Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Appl. Math. 95 (2016) 40688-40692

ISSN: 2229-712X

Soret Effect on Unsteady MHD flow past an Impulsively Started Inclined Oscillating Plate with Variable Temperature and Mass Diffusion

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ARTICLE INFO

Article history:

Received: 13 April 2016;

Received in revised form: 25 May 2016;

Accepted: 31 May2016;

Keywords

MHD, Soret effect, Inclined oscillating plate, Variable temperature and mass diffusion.

ABSTRACT

Soret effect on unsteady MHD flow past an impulsively started inclined oscillating plate with variable temperature and mass diffusion is studied here. The fluid taken is electrically conducting. The Governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity and concentration profile is discussed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof Number, Prandtl number, soret parameter, phase angle, the magnetic field parameter and Schmidt number.

Introduction

The study of MHD flow associated with heat and mass transfer problems play important roles in different area of science and technology, like biological science, petroleum engineering, chemical engineering, mechanical engineering, biomechanics, irrigation engineering and aerospace technology. Lighthill[1] has investigated the response of laminar skin friction and heat transfer to fluctuations in the stream velocity. Free convection effects on the oscillatory flow on an infinite vertical porous plate with constant suction was analyzed by Soundalgekar[2] which was further improved by Vajravelu and sastri[4]. Oscillatory magneto hydrodynamic flow past a flat plate with Hall effect was studied by Datta and Jana[3]. Attia and Katb[5] have considered MHD flow between two parallel plate with heat transfer. Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects was investigated by Postelnicu[6]. Ibrahim[7] has analyzed analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-soret effect and Hall effect. Makinde[8] has worked on MHD mixed convection with Soret and Dufour effects past a vertical plate embedded in a porous medium. MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion was studied by Rajput and Kumar[9]. Ahmed and Sinha[10] have considered Soret effect on oscillatory MHD mixed convective mass transfer flow past an infinite vertical porous plate with variable suction. The Soret effect on free convective unsteady MHD flow over a vertical plate with heat source was investigated by Bhavana et al[11]. Saraswat and Srivastava [12] have analyzed MHD flow past an oscillating infinite vertical plate with variable temperature through porous media. Anuradha and Priyadharshini[13] have considered heat and mass transfer on unsteady MHD free convective flow past a semi-infinite vertical plate with Soret effect. Dufour effect on unsteady MHD flow through porous medium past an impulsively started inclined oscillating plate with variable temperature and mass diffusion was studied by us[14]. In this paper, we are considering Soret effect on unsteady MHD flow past an impulsively started inclined oscillating plate with variable temperature and mass diffusion. The results are shown with the help of graphs.

Mathematical Analysis

In this paper we have considered MHD flow between two parallel electrically non conducting plates inclined at an angle α from vertical. x axis is taken along the plate and y normal to it. A transverse magnetic field B_0 of uniform strength is applied on the flow. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The concentration level C_∞ is same everywhere in stationary condition. At time t > 0, the plate starts oscillating in its own plane with frequency ω and temperature of the plate is raised to T_w. The concentration C near the plate is raised linearly with respect to time.

The flow modal is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta Cos\alpha (T - T_\infty) + g\beta^* Cos\alpha (C - C_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{1}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} + D_m \frac{K_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

with the corresponding initial and boundary conditions:

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$$
\left.\begin{array}{ll}
t \le 0 : u = 0, T = T_\infty, C = C_\infty, & \text{for all y,} \\
t > 0 : u = u_0 Cos\omega t, & \text{at y=0,} \\
T = T_\infty + (T_w - T_\infty)\dfrac{u_0^2 t}{\upsilon}, & \\
C = C_\infty + (C_w - C_\infty)\dfrac{u_0^2 t}{\upsilon}, & \\
u \to 0, T \to T_\infty, C \to C_\infty & \text{as } y \to \infty,
\end{array}\right\} \quad (4)
$$

Here u is the velocity of the fluid, g - the acceleration due to gravity, β- volumetric coefficient of thermal expansion, t- time, T-temperature of the fluid, β^*-volumetric coefficient of concentration expansion, C-species concentration in the fluid, υ- the kinematic viscosity,ρ - the density, C_p- the specific heat at constant pressure, k - thermal conductivity of the fluid, D - the mass diffusion coefficient , D_m - is the effective mass diffusivity rate, T_w-temperature of the plate at $y=0$, C_w- species concentration at the plate $y= 0$, B_0- the uniform magnetic field, σ- electrically conductivity.

The following non- dimensional quantities are introduced to transform equation (1), (2) and (3) into dimensional form:

$$
\left.\begin{array}{l}
\bar{y} = \dfrac{y u_0}{\upsilon}, \bar{u} = \dfrac{u}{u_0}, \theta = \dfrac{(T - T_\infty)}{(T_w - T_\infty)}, S_c = \dfrac{\upsilon}{D}, \\
\mu = \rho\upsilon, P_r = \dfrac{\mu c_P}{k}, G_r = \dfrac{g\beta\upsilon(T_w - T_\infty)}{u_0^3}, \\
\varpi = \dfrac{\omega\upsilon}{u_0^2}, M = \dfrac{\sigma B_0^2 \upsilon}{\rho u_0^2}, G_m = \dfrac{g\beta^*\upsilon(C_w - C_\infty)}{u_0^3}, \\
\overline{C} = \dfrac{(C - C_\infty)}{(C_w - C_\infty)}, \bar{t} = \dfrac{t u_0^2}{\upsilon}, S_r = \dfrac{D_m K_T}{\upsilon T_m (C_w - C_\infty)},
\end{array}\right\} \quad (5)
$$

Here $\bar{u}$ is the dimensionless velocity, y - dimensionless coordinate axis normal to the plates, $\bar{t}$ - dimensionless time, θ - the dimensionless temperature, $\overline{C}$ - the dimensionless concentration, G_r - thermal Grashof number, G_m - mass Grashof number, μ - the coefficient of viscosity, P_r - the Prandtl number, S_c - the Schmidt number, K_T -Thermal diffusion ratio, T_m -The mean fluid temperature , M - the magnetic parameter and S_r - Soret number.

Thus the model becomes

$$
\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + G_r Cos\alpha\theta + G_m Cos\alpha\overline{C} - M\bar{u}, \quad (6)
$$

$$
\frac{\partial \overline{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial \bar{y}^2} + S_r \frac{\partial^2 \theta}{\partial \bar{y}^2}, \quad (7)
$$

$$
\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{y}^2}, \quad (8)
$$

The boundary conditions are:

$$
\left.\begin{array}{ll}
\bar{t} \le 0 : \bar{u} = 0, \theta = 0, \overline{C} = 0, & \text{for all } \bar{y}, \\
\bar{t} > 0 : \bar{u} = Cos\,\bar{\omega}\bar{t}, \theta = \bar{t}, \overline{C} = \bar{t}, & \text{at } \bar{y} = 0, \\
\bar{u} \to 0, \theta \to 0, \overline{C} \to 0, & \text{as } \bar{y} \to \infty.
\end{array}\right\} \quad (9)
$$

Dropping bars in the above equations, we get

$$
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r Cos\alpha\,\theta + G_m Cos\alpha\, C - Mu, \quad (10)
$$

$$
\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + S_r \frac{\partial^2 \theta}{\partial y^2}, \quad (11)
$$

$$
\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}, \quad (12)
$$

The boundary conditions become:

$$
\left.\begin{array}{ll}
t \le 0 : u = 0, \theta = 0, C = 0, & \text{for all y,} \\
t > 0 : u = Cos\,\omega t, \theta = t, C = t, & \text{at y=0,} \\
u \to 0, \theta \to 0, C \to 0, & \text{as } y \to \infty
\end{array}\right\} \quad (13)
$$

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13), are solved by the usual Laplace - transform technique. The solution is as under:

$$
\begin{aligned}
u = {} & \frac{1}{4}e^{-it\omega}(A_{21} + A_{22} - A_{23} - A_{24},) + \frac{1}{2A^2}Cos\alpha \\
& [(2G_r + \xi G_m)(2e^{-\sqrt{M}y}A_1 + 2(Mt + P_r)A_2 e^{-\sqrt{M}y} + \\
& y\sqrt{M}e^{-\sqrt{M}y}A_3 + \frac{(4 - P_r)}{2}A_4 A_7) + 2(\xi(\ _m - G_r) \\
& (A_{11} + A_9(My^2 P_r + 2Mt + 2P_r - 2) + A_7(A_5 - 1) - \\
& \frac{A_5 P_r}{\sqrt{\pi}}) - 2G_m(1 + \xi)(A_{12} + A_{10}(My^2 S_c + 2Mt + \\
& 2S_c - 2) + A_6 A_8(S_c - 1))]
\end{aligned}
$$

$$
\begin{aligned}
C = {} & (1 + \xi)(\frac{2t + y^2 S_c}{2}Erfc[\frac{y\sqrt{S_c}}{2\sqrt{t}}] - \frac{A_{12}}{2M}) + \\
& \xi(\frac{2t + y^2 P_r}{2}Erfc[\frac{y\sqrt{P_r}}{2\sqrt{t}}] - \frac{A_{11}}{2M})
\end{aligned}
$$

$$
\theta = t\{(1 + \frac{y^2 P_r}{2t})erfc[\frac{\sqrt{P_r}}{2\sqrt{t}}] - \frac{y\sqrt{P_r}}{\sqrt{\pi\sqrt{t}}}e^{-\frac{y^2}{4t}P_r}\}
$$

The expressions for the constants involved in the above equations are given in the appendix.

Result and Discussion

The velocity and temperature profile for different parameters like, mass Grashof number Gm, thermal Grashof number Gr, magnetic field parameter M, Soret number Sr, Prandtl number Pr, Schmidt number Sc and time t are shown in figures 1 to 13. It is observed from figure 1, that velocity of fluid decreases when the angle of inclination (α) is increased. It is observed from figure 2, when the mass Grashof number is increased then the velocity gets increased. From figure 3, it is deduced that when thermal Grashof number Gr is increased then the velocity is increased. It is observed from figure 4, that the effect of increasing values of the parameter M results in decreasing u. it is deduced that when velocity is increased then Prandtl number is increased (figure 5).

When the Schmidt number is increased then the velocity gets decreased (figure 6). Further, it is observed that when Soret number is increased then the velocity is increased (figure 7). It is deduced that when phase angle is increased then the velocity is decreased (figure 8). Further, from figure 9, it is observed that velocities increase with time. It is observed that the concentration increases when Soret number, Prandtl number and time are increased (figures 10,12,13) and it decreases with increase in Schmidt number (figure 11).

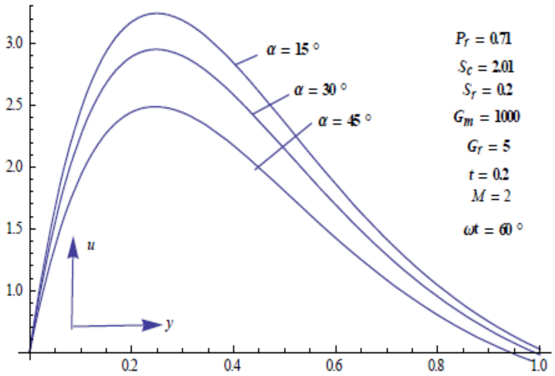

Figure 1. Velocity u for different values of α.

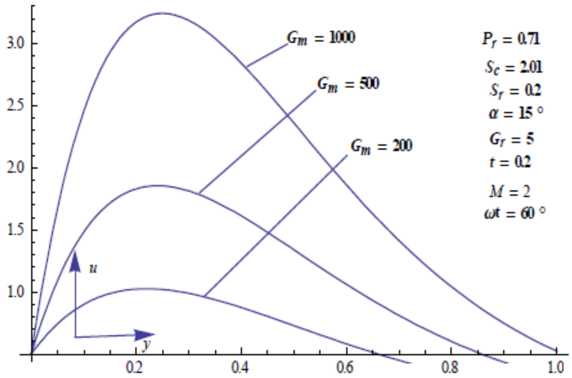

Figure 2: Velocity u for different values of *Gm*.

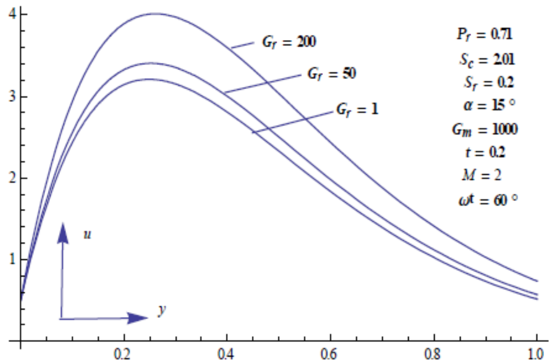

Figure 3. Velocity u for different values of *Gr*.

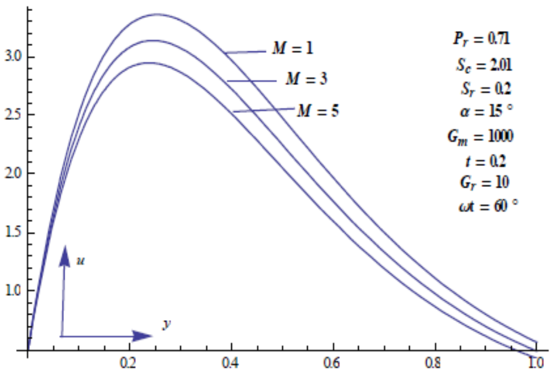

Figure 4. Velocity u for different values of *M*.

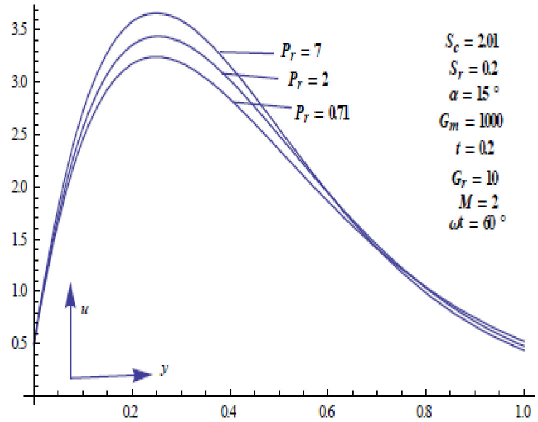

Figure 5. Velocity u for different values of *Pr*.

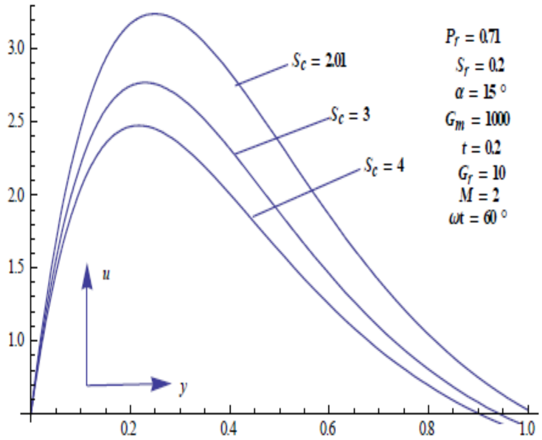

Figure 6. Velocity u for different values of *Sc*.

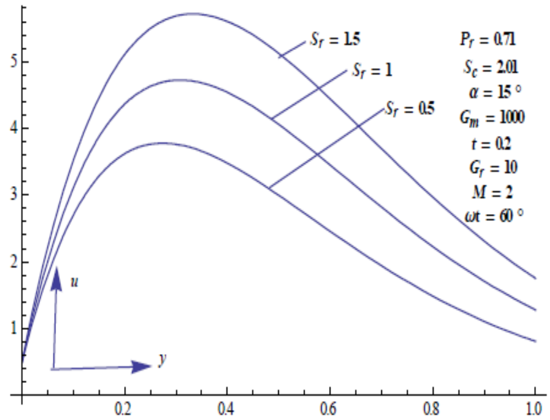

Figure 7. Velocity u for different values of *Sr* .

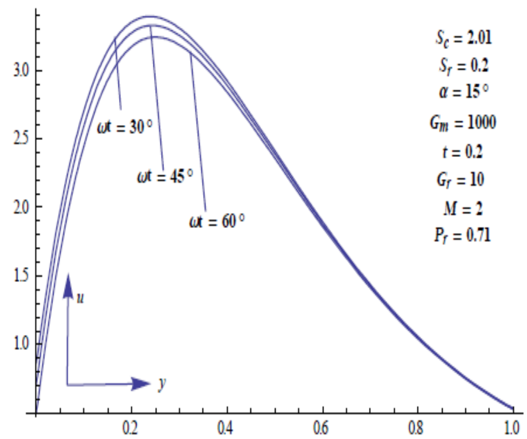

Figure 8. Velocity u for different values of ωt .

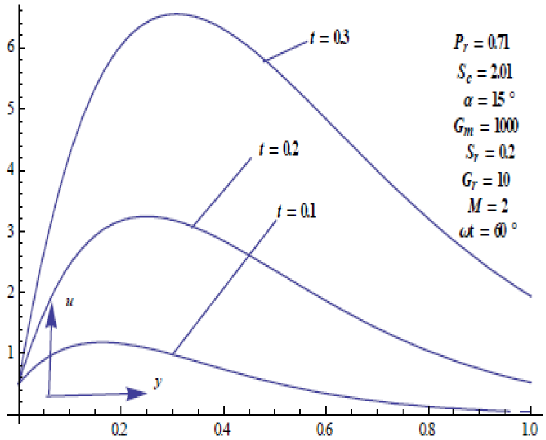

Figure 9. Velocity u for different values of *t*.

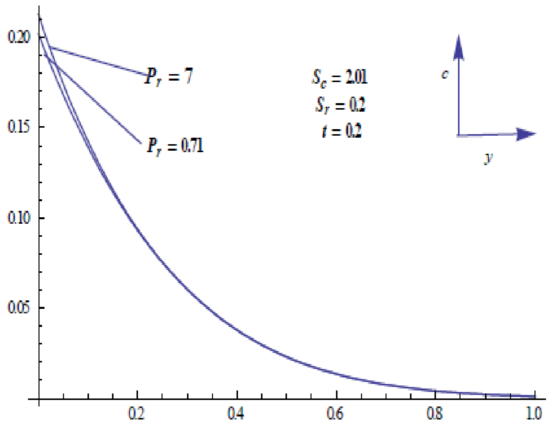

Figure 10. Concentration c for different values of *Pr*.

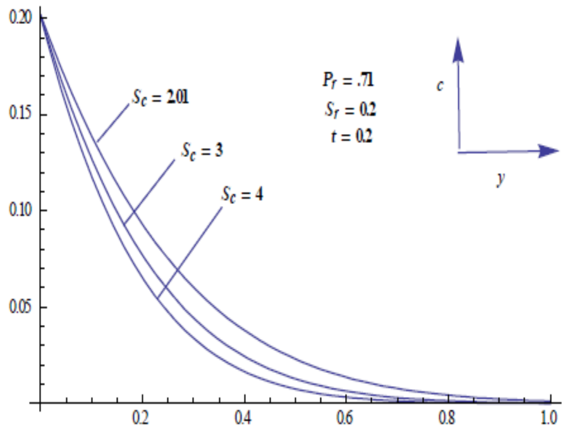

Figure 11. Concentration c for different values of *Sc*.

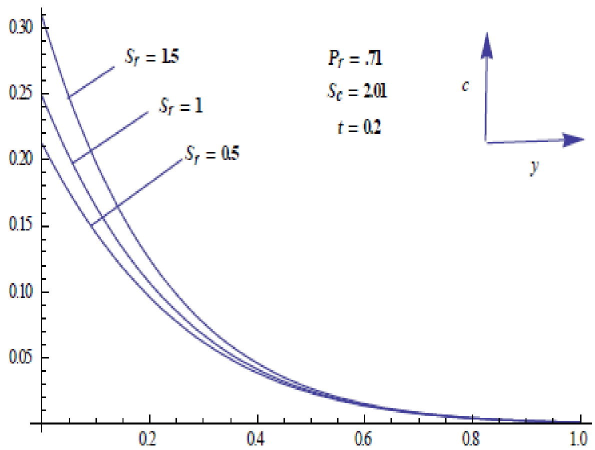

Figure 12. Concentration c for different values of *Sr*

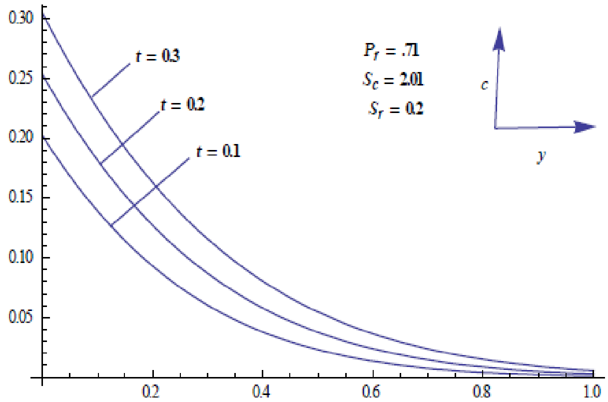

Figure 13. Concentration c for values of *t*.

Conclusion

In this paper a theoretical analysis has been done to study the Soret effect on unsteady MHD flow past an impulsively started inclined oscillating plate with variable temperature and mass diffusion. Solutions for the model have been derived by using Laplace - transform technique. Some conclusions of the study are as below:

- Velocity increases with the increase in thermal Grashof number , mass Grashof Number, Prandtl number, Soret number and time.
- Velocity decreases with the increase in the angle of inclination of the plate, the magnetic field, phase angle and Schmidt number.
- Concentration near the plate increases with the increase in Prandtl number, Soret number and time.
- Concentration near the plate decreases with Schmidt number is increased.

Appendix

$$A_1 = (1 + A_{13} + e^{2\sqrt{M}y}(1 - A_{14})),\ A_2 = -A_1,$$

$$A_3 = (1 + A_{13} + e^{2\sqrt{M}y}(A_{14} - 1)),$$

$$A_4 = (-1 + A_{15} + A_{19}(A_{16} - 1)),$$

$$A_5 = (-1 + A_{17} + A_{19}(A_{18} - 1)),$$

$$A_6 = (1 + A_{17} + A_{20}(1 - A_{18})),$$

$$A_7 = e^{\frac{Mt}{-1+P_r} - y\sqrt{\frac{Mt}{-1+P_r}}},\ A_8 = e^{\frac{Mt}{-1+S_c} - y\sqrt{\frac{Mt}{-1+S_c}}},$$

$$A_9 = \left(-1 + Erf\left[\frac{y\sqrt{P_r}}{2\sqrt{t}}\right]\right),$$

$$A_{10} = \left(-1 + Erf\left[\frac{y\sqrt{S_c}}{2\sqrt{t}}\right]\right),$$

$$A_{11} = \frac{2Mye^{-\frac{y^2 P_r}{4t}}\sqrt{tP_r}}{\sqrt{\pi}},$$

$$A_{12} = \frac{2Mye^{-\frac{y^2 S_c}{4t}}\sqrt{tS_c}}{\sqrt{\pi}},$$

$$A_{13} = Erf\left[\frac{2\sqrt{M}t - y}{2\sqrt{t}}\right],$$

$$A_{14} = Erf\left[\frac{2\sqrt{M}t + y}{2\sqrt{t}}\right],$$

$$A_{15} = Erf\left[\frac{y - 2t\sqrt{\frac{MP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{16} = Erf\left[\frac{y + 2t\sqrt{\frac{MP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{17} = Erf\left[\frac{2t\sqrt{\frac{M}{-1+P_r}} - y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{18} = Erf\left[\frac{2t\sqrt{\frac{M}{-1+P_r}} + y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{19} = e^{2y\sqrt{\frac{MP_r}{-1+P_r}}},\ A_{20} = e^{2y\sqrt{\frac{MS_c}{-1+S_c}}},$$

$$A_{21} = e^{-y\sqrt{M-iw}} + e^{y\sqrt{M-iw}},$$

$$A_{22} = e^{-y\sqrt{M+iw}+2itw} + e^{y\sqrt{M+iw}+2itw},$$

$$\zeta = \frac{S_c P_r S_r}{P_r - S_c},$$

$$A_{23} = e^{-y\sqrt{M-iw}} erf\left[\frac{y - 2t\sqrt{M - iw}}{2\sqrt{t}}\right] + e^{y\sqrt{M-iw}} erf\left[\frac{y + 2t\sqrt{M - iw}}{2\sqrt{t}}\right],$$

$$A_{24} = e^{-y\sqrt{M+iw}+2itw} erf\left[\frac{y - 2t\sqrt{M + iw}}{2\sqrt{t}}\right] + e^{y\sqrt{M+iw}+2itw} erf\left[\frac{y + 2t\sqrt{M + iw}}{2\sqrt{t}}\right].$$

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