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Discrete Mathematics

Elixir Dis. Math. 95 (2016) 40685-40687

ISSN: 2229-712X

Total Dominating Color Transversal number of Some familiar Graphs

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ARTICLE INFO

Article history:

Received: 29 April 2016;

Received in revised form: 25 May 2016;

Accepted: 31 May 2016;

Keywords

Total Dominating Color Transversal number,

χ – Partition of a graph,

Transversal of a χ – Partition.

ABSTRACT

A Total Dominating Color Transversal Set of a Graph G is a Total Dominating Set which is also Transversal of Some χ - Partition of vertices of G. Here χ is the Chromatic number of the graph G. Total Dominating Color Transversal number of a graph is the cardinality of a Total Dominating Color Transversal Set which has minimum cardinality among all such sets that the graph admits. In this paper we find this number for Generalized Wheel Graph, Petersen Graph, Herschel Graph, Grotszch graph and Helm Graph.

1.Introduction

In [1], we introduced the concept of Total Dominating Color Transversal number of a graph. We determined this number for Path Graph, Cycle Graph, Complete graph etc. We also found some characterizations of this number. In this paper we determine this number for very well known graphs called Petersen Graph, Generalized Wheel Graph, Herschel Graph, Grotszch graph and Helm Graph.

2. Definitions

Definition 2.1[3]: (Total Dominating Set)

Let G = (V, E) be a graph. Then a subset S of V (the vertex set of G) is said to be a Total Dominating Set of G if for each v ∈ V, v is adjacent to some vertex in S.

Definition 2.2[3]: (Minimum Total Dominating Set/Total Domination number)

Let G = (V, E) be a graph. Then a Total Dominating set S is said to be a Minimum Total Dominating set of G if $|S|$ = minimum { $|D|$: D is a Total Dominating set of G}. Here S is called γ_t –set and its cardinality, denoted by γ_t(G) or just by γ_t, is called the Total Domination number of G.

Definition 2.3[1]: (χ -partition of a graph)

Proper coloring of vertices of a graph G, by using minimum number of colors, yields minimum number of independent subsets of vertex set of G called equivalence classes (also called color classes of G). Such a partition of a vertex set of G is called a χ - Partition of the graph G.

Definition 2.4[1]: (Transversal of a χ - Partition of a graph) Let G = (V, E) be a graph with χ – Partition $\{V_1, V_2, \ldots, V_\chi\}$. Then a set S ⊂ V is called a Transversal of this χ – Partition if $S \cap V_i \neq \emptyset$, $\forall\, i \in \{1, 2, 3, \ldots, \chi\}$.

Definition 2.5[1]: (Total Dominating Color Transversal Set)

Let G = (V, E) be a graph. Then a Total Dominating Set S ⊂ V is called a Total Dominating Color Transversal Set of G if it is Transversal of at least one χ – partition of G.

Definition 2.6[1]: (Minimum Total Dominating Color Transversal Set)

Let G = (V, E) be a graph. Then S ⊂ V is called a Minimum Total Dominating Color Transversal Set of G if $|S|$ = minimum {$|D|$: D is a Total Dominating Color Transversal Set of G}. Here S is called γ_{tstd} –Set and its cardinality, denoted by by γ_{tstd}(G) or just by γ_{tstd}, is called the Total Dominating Color Transversal number of G.

Definition 2.7:(Join of Two Graphs)

Let G and H be two graphs. Then their join graph, denoted by G $\vee$ H, is the graph with all the edges that connect the vertices of the first graph with the vertices of the second graph. It is a commutative operation (for unlabelled graphs).

Definition 2.8: (Generalized Wheel Graph)

The Generalized Wheel graph is denoted and defined as follows:

$W_{p+q} = K_p \vee C_q$, where $\vee$ denotes the Join operation of graphs, K_p denote the Complete graph with p vertices and C_q denote the Cycle graph with q vertices.

Definition 2.9: (Helm Graph H_n) (n ≥ 4)

A Helm graph, denoted by H_n, is a graph obtained by attaching a single edge and vertex to each vertex of the outer circuit of a wheel graph W_n.The number of vertices of H_n is 2n − 1 and the number of edges 3(n − 1).

3. Main Results

First we determine the Total Dominating Color Transversal number of Generalized Wheel Graph.

Theorem 3.1[1]: If γ_t(G)= 2 then γ_{tstd}(G) = χ(G). ■

Theorem 3.2 : γ_{tstd} (W_{p+q}) = p + 3 , when q is odd

= p + 2 , when q is even

Proof

Since each vertex of K_p is adjacent to every vertex of C_q , γ_t (W_{p+q}) = 2. So by theorem 3.1, γ_{tstd} (W_{p+q}) = χ(W_{p+q}). So let us determine the Chromatic number of W_{p+q}. Since K_p is a subgraph of W_{p+q} , γ_{tstd} (W_{p+q}) ≥ p. Now as each vertex of K_p is adjacent to every vertex of C_q , no vertex of C_q can be

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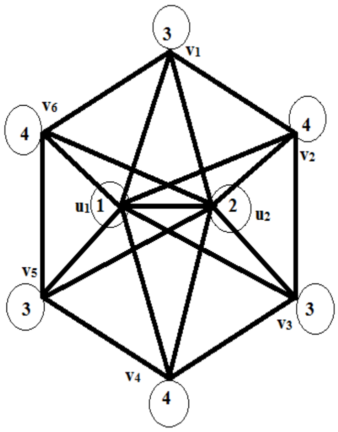
assigned color that is assigned to any vertex of K_p. So chromatic number of W_{p+q} is actually the addition of Chromatic number of K_p and Chromatic number of C_q. That is, $\chi(W_{p+q}) = \chi(K_p) + \chi(C_q)$. We know that Cycle C_q is bipartite or tripartite, respectively, when q is even or odd.

Hence we have $\gamma_{tstd}(W_{p+q}) = p + 3$, when q is odd

$= p + 2$, when q is even ■

Example 3.3:



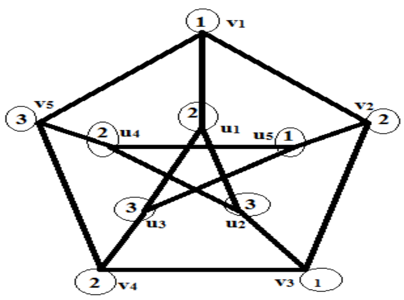
Generalized Wheel graph - $W_{2+6} = K_2 \vee C_6$

Fig. 1

$\gamma_{tstd}(W_{p+q}) = 2 + 2 = 4.$

Theorem 3.4: γ_{tstd} (P) = 4, where P Stands for Petersen Graph.

Proof:



P: Petersen Graph

Fig. 2

We know that γ_t (P) = 4. So $\gamma_{tstd}(P) \geq 4$. As $\{v_1, u_1, u_2, u_3\}$ is a Total Dominating Color Transversal Set of G, we have γ_{tstd} (P) = 4. ■

Now we find the Total Dominating Color Transversal number of other well known graph called Herschel graph. But before that we need the following results.

Result 3.5: Let G = (V, E) be a graph and S be a subset of V with diameter of < S > be k (k is a finite positive integer) (< S > is the subgraph of G induced by the vertices in S). If v ∈ V is adjacent to some vertex in S then d (u, v) ≤ k + 1, ∀ u ∈ S.

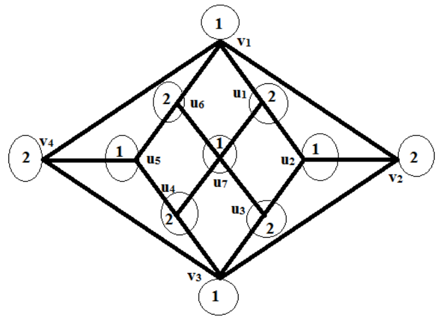
Proof: Consider the given graph G and set S. Suppose a vertex v is adjacent to some vertex in S.

Let d (u, v) > k + 1 for some u ∈ S. Definitely v ∈ V \ S, as diameter of < S > is k. Obviously v is not be adjacent to u. So suppose v is adjacent to some other vertex, say w, in S . Now k + 1 < d(u, v) ≤ d (u, w) + d (w, v) ≤ k + 1(as d (u, w) ≤ k). So we get k + 1 < k + 1, which is a contradiction. Therefore d (u, v) ≤ k + 1, ∀ u ∈ S.

Theorem 3.6 [1]: If $\chi(G) = 2$ then $\gamma_{tstd}(G) = \gamma_t$ (G). ■

Theorem 3.7: γ_{tstd}(H) = 4, where H stands for Herschel Graph.

Proof:



H: Herschel Graph

Fig.3

Clearly from the fig. 3, the Herchel graph is bipartite. So by theorem 3.6, $\gamma_{tstd}(H) = \gamma_t(H)$.

Claim: γ_t(H) = 4.

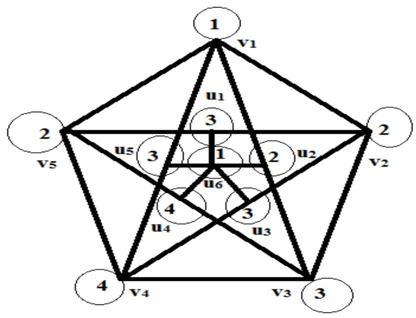
We first note that the diameter of H is 4 and d $(u_2, u_5) = 4$. Let S be a Total Dominating Set of H. Assume |S| = 3. Since S is a total dominating set of G, < S > is connected. So if D is the diameter of < S > then D ≤ 2. If v ∈ V is adjacent to some vertex in S then by result 1.4.1, d (u, v) ≤ 2 + 1 = 3, ∀ u ∈ S. Obviously both u_2 and u_5 cannot be in S as D ≤ 2. If any one of u_2 or u_5 is in S, say u_2 (without loss of generality), then the vertex u_5 cannot be adjacent to any vertex in S, by result 3.5. Note that there does not exists any vertex in H such that it is adjacent to both u_2 and u_5. Also there are no adjacent vertices x and y, in S, such that x is adjacent to u_2 and y is adjacent to u_5 , for otherwise d $(u_2, u_5) < 4$. So the only possibility is both u_2 and u_5 are out side S and there exists non adjacent verttices x and y in S such that x is adjacent to u_2 and y is adjacent to u_5. The only possible sets S are $\{u_1, v_1, u_6\}$, $\{u_1, u_7, u_6\}$, $\{u_1, u_7, u_4\}$, $\{u_1, v_1, v_4\}$, $\{u_4, u_7, u_3\}$, $\{u_4, v_3, u_3\}$, $\{u_4, v_3, v_2\}$, $\{v_1, v_2, v_4\}$ and $\{v_2, v_3, v_4\}$. None of these sets totally dominates all the vertices of H. So our assumption that |S| = 3 is wrong. Hence if S is a total dominating set of H then |S| > 3. So γ_t(H) > 3.

Note that as $\{u_2, u_3, u_5, u_6\}$ is a Total Dominating Set of H. So γ_t(H) = 4. Therefore γ_{tstd}(H) = 4.

Now we find the Total Dominating Color Transversal number of another graph called Grotszch graph, denoted by G_4, which is triangle free 4 – Chromatic graph.

Theorem 3.8: $\gamma_{tstd}(G_4) = 4$, where G_4 stands for Grotszch graph.

Proof:



Grotszch Graph – G_4

Fig. 4

We first note that Grotszch graph has Chromatic number 4.

Hence $\gamma_{tstd}(G_4) \geq 4$. Trivially $\{v_2, v_3, u_4, u_6\}$ is a γ_{tstd} - Set of G with $\gamma_{tstd}(G_4) = 4$.

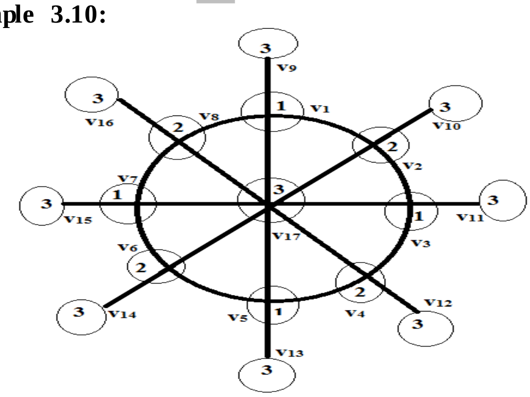
Theorem 3.9: Let H_n be a Helm Graph. Then $\gamma_{tstd}(H_n) = n$.

Proof: Note that W_n , the Wheel Graph with n vertices, is a subgraph of H_n and $\chi(W_n) = \chi(H_n)$.

Let $v_1, v_2, \ldots, v_{n-1}, v_n$ be the vertices of the W_n where $v_1, v_2, \ldots, v_{n-1}$ are the vertices on the outer cycle of W_n and v_n is a hub vertex of W_n . Clearly $v_1, v_2, \ldots, v_{n-1}$ are all support vertices of H_n.

Since any total dominating set must contain all its support vertices, $\gamma_{tstd}(H_n) \geq n - 1$. Note that v_n has color class different from all the support vertices. So $\gamma_{tstd}(H_n) \geq n$. Trivially $S = \{v_1, v_2, v_3, \ldots, v_{n-1}\} \cup \{v_n\}$ is a γ_{tstd} – Set of H_n with $\gamma_{tstd}(H_n) = n$.

Example 3.10:



Helm Graph - H_8
Fig. 5

4. Concluding Remarks

Total Dominating Color Transversal number of some familiar graphs is obtained. This number is actually is an amalgam of three very well known concepts of graph theory, viz., Total Dominating Set. Transversal and Proper Coloring of vertices of a graph. Great amount of research work is going on in these concepts, in India as well as abroad. Talking about all the three concepts together will definitely bring more interesting and amazing relations among these concepts in future.

5. References

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