh[\alpha_1 h_1]Cosh[\alpha_1 h_2] - Sinh[\alpha_1 h_2]Cosh[\alpha_1 h_1]}\right)$$

$$\theta = a_9 Cos[N y] + a_8 Sin[N y] - \left(\frac{P_r\gamma}{N^2}\right) - \left(\frac{M^2 B_r a_3}{4\alpha_1^2 + N^2}e^{2\alpha_1 y}\right) - \left(\frac{M^2 B_r a_4}{4\alpha_1^2 + N^2}e^{-2\alpha_1 y}\right) - \left(\frac{M^2 B_r a_5}{\alpha_1^2 + N^2}e^{\alpha_1 y}\right) - \left(\frac{M^2 B_r a_6}{\alpha_1^2 + N^2}e^{-\alpha_1 y}\right) - \left(\frac{M^2 B_r a_7}{N^2}\right) \tag{19}$$

Where

$$a_3 = \left(\frac{a_1^2}{4} + \frac{a_2^2}{4} + \frac{a_1 a_2}{2}\right) \quad a_4 = \left(\frac{a_1^2}{4} + \frac{a_2^2}{4} - \frac{a_1 a_2}{2}\right) \quad a_5 = A(a_1 + a_2) \quad a_6 = A(a_1 - a_2) \quad a_7 = \left(\frac{a_2^2}{2} - \frac{a_1^2}{2}\right) + A^2$$

$$a_8 = \left(\frac{-\cos[N h_1] - \left(\frac{P_r\gamma}{N^2}\right)\left(\cos[N h_2] - \cos[N h_1]\right)}{Sin[N h_1]\cos[N h_2] - Sin[N h_2]\cos[N h_1]}\right) - \left(\frac{\frac{M^2 B_r a_3}{4\alpha_1^2 + N^2}\left(e^{2\alpha_1 h_2}\cos[N h_1] - e^{2\alpha_1 h_1}\cos[N h_2]\right)}{Sin[N h_1]\cos[N h_2] - Sin[N h_2]\cos[N h_1]}\right) - \left(\frac{\frac{M^2 B_r a_4}{4\alpha_1^2 + N^2}\left(e^{-2\alpha_1 h_2}\cos[N h_1] - e^{-2\alpha_1 h_1}\cos[N h_2]\right)}{Sin[N h_1]\cos[N h_2] - Sin[N h_2]\cos[N h_1]}\right) - \left(\frac{\frac{M^2 B_r a_5}{\alpha_1^2 + N^2}\left(e^{\alpha_1 h_1}\cos[N h_2] - e^{\alpha_1 h_2}\cos[N h_1]\right)}{Sin[N h_1]\cos[N h_2] - Sin[N h_2]\cos[N h_1]}\right) -$$

$$\left(\frac{\frac{M^2B_ra_6}{\alpha_1^2+N^2}\left(e^{-\alpha_1h_2}\cos[N\,h_1]-e^{-\alpha_1h_1}\cos[N\,h_2]\right)}{Sin[N\,h_1]\cos[N\,h_2]-Sin[N\,h_2]\cos[N\,h_1]}\right)-\left(\frac{\frac{M^2B_ra_7}{N^2}\left(\cos[N\,h_1]-\cos[N\,h_2]\right)}{Sin[N\,h_1]\cos[N\,h_2]-Sin[N\,h_2]\cos[N\,h_1]}\right)$$

$$a_9=\left(\frac{-a_8Sin[N\,h_1]+\left(\frac{P_r\,\gamma}{N^2}\right)+\frac{M^2B_ra_3}{4\alpha_1^2+N^2}\left(e^{2\alpha_1h_1}\right)+\frac{M^2B_ra_4}{4\alpha_1^2+N^2}\left(e^{-2\alpha_1h_1}\right)}{\cos[N\,h_1]}\right)+$$

$$\left(\frac{-\frac{M^2B_ra_5}{\alpha_1^2+N^2}\left(e^{\alpha_1h_1}\right)+\frac{M^2B_ra_6}{\alpha_1^2+N^2}\left(e^{-\alpha_1h_1}\right)+\frac{M^2B_ra_7}{N^2}}{\cos[N\,h_1]}\right)$$

The coefficients of the heat transfer Zh_1 and Zh_2 at the walls $y = h_1$ and $y = h_2$ respectively, are given by

$$Zh_1=\theta_y h_{1x} \tag{20}$$

$$Zh_2=\theta_y h_{2x} \tag{21}$$

The solutions of the coefficient of heat transfer at $y = h_1$ and $y = h_2$ are given by

$$Zh_1=\theta_y h_{1x}=$$

$$\begin{aligned}&\left(-Na_9Sin[N\,y]+Na_8Cos[N\,y]-2\alpha_1\left(\frac{M^2B_ra_3}{4\alpha_1^2+N^2}\right)e^{2\alpha_1y}+2\alpha_1\left(\frac{M^2B_ra_4}{4\alpha_1^2+N^2}\right)e^{-2\alpha_1y}-\alpha_1\left(\frac{M^2B_ra_5}{\alpha_1^2+N^2}\right)e^{\alpha_1y}+\right.\\&\left.\alpha_1\left(\frac{M^2B_ra_6}{\alpha_1^2+N^2}\right)e^{-\alpha_1y}-\frac{M^2B_ra_7}{N^2}\right)\left(-2\pi\,\varepsilon\,Cos\,[2\pi(x-t)+\phi]-k_1\right)\end{aligned} \tag{22}$$

$$Zh_2=\theta_y h_{2x}=$$

$$\begin{aligned}&\left(-Na_9Sin[N\,y]+Na_8Cos[N\,y]-2\alpha_1\left(\frac{M^2B_ra_3}{4\alpha_1^2+N^2}\right)e^{2\alpha_1y}+2\alpha_1\left(\frac{M^2B_ra_4}{4\alpha_1^2+N^2}\right)e^{-2\alpha_1y}-\alpha_1\left(\frac{M^2B_ra_5}{\alpha_1^2+N^2}\right)e^{\alpha_1y}+\right.\\&\left.\alpha_1\left(\frac{M^2B_ra_6}{\alpha_1^2+N^2}\right)e^{-\alpha_1y}-\frac{M^2B_ra_7}{N^2}\right)\left(2\pi\varepsilon\,Cos[2\pi(-t+x)]+k_1\right)\end{aligned} \tag{23}$$

The volumetric flow rate in the wave frame is defined by

$$\begin{aligned}&q=\int_{h_1}^{h_2}u\,dy=\int_{h_1}^{h_2}\left(a_1Sin\,h[\alpha_1y]+a_2Cos\,h[\alpha_1y]-a\right)dy\\&p\left(-b_1+b_2-\frac{(h_2-h_1)}{A}\right)-(h_{2.}-h_1)\end{aligned} \tag{24}$$

$$b_1=\left(\frac{Cosh[\alpha_1h_2]-Cosh[\alpha_1h_1]}{\alpha_1\,A\,Sinh\;[\alpha_1\,h_2]}\right)\left(\frac{Sinh[\alpha_1h_2]\left(Cosh[\alpha_1h_1]-Cosh[\alpha_1h_2]\right)}{Sinh[\alpha_1h_1]Cosh[\alpha_1h_2]-Sinh[\alpha_1h_2]Cosh[\alpha_1h_1]}\right)$$

$$Q_6=\left(\frac{\left(Sinh[\alpha_1h_2]-Sinh[\alpha_1h_1]\right)}{\alpha_1\,A}\right)\left(\frac{\left(Sinh[\alpha_1h_1]-Sinh[\alpha_1h_2]\right)}{Sinh[\alpha_1h_1]Cosh[\alpha_1h_2]-Sinh[\alpha_1h_2]Cosh[\alpha_1h_1]}\right)$$

The pressure gradient obtained from equation (23) and we can expressed as

$$\frac{dp}{dx}=\frac{q+(h_2-h_1)}{\left(-b_1+b_2-\left(\frac{(h_2-h_1)}{A}\right)\right)} \tag{25}$$

The instantaneous flux Q (x, t) in the laboratory frame is given by

$$Q=\int_{h_2}^{h_1}(u+1)dy=q+(h_1-h_2) \tag{26}$$

The average volume flow rate over one wave period (T=λ/ *c)* of the peristaltic wave is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1+d \tag{27}$$

From the equations (25) and (27), the pressure gradient can be expressed as

$$\frac{dp}{dx}=\left(\frac{\left((\overline{Q}-1-d)+(h_2-h_1)\right)}{-b_1+b_2-\frac{(h_2-h_1)}{A}}\right) \tag{28}$$

4. Numerical Solution and Discussion of the Problem

Fig. 1 depicts the variation in temperature profile for the variation in the radiation parameter N. It shows that the temperature increases when the radiation parameter increases. The variation in temperature profile with different values of Prandtl number Pr (Pr = 1, 1.5, 2) is shown in Fig.2. We observe that the increasing the values of Prandtl number, temperature profile gradually increases in the entire tapered channel. An influence of various values of heat source/sink parameter γ (γ = 0.1, 0.2, 0.3) on temperature profile as shown in Fig.3. It has been inferred that the value of θ increases with heat source/sink parameter increase. The increase of Brinkman number Br (Br = 0.1, 0.3, 0.5) on the temperature distribution is shown through Fig. 4.We notice from the graph that θ increases with increasing Brinkman number Br. Fig.5 depicts that the temperature profiles θ for various values of Hartmann number M (M = 1, 2, 3) with fixed values of other parameters N = 0.5, Br = 0.3, Pr = 1, k_1= 0.1, γ = 0.5, Da = 0.5, γ = 0.1, x =0.6, t =0. 4, ε = 0.2, ϕ = π/6.Indeed, the temperature (θ) increases with increase in Hartmann number. Variation non-uniform parameter K_1 (K_1 = 0.1, 0.2, 0.3) with temperature (θ) has been presented in Fig.6. This figure indicates that an increase in K_1, the temperature increases in entire flow channel. An influence of non-dimensional amplitude ε (ε = 0.2, 0.3, 0.4) on the temperature distribution is shown through Fig. 7. We notice that θ increases with an increase in ε.

Hence, we conclude that from Fig.1-7, the temperature profiles are almost parabolic in behaviour.

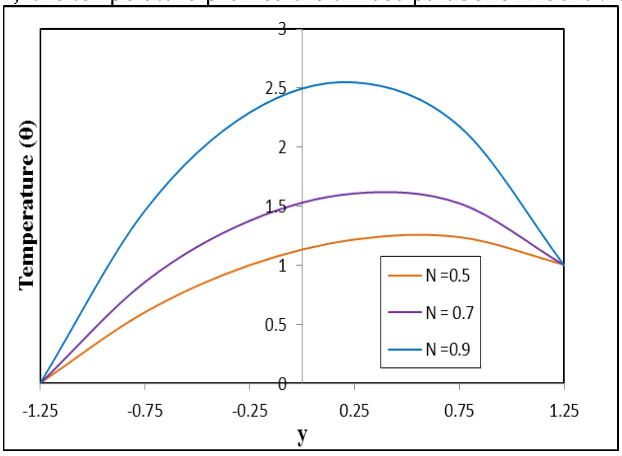

Figure (1). The variation of temperature (θ) with different values N with Pr = 1, Br = 0.3, γ = 0.1, k_1= 0.1, Da = 0.6, M = 1, x = 0.6, t =0. 4, ε = 0.2, ϕ = π/4.

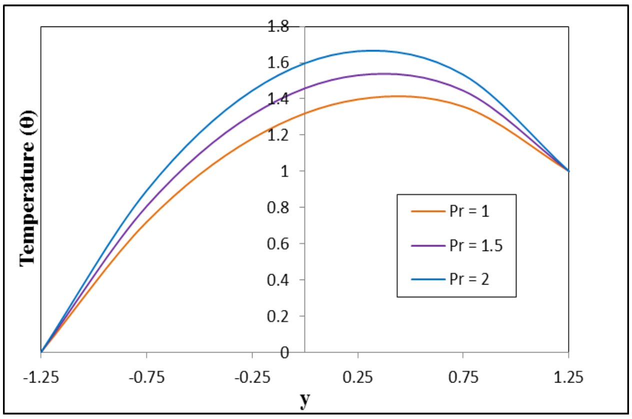

Figure 2. The variation of temperature (θ) with different values of Pr with N = 0.5, Br = 0.3, γ = 0.3, k_1= 0.1, Da = 0.6, M = 1, x = 0.6, t =0. 4, ε = 0.2, ϕ = π/4.

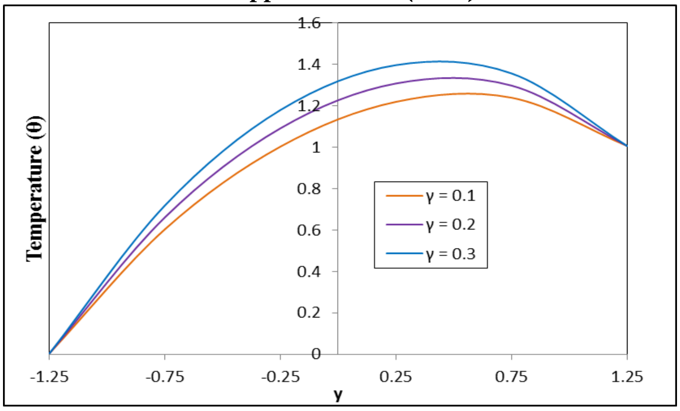

Figure 3. The variation of temperature (θ) with different values γ with $Pr = 1$, $Br = 0.3$, $N = 0.5$, $k_1 = 0.1$, $Da = 0.6$, $M = 1$, $x = 0.6$, $t = 0.4$, $\varepsilon = 0.2$, $\phi = \pi/4$.

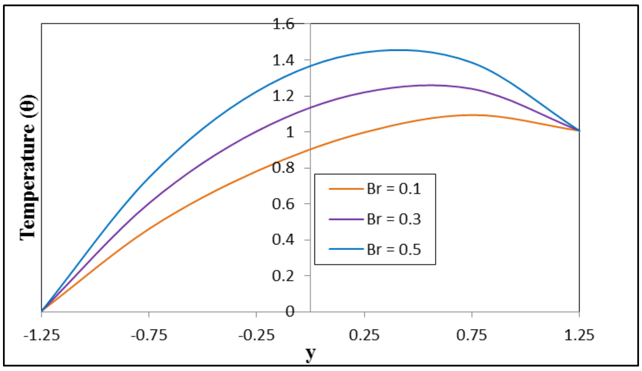

Figure 4 .The variation of temperature (θ) with different values of Br with $Pr = 1$, $\gamma = 0.1$, $N = 0.5$, $k_1 = 0.1$, $Da = 0.6$, $M = 1$, $x = 0.6$, $t = 0.4$, $\varepsilon = 0.2$, $\phi = \pi/4$.

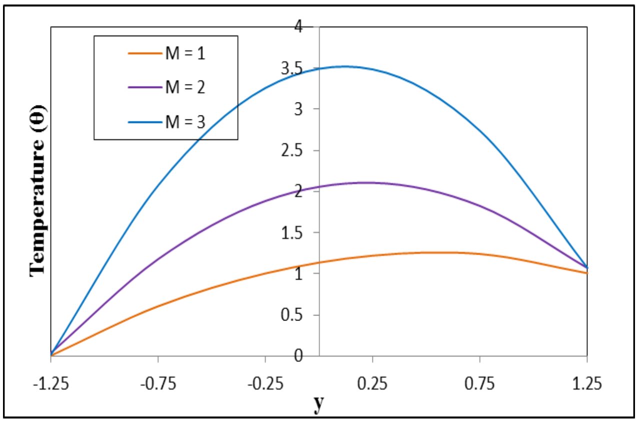

Figure 5. The variation of temperature (θ) with different values of M with $Pr = 1$, $\gamma = 0.1$, $Br = 0.3$, $N = 0.5$, $k_1 = 0.1$, $Da = 0.6$, $x = 0.6$, $t = 0.4$, $\varepsilon = 0.2$, $\phi = \pi/4$.

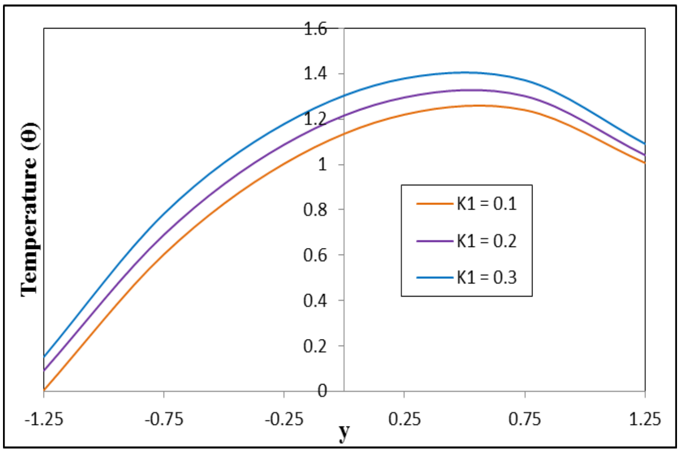

Figure 6. The variation of temperature (θ) with different values of k_1 with $Pr = 1$, $\gamma = 0.1$, $Br = 0.3$, $M = 1$, $N = 0.5$, $Da = 0.6$, $x = 0.6$, $t = 0.4$, $\varepsilon = 0.2$, $\phi = \pi/4$.

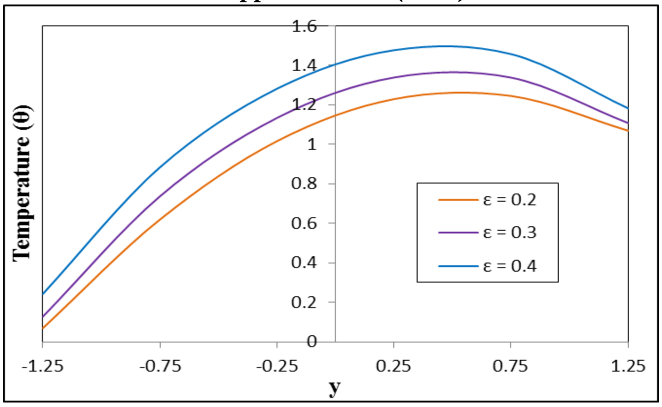

Figure 7. The variation of temperature (θ) with different values of ε with Pr = 1, γ = 0.1, Br = 0.3, M = 1, k_1 = 0.1, N = 0.5, Da = 0.6, x = 0.6, t = 0. 4, ϕ = π/4.

The results presented in Figures 8 - 14 indicates the behavior of radiation parameter (N), Prandtl number (Pr), porous medium (Da), , Magnetic field (M), Brinkman number (Br) and heat source/sink parameter (γ).on the heat transfer coefficient (Z) at y $=h_1$. These figures reveals that the oscillatory behavior of heat transfer which may be due to the phenomenon of peristalsis. Fig. 8 illustrates the variation in heat transfer coefficient at the wall y = h_1 for different values of radiation parameter N (N = 0.5, 0.7, 0.9). We observe that as the values of radiation parameter increases the heat transfer coefficient decreases in the portion of the channel x ε [0, 0.54] and then it is increases in other portion of the channel x ε [0.54, 1] with fixed other parameters. Influence of Prandtl number Pr (Pr = 1, 1.5, 2) on heat transfer coefficient at the wall y $=h_1$ as shown in Fig. 9. This figure indicates the heat transfer coefficient decreases in the portion of the tapered channel x ε [0, 0.54] and then it is increases in the rest of the tapered channel x ε [0.54, 1] with fixed other parameters. Fig. 10 depicts that the temperature distribution (θ) for various values of heat source/sink parameter γ (γ = 0.1, 0.2, 0.3). We notice from this graph that the heat transfer coefficient decreases in x ε [0, 0.54] and then it is increases in another portion of the channel x ε [0.54, 1] with fixed other parameters. However, from Fig. 11 we observe that the heat transfer coefficient decreases in the portion of the channel x ε [0, 0.54] and then it is increases in the rest of the channel x ε [0.54, 1] with various values of Br (Br = 0.1, 0.2, 0.3) with fixed other parameters. In Fig. 12, dispersion of magnetic field M (M = 1, 2, 3) on the heat transfer coefficient at the wall y = h_1 is shown and it is implies that the heat transfer coefficient decreases in the portion of the vertical tapered channel x ε [0, 0.54] and then it is increases in another portion of the vertical tapered the channel x ε [0.54, 1] with fixed other parameters. An important result presented in Fig. 13 that the variation in non-uniform parameter K_1 (K_1 = 0.1, 0.2, 0.3) on the heat transfer coefficient. It may be noted from this figure the heat transfer coefficient decreases in an entire vertical tapered channel x ε [0, 1].Influence of non-dimensional amplitude (ε) on heat transfer coefficient at the wall y = h_1 as shown in figure 14.It was notice that the heat transfer coefficient decreases in the portion of the vertical tapered channel x ε [0, 0.54] and then increases in another portion of the vertical tapered the channel x ε [0.54, 1] with fixed other parameters.

Therefore, we notice that due to peristalsis, the heat transfer coefficient is in oscillatory behavior.

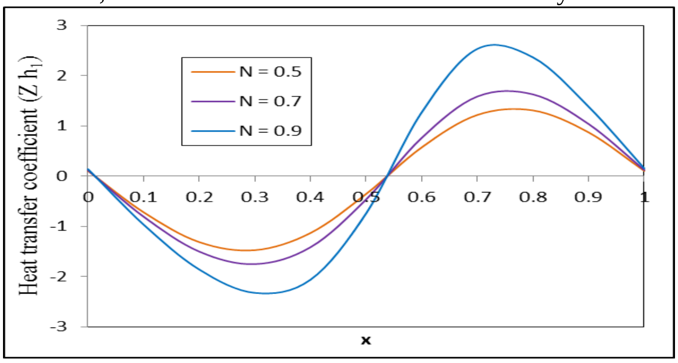

Figure 8. Effect of N on heat transfer coefficient at the wall y = h_1 with fixed Pr = 1, Br = 0.3, γ = 0.1, k_1 = 0.1, Da = 0.6, M = 1, x = 0.6, t = 0. 4, ε = 0.2, ϕ = π/4.

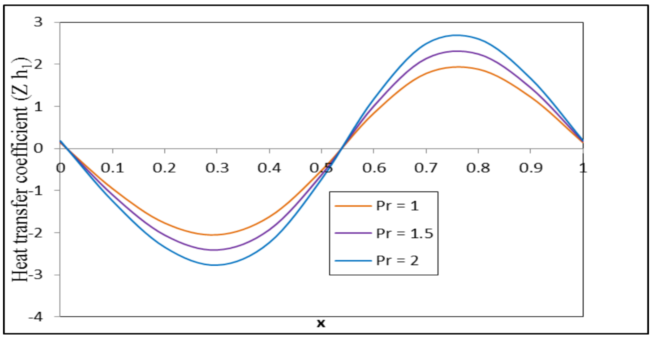

Figure 9. Effect of Pr on heat transfer coefficient at the wall y = h_1 with fixed N = 0.5, Br = 0.3, γ = 0.3, k_1 = 0.1, Da = 0.6, M = 1, x = 0.6, t = 0. 4, ε = 0.2, ϕ = π/4.

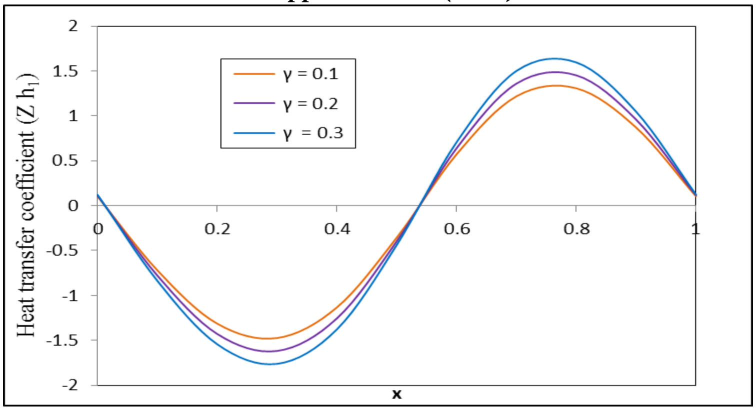

Figure 10. Effect of γ on heat transfer coefficient at the wall $y = h_1$ with fixed Pr = 1, Br = 0.3, N = 0.5, k_1 = 0.1, Da = 0.6, M = 1, x = 0.6, t =0. 4, ε = 0.2, ϕ = π/4.

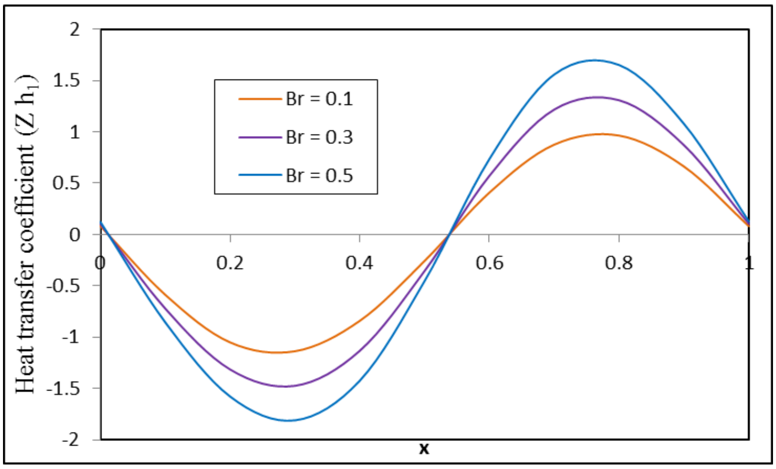

Figure 11. Effect of Br on heat transfer coefficient at the wall $y = h_1$ with fixed Pr = 1, γ = 0.1, N = 0.5, k_1 = 0.1, Da = 0.6, M = 1, x = 0.6, t =0. 4, ε = 0.2, ϕ = π/4.

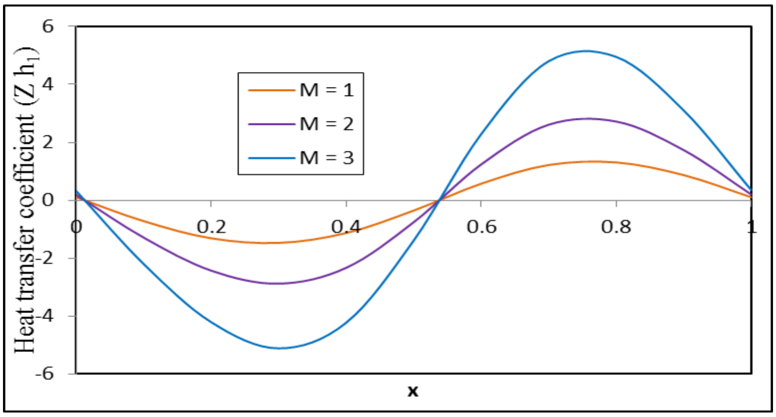

Figure 12. Effect of M on heat transfer coefficient at the wall $y = h_1$ with fixed Pr = 1, γ = 0.1, Br = 0.3, N = 0.5, k_1 = 0.1, Da = 0.6, x = 0.6, t =0. 4, ε = 0.2, ϕ = π/4.

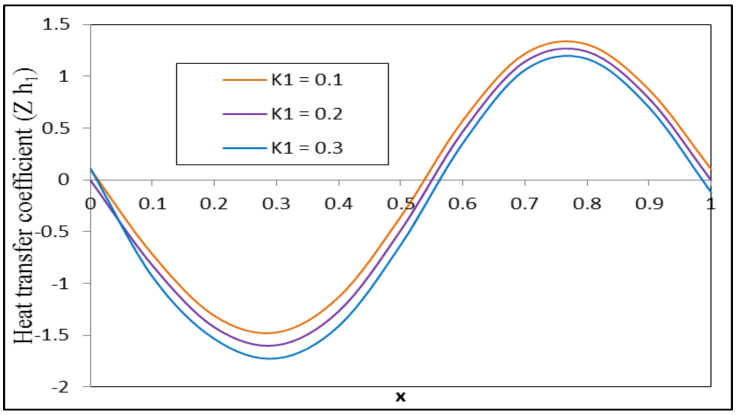

Figure 13. Effect of k_1 on heat transfer coefficient at the wall $y = h_1$ with fixed Pr = 1, γ = 0.1, Br = 0.3, M = 1, N = 0.5, Da = 0.6, x = 0.6, t =0. 4, ε = 0.2, ϕ = π/4.

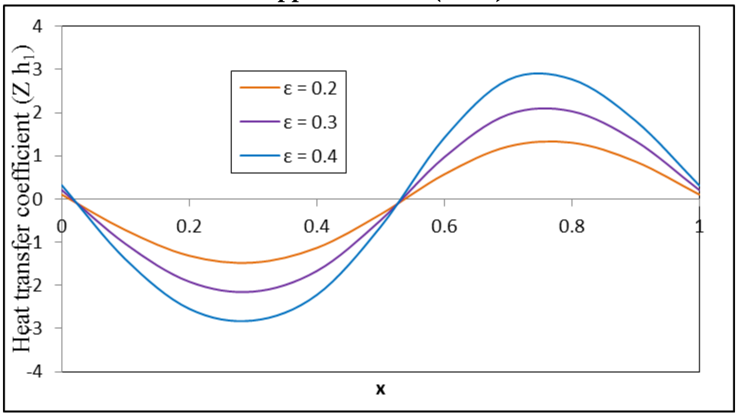

Figure 14. Effect of ε on heat transfer coefficient at the wall $y = h_1$ with fixed Pr = 1, γ = 0.1, Br = 0.3, M = 1, N = 0.5, Da = 0.6, k_1 = 0.1, x = 0.6, t =0. 4, ϕ = π/4.

Conclusions

Magnetohydrodynamic peristaltic transport with porous medium through a coaxial asymmetric vertical tapered channel and Joule heating with radiation is examined. The important findings of the present study are summarized below.

(a) We notice that the temperature increases when increase in Radiation parameter (N), Prandtl number (Pr), heat source/sink parameter (γ), Brinkman number (Br), Hartmann number (M), non-uniform parameter (K_1) and non-dimensional amplitude (ε) in entire tapered channel.

(b) Temperature profiles are almost a parabolic in behaviour.

(c) Heat transfer coefficient decreases in the portion of the channel x ε [0, 0.54] and then it is increases in another portion of the channel x ε [0.54, 1] when increase in Radiation parameter (N), Prandtl number (Pr), heat source/sink parameter (γ), Brinkman number (Br), Hartmann number (M) and non-dimensional amplitude (ε).

(d) We observe that the heat transfer coefficient decreases when non-uniform parameter (K_1) is assigned higher values.

(e) Heat transfer coefficient is in oscillatory behavior.

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