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Numbers, its inception, development and operations on it

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ABSTRACT

Arabic numerals, also called Hindu-Arabic numeral, the ten digits:0,1,2,3,4,5,6,7,8,9,(0,1,2,3,4,5,6,7,8,9)the most common system for the sympolic representation of numbers in the world today. the system was adopteb by Arab mathematicians in Baghdad and passed on to the Arabs farther west. In this article we give the history of numbers and its origin.

Introduction

Number is the language of science and the best way to express it is symbols, and numrals are forms(symbols) written by the numbers codes.Alsumariun expressed Semites for numbers letters alphabet, and the Babylonians expressed them in cuneiform, and Egyptians wrote numbers in the form of horizontal and vertical lines and fees hieroglyphics and the Greeks used first letters of words to denote numbers, as did the Phoenicians and Hebrews, while Romans used vertical lines and then evolved to take the form of letters of the alphabet. Arabs before Islam expressed numbers in terms of Alphabet and after the advent of Islam and the descent of the Holy Quran and the receipt of a lot of numbers in it written in Arabic language Muslims expressed numbers writing method instead of symbols.

After the expansion of the Islamic state Arab Alsham used the Levant Romania and Coptic figures in Egypt, and continued until the time of Caliph al-Walid Ibn Abdul Malik , who ordered not to use foreign numbers in the collections of the state , makking the Arabs are returning to the use of the letters of the Arabic alphabet; أبجد ، هوز ، حطي ، كلمن ، سعفص ، قرشت ، ثخذ ، ضظغ and the number allocated to each character , as in the following table :

60	س
70	ع
80	ف
90	ص
100	ق
200	ر
300	ش

8	ح
9	ط
10	ي
20	ك
30	ل
40	م
50	ن

1	أ
2	ب
3	ج
4	د
5	ه
6	و
7	ز

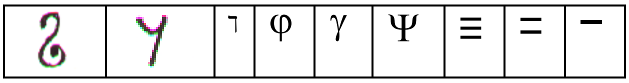
400	ت
500	ث
600	خ
700	ذ
800	ض
900	ظ
1000	غ

The rest of the thousands even nine hundredand thousand , the letters they wrote to merge with the character (غ) in the following manner :

3000	ضغ
9000	طغ
20000	كغ
90000	صغ
200000	رغ
900000	ظغ

2000	بغ
4000	دغ
10000	يغ
30000	لغ
100000	قغ
300000	شغ

In the era of al-Mansur (Abu Abdullah Muhammad Ibn Ibrahim al-vzazi first astronomical in Islam, translate the book (Send hend) of the Indian astronomer Ganga, who is looking at the stars with a way been done in India and the new numbers written:



Going to decimal system adaptation of Alsomirin and Babylonians, and sees some scientists that it is derived from the nine letters first from the ancient Aramaic alphabet sloping from the Phoenician before (450 BC), and after briefing the Arabs on these forms and studied and refined it they came out with two series of numbers called the first Indian numbers originate in that chain .The Arabic formats came as the following:

9	8	7	6	٥	٤	3	2	1

Note that these forms shows it's no different from the forms of used numbers in the Arab Mashreq especially Iraq, Egypt and Syria only slightly where there has been the number(٤) simple modification of bringing the written form (4) as well as the number (٥) lifted Alrkzh from it and became (5). It is worth noting that Muhammad Ibn Musa al-Khwarizmi[1][5,22,23] is the first one using this series in

1 Al -Khawarizmi : Abu Abdullah Muhammad ibn Musa , born in Khorezm in the country Turkistan , and amohe people, and emerged in mathematics , astronomy, his legacy includes the words and concepts algebra, algorithms, zero, cipher

writing his books and he is the first one who compose (represent) numbers on the basis of the decimal system, and gave the idea of ranks (individual, tens, hundreds, ...) also gave valuable by numbers place in these ranks.

The second series, which is called the dust numbers now known as Arabic numbers stated in aL-Kulsadi Andalusi book" the unveiling of dust aware ". Abu Muhammad Abdullah Ibn Mohammed Alozna known as Ibn al-yasmine[2][22], died in (601 AH), where he said that the number has nine forms composed by a so- called form of dust it is on the following image:

9	8	7	6	Ч	عو	3	2	1

It may also be as follows:

9	8	7	ч	ص	ع	3	2	1

Mohammed Ibn Ahmed Ibn Yahya al-hebbak al-tlemcani (d 867 AH) was drawing figures in his commentary to summarize Ibn al-banna al-marrakechi as follows:

9	8	7	6	ч	ـــع	لج	2	1

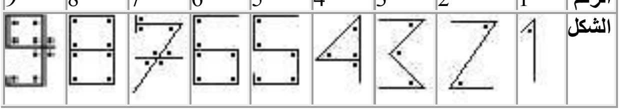
He said in his book (Collector assets numbers) that dust letters that behave in the account image nine characters as follows:

9	8	7	6	ч	ـــع	3	2	1

Some scientists believe that dust series arranged on the basis of angles number [28, p 48],and some said al-Khwarizmi was designed figures the number of angles contained in each number (with no refernce).

One includes one corner , and number two includes two angles , and so on the Arab shapes came on the form:

9	8	7	6	5	4	3	2	1	الرقم



Then there has been simple changes on in shape of this series and became in the following shapes :

	9	8	7	6	5	4	3	2	1

Other scientists believe that the idea of these numbers may be adapted for the Indian figures . However, the Arabs have developed and they gave them forms of Arabic letters :

9	8	6	5	عو	ء	خ	ح	1

After entering some modification it became what it is today.

Some researchers believe that the dust numbers devised by Arabs since the first covenant to learn Arabic writing, between the middle of the third centuries AD and the end of the sixth century, a time in which they are calligraphy transformation of his image of Nabatia to the current Arab image, which does not stray too far from the image Nabataean Line discovered in the town of Alnamardh Bhoran year 828 AD, and where male man measuring]. and some of them believe that naming this series figures dust does not mean that Indians are by their authors in origin , because these numbers are forms remained convergence features Arabic letters and reserves within the meaning of some of them as the number one , there is no difference between it and the character (ا) and somewhat in number (ч) and the number (6) which is similar to (reverse waw و) and number nine similar to the character (ط) reversed , and the number (7), it looks like inverted (ل).

Some scientists believe that the numbers defined by al-Khawarizmi whether they are Indian or Gbarbia have one(same) origen , but Modulation hit some of them evolution and change and keep the rest on his first , one and nine kept intact , while the two and three have never changed written by Arab Almashreq vertical writing and horizontal writing by Morocco Arabs and the four resemble that , while the five that are written in the form (d) is a (5) upside down with some modification , and the number (7) is taken from (7) dusty but reversed.

Note that David Eugene Smith [26], mension that the forms of Gbarbia numbers a relatively different, but essentialy have same origin.

Zero and its development:[15,16,19,25] Zero is both a number and the numerical digit used to represent that number in numerals.

Zero's origins most likely date back to the "fertile crescent" of ancient Mesopotamia. Sumerian scribes used spaces to denote absences in number columns as early as 4,000 years ago. Ancient Egyptian numerals were base 10. They used hieroglyphs for the digits and were not positional. By 1740 BC, the Egyptians had a symbol for zero (𓄤) in accounting texts. The Babylonians employed a number system based around values of 60, and they developed a specific sign—two small wedges—to differentiate between magnitudes in the same way that modern decimal-based systems use zeros to distinguish between tenths, hundreds and thous Perhaps we should note at this point that there was another civilisation which developed a place-value number system with a zero. This was the Maya people who lived in central America .The Mayans began using a zero marker in their calendars around 350 A.D. These early counting systems only saw the zero as a placeholder—not a number with its own unique value or properties.

The ancient Greeks began their contributions to mathematics around the time that zero as an empty place indicator was coming into use in Babylonian mathematics.Records show that the ancient Greeks seemed unsure about the status of zero as a number. They asked themselves, "How can nothing *be* something ".By 130 AD, Ptolemy, influenced by Hipparchus and the Babylonians, was using a symbol for zero (a small circle with a long overbar) within a sexagesimal numeral system otherwise using alphabetic Greek numerals.

In around 500AD Aryabhata devised a number system which has no zero yet was a positional system. He used the word "kha" for position and it would be used later as the name for zero. The mathematician Brahmagupta and others used small dots under numbers to show a zero place-holder, but they also viewed the zero as having a null value, called "sunya".Another zero was used in tables alongside Roman numerals by 525 (first known use by Dionysius Exiguus), but as a word, *nulla* meaning "nothing", not as a symbol.). The initial "N" was used as a zero symbol in a table of Roman numerals by Bede or his colleague around 725. Later Indian mathematicians had names for zero in positional numbers yet had no symbol for it. The first record of the Indian use of zero which is dated and agreed by all to be genuine was written in 876. Note that zero known among the Arabs in the past and

2 Ibn al-yasmine: From the people of Fez died in Marrakesh in 1204AD , poet,mathematician and astronomer.

used their language to denote the word (except) as can be seen in the words of Hatim al-Tai:

(See that what has not had decimated Dharna though it seemed evident than zero).

And in the words of Prophet Muhammad, peace be upon him: " The Lord shy cream , ashamed of his servant if he raised his hands to the sky bring the hands back to zero. Narrated by Abu Dawood in his Sunan. From Babylonians or India or other civilizations, the zero made its way to China and back to the Middle East, where it was taken up by the mathematician Mohammed Ibn-Musa al-Khowarizmi around 773. It was al-Khowarizmi who first synthesized Indian arithmetic and showed how the zero could function in algebraic equations, and by the ninth century the zero had entered the Arabic numeral system in a form resembling the oval shape we use today. The first recording of the Arab zero back to 873 AD , while the first Indian zero back to 876 AD.

Muslims expressed(denoted) zero as small circle with appoint inside it ⊙ and then chose the Levant point to reflect their importance to zero in writing came (0,1,2,3,4,5,6,7,8,9) their numbers

Maghreb and al –Andalus remove the point inside the circle to indicate zero and their numbers came (0,1,2,3,4,5,6,7,8,9). Slightly later in the 12^{th} century al-Samawal was writing:-

If we subtract a positive number from zero the same negative number remains. ... if we subtract a negative number from zero the same positive number remains.

The Indian ideas spread east to China. In 1247 the Chinese mathematician Ch'in Chiu-Shao wrote *Mathematical treatise in nine sections* which uses the symbol O for zero. A little later, in 1303, Zhu Shijie wrote *Jade mirror of the four elements* which again uses the symbol O for zero.

In 1200 m Fibonacci taken from the Arabs their numbers as taking them to zero and he said, "We can by nine Arabic numerals and that mark (0) zero-called Arab to write any number whatever."

Arithmetic Operations (Calculation)

It was the emergence of the fundamental operations (addition, subtraction, multiplication and division) naturally, and in many places. The Babylonians and Egyptians were interested in the development of tables to teach mathematics in the extraction of the results of calculations which provided them with a lot of effort and time, and enabled them to perform multiplication and division in large numbers.

Greek scientists have quote mathematical facts and ways of Babylonian treasures such as spread sheets and sixtieth division , and they quote Egyptian way of multiplication and division. Arabs came in new ways and style in the calculations and al-Khawarizmi's Arithmetic[5], was the first systematic treatment of arithmetical operations. al-Khwarizmi discussed the place-value system and rules for performing the four arithmetical operations (addition, subtraction, multiplication and division).

1. Addition

Addition of two numbers is the total amount of those quantities combined

The method is to write the two numbers under each other so that individual one place to be in a row my head, and so dozens, and starts a combination of right-to-left and increases every rank on the beneath.If the earned less than ten paint underneath.If it is more than ten (or ten)put zero in these two pictures of ten and one to increase it to what in second place, or you draw it beside its predecessor and every rank not beside number, trnsilate (move) it to line combination, and this is its image as al-Amili[3], put it.

$$\begin{array}{|ll} 20372 & \\ 07656 & + \\ \hline \end{array}$$

28028

Ghayasuddin al-Kashi situation is as follows:

64534 5394853	Numbers we want to add
5459387	The Sum

To find out the validity of the calculation Arabs used what they call it the balance of the number base , known in the West, the (Golden Rule), referred to it by al-Amili[22,23,24] and al-Kashi[4][12].

(we collect constituent number of digits and rule out all the right Nines what remains of it after that is the balance of the number) .

Therefore the balance of the number (64 534) is (4) and the balance of the number (5,394,853) is (1) Thus, the total budget is (5) but the balance of (5,459,387) equal to (5), which holds the balance of the same combination which means that the combination process is correct .

2. **Difference**:

The Difference is to decrease a number from a number which is not less than from it. The method is as we mentioned in a particular combination, and we start from the right side and (subtract) everything in rank his image of undiminished what Ahave of undiminished him and put the rest under it if remained something. If there is nothing left, we will place there zero. If not decrease what in rank than ahave take one of its tens in any of the subsequent left shall be for that place ten subtract them and increase the rest on the adjacent of undiminished him although not in tens number take from its hundred one a ten for tens(ashrat) and put nine in its tens(ashrath) in writing or in mind to keep one and we do what we said on that way.

Example : To (subtract) , 5479 from (846 592),we arrange them as follows

846592	المنقوص منه
-5479	المنقوص
841113	الباقي

3 al-Amili : Bahaa Eddin Mohammed Ibn Hussein Ibn Abdul Samad al-Harthy ,(1547-1622), Mathematician , astronomer and writer . The most famous writings : Khlast Alhesab(account summary) , Anatomy of the universe , A message in Alostrlabah , message in algebra and almuqabla.

4 al -Kashi: Jamshid bin Mahmoud ibn Masood, nicknamed ghayasuddin, was born in Kashan ment 654 AH, and lived in Samarkand, where excelled in arithmetic, astronomy, physics. He is the first to introduce decimal sign in Arithmetic operations.He wrote many letters and books in mathematics, astronomy including: “Alrisala Almuhitiya”in which he calculate the ratio between the circumference of a circle and its diameter $\pi = 3.14259265358979325$ and “mifthah Alhisab” containing some of his discoveries in Arithmetics, and a book”Nuzhat Alhadaek” in which he show that the orbits of the moon and Mercury are elliptical rather than circular.

3. Multiplication

The multiplication of two integers is equivalent to adding as many copies of one of them, as the value of the other one:

$$a \times b = \underbrace{b + \cdots + b}_{a}$$

Arab introduced many rules for Multiplication , including the following:

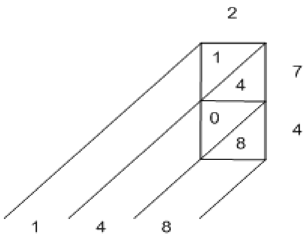
$$ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

Example:

$$36 \times 24 = \left(\frac{36+24}{2}\right)^2 - \left(\frac{36-24}{2}\right)^2 = (30)^2 - (6)^2 = 900 - 36 = 864$$

ii) Grid method (The 'Sieve' or 'Lattice' Method for Long Multiplication **)**

The Arabs popularised a method for long multiplication known as the 'Sieve' or 'Lattice' method, which is of historical significance. It has been proved that, even today, the 'Lattice' provides a useful diversion in learning long multiplication.Here the 'Lattice' method is illustrated in a very simple case by using it to multiply 2 by 74:



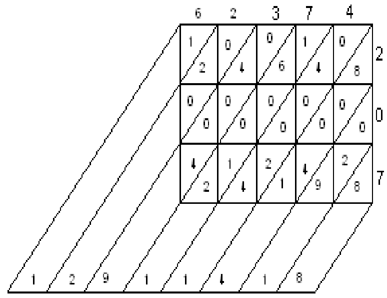
This 'Lattice' or 'Sieve' multiplication shows how al-Karkhi[5][3] (successor of al-Khwarizmi) contributed to arithmetic . This method. explained in al- Amili [24 , p. 56] This multiplication procedure can be explained to a student as follows:

1- the multiplicand is arranged as the horizontal heading in the tableau;

2- the multiplier is arranged vertically in the extreme right column;

3- the products of respective single digits are the decomposed in terms of tens and units within subdivided rectangles arranged horizontally;

4- the product of the multiplication is then performed by the addition in the diagonal channels of operation, starting with diagonal channel on the right representing ones, progressively through channels further left representing tens hundreds, thousands etc. There is space in the diagonal channels to allow for "carrying over" as necessary, and the results are written at the bottom. Here is a longer multiplication to illustrate the technique:

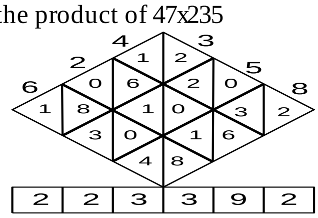


This technique is actually easier than the technique generally taught currently in schools. It involves much less movement of the eyes and it is obvious to see where you are in the calculation at any time and there are less carrying operations. It can also be verified far more easily.

iii) Morbh network method

This method introduced by al-Kashi [Miftah Alhisab [12],pp. 53-54] and to make the process of multiplication in this way divide the Rectangle to squares and divide each square into two cell by longitudinal lines passes through its opposite angles (i.e: the upper and lower), then put one of number you want to multiplty outside of the right upper leg , and the other on the left upper back in the right-to- left , and multiplty each of the digits of one number in each and every one of the multiplier ,and keep its result in the square, which occurred in their meeting , Individual in the right triangle and dozens in the triangle left to be done , and then we plan the network lines stretched down and then collect all that is between the two lines so that the number in between the two lines down the network is a sum of the numbers confined between them across the network , and so on with the rest of the lines on its left to be done .

Example: Find the product of 47x235



1V) Divisibility :

Division is a collection relative to the number one as a divisor to any divisor .That is

$$a = bc \Leftrightarrow \frac{c}{1} = \frac{a}{b}$$

Example : Divide (20325) by (135) ?

Al- Karkhi asks for the largest number of hits in the hundreds (135) to produce the nearest number to the divisor .He find it hundred and raises product of the divisor

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5 Al- Karkhi (Alkrgi): Abu Bakr Muhammad ibn al-Hasan was born in Karkh from the outskirts of Baghdad, lived and put the most important production in Baghdad at the end of the tenth and the begining of Eleventh century.He has spent apart of his life in the mountainous areas, where he worked in engineering, this work appeare in his book "About drilling of wells,.He "died in Baghdad in (421 AH = 1020 AD), considered by some as one of the greatest Mathematician who have had a real impact in the progress of Mathematical Sciences, he has several books, including: a book in the Indian account, which speaks for the extraction approximate polynomial roots, and a book in the induction , and Alkafi book which contains rules of the product signs and unknowns, sums of the Algebraic terms and the laws of the last term and the total sum in numerical sequence , and the square root of Algebraic amounts. while in his book Alfkhry in algebra he study many problems, he is the first Arab proved that: :

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2, (a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

and $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$ .In his book ‘Ellal Algeber wa almukabla” ،and, in het set out rules to solve the equations of the second degree as well as multiplication and division and addition and subtraction rules formulas for two rational numbers and proved those rules algebraically .

And al- Karkhi asks for the largest number of dozens if beaten in the (135) gives the closest result to the (6825) ,he found it (50) ,then he multiply 50 by 135which is equal 6750 and subtracting the output of the rest which is6825-6750=75.He deduce that the quotient is(100+50) and the remainder (75) That is (150) and the remainder is (75) .

Note that a manuscript founded in 1971 value , in the Indian Library in London describes the oldest method of lengthy divisible known in the Islamic state and requires expert skill in mathematics discribed in the following example :

Example

If we want to divide the number (22 806) on (543) we follow the following:

(A) divide the page paper to several of the vertical columns equal to the number of digits in the current divided by the number who writes at the top of the page, while writes number divisor so that the bottom of the page shows the last digit of each of the two numbers at the left side of the page:

We start by dividing the column to the far left . Divide (2) by (5) you got (zero) , so the first number from the left of the quotient is (zero) , and writes under the first divisor numbers as it is in the form :

2	2	8	0	6
5	4	3		

Then write divisor (543) Top Previous position directly with unseat him betrayed and one to the right , as in the following figure :

2	2	8	0	6
	5	4	3	
5	4	3		
		0		

After that , we find that (5) is divided (22) four times , and be the first number (left) to outside the division , and who writes down the first divisor numbers it (in and put a joke) next to the number to the outside of dividing the previous step, and then multiply the divisor in (4) and ask the product in the current number to be divided by the rest (2086) as it shows the following table:

2 2	2 0	8	0	6
	2 1	8 6	0	6
	1	2 1	0 2	6
	1	0	8	6
	5	4	3	
5	4	3		
		0	4	

Repeat this process for the apportionment (2086) to (543) to get the final out of the division (42) , as detailed in the following table

((المقسوم)) Divisor output of multiplying(5×4)	2 2	2 0	8	0	6
output of multiplying(4×4)		2 1	8 6	0	6
output of multiplying(4×3)		2 1	8 2	0	6
output of multiplying(5×2)		1 1	0 0	8	6
output of multiplying(4×2)			0	8 8	6
output of multiplying(3×2)				0	6 6
الباقيRemainder					0
			5	4	3
		5	4	3	
((المقسوم عليه))	5	4	3		
خارج القسمة The Result			0	4	2

Finally we would like to mension that a manuscript written by Bahauddin Alamili describe global method of division similar to the current method.

Note that to ensure the validity of the division process Muslims used balance counting also " The balance of division " See (Meftah alhesab) for al- Kashi,[12].

Fractions (Rational Numbers)

Fraction is a part or two parts or portions of one. The compound fraction or number fractional is what is of integer and fraction , and the oldest knowledge of ordinary fraction attributed to the Babylonians and Egyptians . Babylonians has created the fractions based on the sixtieth system : half = 30 , one-third = 20 , quarter = 15 .

The Egyptians numbering ordinary fraction

$$\frac{1}{4}\ ‘\ \frac{1}{98}\ ‘\ \frac{1}{21}\ ‘\ \frac{1}{6}\ ‘$$

has Djalo oval sign above, the number to indicate the fraction ,like 111 to one-third in uhmer days were writing the price so and write one to twenty $\overset{.\ .}{\underset{\wedge}{—}}$. Al- Khawarizmi,[5] describe fractions on the basis of sexagesimal then describes their multiplication and division operations in ways similar to the ways of the Babylonians which are known to the Greeks , then moves to the square root extraction .

Al- Buzjana[6] (940-998) dealt with fractures theory in his book (The book needs of arithmetic) distinguishing between three types of fractions: regular ($\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},...,\frac{1}{10}$) , Compound fractions which are of the from a to b, where $a < b \le 10$ and Alwhdiah Fractions which are a multiplying of regular fractions, .Note that Abu al-Wafa called major or regular fractures and fractures occurring from the sum or multiplying major fractures " speaking fractures " the other is called the endocrine fractures.

Note that the Indian Liflati (1150)write normal(regular) fraction in the form, $\frac{a}{b}$ while the fractional number containinb integer and rational written in the following form: $\begin{matrix} a \\ b \\ c \end{matrix}$,hence $\begin{matrix} 4 \\ 2 \\ 3 \end{matrix}$ meane four and two thirds,. .

The horizontal fraction bar was introduced by Arabs.Several sources attribute the horizontal fraction bar to al-Hassar[7] arround 1200.Other sources attributed to Ibn al-banna al-Marrakichi[8] ,[9,10], they write the rational number as follows: $\frac{a}{b}$, and expressed as fractional number $\begin{matrix} a \\ b \\ c \end{matrix}$ as fellows $a\frac{b}{c}$.

and we find in Arithmetic of Ibn al-Banna al Marrakechi, , and Abu al-hssan al-Kulsadi[9] [7],patterns of fractions like Almuntaseb fraction like five ninths and four -sevenths of the nine and one-third of seven of the nine and three-quarters of one-third of seven of the nine($\frac{475}{756}$) and the different fraction likeandquarters of one-third of any seven of the nine and two thirds andfour-fifths of one-third fifths of one- -third $\frac{77}{45}$ and Alamubath or fraction of fraction like one-thirds of four-fifths of six sevenths ($\frac{24}{105}$)

Decimal fractions : Muslims contributed and invented the present arithmetical decimal system and the fundamental operations connected with it: addition, subtraction, multiplication, division, exponentiation, and extracting the root .

Thus neither the Babylonians nor the Chinese had a device or symbol for separating integers from fractions. That symbol is the decimal fraction, which is the invention of Arabs and is credited to them.Decimal fractions, however, are part of just one of the arithmetic systems used within Islamic science where there were at least three different kinds of arithmetic: the base-60 system of astronomers, the decimal arithmetic and the finger arithmetic.

It has been mentioned that decimal fractions made their first appearance in Arab mathematics in the Book of Chapters on Indian Arithmetic, written in Damascus in the year 952 or 953 by Abul Hassan Al-Uqlidisi. Al-Uqlidisi considered the problem of successively halving 19 five times, and gave the answer as $\overset{1}{0}$ 59375 . The vertical mark on 0 indicates that the decimal fraction part of the number starts with the digit to the right. This notation is perfectly general, so that what in England they write as 0.059375 and in Germany as 0,059375 . There is evidence that al-Uqlidisi was aware of the method of multiplying decimal fractions by whole numbers. In his Treatise of 1172, al-Sumaal[10][8,24], established a usage of

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6 al-Buzgani ; Abu al-Wafa Mohammed bin Yahya bin Ismail Abbas , a disciple of al-Battani was born in Buzgan near Ansabour year 328 AH and moved to Baghdad and lived there working in the authoring, monitoring and teaching, until he died the year 388 AH, is one of the imams of Sciences astronomical, sports and fired him Encyclopedia, explaining the works of Euclid and Diovents and al-Khawarizmi , enter geometry to algebra paved for the emergence of analytic geometry and created new solutions to some of the equations of the fourth degree and as a result of his interest in he find new solutions to some equations of the fourth degree and as a result of his interest in engineering and applied engineering drawing, a book Maihtaj him craftsmen from the engineering also put a message in the work of the ruler and Alfrckar and Alcua (based Triangle corner) translated into English .He Scored the lead in trigonometry , enter cutter and cutter fully, and mathematical tables for its own account and devised a new way to calculate sinus tables, Use the trigonometric ratio in the shade mathematical problems and solutions concerned with trigonometric Palmttabqat which plays an important role in trigonometry and invented many of them He has written several books , including extract strings, comprehensive plummet, entrance Alarosmaatifi, know the circuit in astronomy, Maihtaj him the workers and the book industry in the account , the extraction side of a square Pmamal who replaced him equations, discovered kinds of bugs in the movement of the moon . Given the importance of the work in astronomy al-Buzjana fired astronomers of the twentieth century his name on a volcano on the surface of the moon.

7 al-Hassar or Abu Bakr Muhammad ibn Abdallah ibn Ayyash al-Hassar was a Muslim mathematician from Morocco, living in the 12th century.

8 Ibn al-Banna Marrakechi (1256-1321) Ahmed bin Mohammed bin Othman al-Azdi infection, Abu Abbas , was born in Marrakech , his father was builder , excelled in mathematics, astronomy and has them : to summarize the account works , and the message in the weights , and part of the space , and the book in the stars, and an article in the science of the astrolabe , and the law to know the times by calculation(Arithmetic).

9 al- Kulsadi (1412-1496m): Abu Hassan Ali Ibn Mohammed Ibn Ali al- Qurashi , the world of Andalusian account and inheritance , was born in unrolled the year 125 AH and the Tvgah then moved to Granada Fastotnha and driven them before falling to Tunisia where he died in Bagh year 891 AH , was inclined to simplify matters and the use of function icons on the marks and unknowns , his many books, including: Account Act , is necessary in the science of inheritance , colleges statutes , explain to summarize the work of the Arithmetics of Ibn al-Banna al-Marrakech.

10 Al-sumaal Ibn Abbas al-Maghrebi , best known in Mathematical sciences and medicine, originally from the

decimal fractions. In his work on decimal fractions, al-Sumawal established a rule for approaching the irrational, square and cubic roots; a rule that is called the "rule of zero". The general formulation of this rule is to be found in al-Sumawal's Treatise is

$$a^{\frac{1}{n}} = \frac{\left(a \times 10^{nr}\right)^{\frac{1}{n}}}{10^{r}}, \quad r = 1, 2, ...$$

Obviously, the approximation obtained according to this rule includes a decimal fraction. This sporadic usage of decimal fractions according to al-Samawal's theoretical exposition came at the end of his working following an exposition of One of al-Sumawal's aims was to unify and generalise.

He aimed to have "indefinite correction of fractions". He meant to give the fractions a form so that they can be calculated like integers, and from then on, it is possible to correct the approximation of different operations indefinitely.

We would like to point out that Alsumaal assumption $10^0 = 1$ $\left(10, 10^2, \cdots\right)$, $10^0 = 1\left(\cdots, \frac{1}{10^2}, \frac{1}{10}\right)$, and writting means that there is limited or unlimited representation of any real number

$$r = \sum_{k=-m}^{n} a_k (10)^k, m, n \in Z^+$$

al-Kashi, astronomer royal for the Timurid Ulugh Beg at Samarkand, used decimal fractions with facility, and several of his remarkable feats of calculations were carried out in both decimal and sexagesimal systems. In his Treatise on the Circumference of a cicle (al-Risala almuhitiya , translated and published by German historian), al-Kashi rsorts to decimal fraction for the approximation of π (The calculation of the perimeter of inscribed and circumscribed polygons of 3×2^{28} sides). He gives an approximation of 2π in sexagesimal notation first the value

50,14,46,51,34,1,28,59,16,=62π

Then he converted to the decimal system and found that :

$$= 6.28318530717958650\ 2\pi$$

$$\pi = 3.14159265358979325$$

Maghreb and housing Baghdad, then deported to the country Persians, and remained there until he died in Maragha Azerbaijan about the year 570 AH, he wrote many many books in mathematics The best known "Albaher in algebra" explaining the rules of signs cited by al- Karkhi and prove it in algebraic ways , he prove that $a^m . a^n = a^{m+n}$ for all $m, n \in Z$.he generalize divisins rules to polynomails witth rational coefficients.He introduced decimal fractions, finding square roots of the algebraic amounts. He found the nth root of any number in a general way before Rofaini and Horner, he gave new proofs

to $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $and \sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$.

Note that with the number of digits in the two systems sixtieth and decimal are equal which indicates the existence of similarity (1-1 correspondance) between them.

al-Kashi [12], provided a comprehensive and systematic treatment of Arithmetic operations with decimal fractions.

Finally, we would like to point out the importance of the recent discovery of hangar and Huggel in 1963 for the Byzantine manuscript brought to Vienna in 1562 AD confirms knowledge of the West in a way or another, the results of the Arabs and Muslims in multiplication, division, fractions.

Note the Dutch mathematician Simon Stevin (1548-1620) who came more than 185 years after al- Kashi, learnt of the Arab methods of treating decimal fractions. In place of the short vertical line over the last digit of the integer part of the number that was the original notation of al-Uqlidisi, Stevin use Zero (Sifr in Arabic).

References

1. Falih A.M. Al-Dosray: Introduction to Mathematics Islamic civilization and its applications - Safa Press, 2003.

2. Ali Abdullah Al-Daffa" Nawabigh ulama al-Arab wa-al-Muslmin fi al-riyadiyat."John Wiley and Sons, Inc.(1978)

3. Abu Bakir Muhammad al-Karkhi : Al-Badi fi alhisab : authenticated by Adel Enbuba , Beirut, the Lebanese University , 1964 .

4. Abu Bakir Muhammad al-Karkhi " Alkafi fi Alhisab" Explanation and authenticated bySami Shalhoub ,Haleb University, 1986 .

5. Muhammad ibn Musa al-Khwarizmi'Kitab Algaber wa almuqabala " authenticated (investigated) by Ali Mustafa Musarifa and Mohamed Moussa Ahmed,Egyptian University.1939.

6. Mohammed Salem Al-hemiada "Numerals journey through history" publications Iraqi information ministry, 1975.

7. Abu Hassan al- Kulsadi "Sarh(explain) Talkhis(summarize)amal al-hisab " translated into French by Fares bin Taleb , DMI West, 1999 .

8. al-Sumawal al-Maghribi " al-Bahir fi algeber" investigation and analysis Salah ahmad and Rushdi Rashed Damishq university 1980.

9. Ibn al-Banna al- Marrakechi " Rafue (lift) alhigab an wiguh (faces) amal alhisab " Forward , study and investigation,by Dr. Mohamed iblakh, publications Faculty of Arts and Sciences, Fass.

10. Ibn Al-Banna al-Marrakechi "Telkhis alhisab(summarize Arithmetic)" translating and commented upon by Dr. Mohamed Souissi, 1969 Tunisian university publications.

11. Ibn Al-Banna al-Marrakechi "articles in Arithmetic" a historic and comments of Dr. Ahmed Salim Saidan and Introduction, Dar Al-Furqan for publication 1984.

12. Ghayasuddin al-Kashi " Miftah Alhisab " Investigation(authenticated)Ahmed Saeed Eldemerdash and Mohammed Hamdi Hefny al-Sheikh, Dar Al Arab writers for printing and publishing, 1967 .

13. Abu al-Hassan ibn Ibrahim al-Uqlidisi " *Kitab al-fusul fi al-hisab al-Hindi* (chapters in the Indian Arithmetics)" investigation(authenticated)Ahmed Salim saidan Jordanian Committee for Arabization, publishing and translation, Amman.

14. H. A.15 al-Tayar and Yahya Eid Saaid "A Brief History of Mathematics: Mosul University, 1977.

15. J.L. Berggren: History of Mathematics in The Islamic World: The Present State of The Art. *Middle East Studies Association Bulletin* 19 (1985), pp. 9-33.

16. N. Bourbaki (1998). *Elements of the History of Mathematics*. Berlin, Heidelberg, and New York: Springer-Verlag.
17. L. Dikson, "History of the Theory of Numbers" Vols I , II , III, Chelsea
18. O. Farukh "History of Science at the Arabs," House of science to millions, Beirut, 1914.
19. R. Kaplan, (2000) *The Nothing That Is: A Natural History of Zero*, Oxford: Oxford University Press.
20. M. Charbel "Mathematics in the Islamic civilization" Groy Bri Tripoli, Lebanon.1988 m.84.
21. J. Okasha, Hamada Abu Awad, Mustafa Asaad and Samir Abu Ali "History of Mathematics" Future House of Publishing and Distribution, Amman 1990.
22. R. Rashed : Arab mathematics history between algebra and arithmetic - the Center for Arab Unity Studies , Beirut , 1989 .
23. R. Rashed : harmonic analysis , numerical analysis , analysis Aldioventa and number theory - Encyclopedia of History of the Arab Sciences (491-538) , the Center for Arab Unity Studies Beirut 1997.
24. J. Shawqi "al-amal al-riyadia for Bahauddin al-Amili" Sunrise House 0.1981.
25. C . Seife, (2000) *Zero: The Biography of a Dangerous Idea*, Penguin USA (Paper). ISBN 0-14-029647-6
26. D. E. Smith, "History of Mathematic", Gins and company.
27. A. Solomon and S. Hallaq" Studies in the History of Science at Arab"university knowledge House 1998.
28. K. H. Toukan "Turath al-Arab al-Ilmi in mathematics and astronomy," Dar Alsuroq Beirut.