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Isomorphism and Anti isomorphism in Q-Fuzzy Translation of Interval Valued Q- Fuzzy Subhemirings of a Hemiring

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of a Interval valued fuzzy subhemiring of a hemiring.

Introduction

There are many concepts of universal algebras generalizing an associative ring (R; +; .). Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras (R; + ;.) share the same properties as a ring except that (R; +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R; +;.) is said to be a semi ring (R; +) and (R; .) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a,b and c in R. A semiring R is said to be additively commutative if a+b=b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1.a=a=a.1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. A semiring R is said to be a hemi ring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh(19), Gratten-Guiness(6), Jahn.K.U(7), in the seventies, in the same year.An interval valued fuzzy set(IVF) is defined by an interval-valued membership function Jun.Y.B and Kin.K.H,(8) defined an interval valued fuzzy R-subgroups of nearrings. Solairaju .A and R.Nagarajan(18), defined the charactarization of interval valued Anti fuzzy Left h-ideals over Hemirings. Azriel Rosenfield (5) defined a fuzzy Groups. Solairaju .A and R.Nagarajan,(22) have given new structure and construction of Q-fuzzy groups,In this paper, we introduce some properties and theorems in Interval valued Q-fuzzy subhemirings of a hemiring under Isomorphism and Antiisomorphism and established some results.

1. Preliminaries

1.1Definition

Let X be any nonempty set. A mapping $[M]: X \to D[0,1]$ is called an interval valued fuzy subset(briefly, IVFS) of X, where D[0,1] denoted the family of all closed subintervals of [0,1] and $[M](x)=[M^-(x), M^+(x)]$,for all x in X, where $M^-$and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus [M](x) is an interval (a closed subset of [0,1]) and not number from the intervak [0,1] as in the case of fuzzy subset. Note that [0] = [0,0] and [1]=[1,1].

1.2 Remark

Let D^X be the set of all interval valued fuzzy subsets of X, where D means D[0,1].

1.3Definition

Let

$[M] = \{\langle x, [M^-(x), M^+(x)]\rangle / x \in X\}, [N] = \{\langle x, [N^-(x), N^+(x)]\rangle / x \in X\}$

be any two interval valued fuzzy subsets of x. We define the following relations and operations:

i)$[M] \subseteq [N]$ if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$,for all x in X.

ii)$[M] = [N]$ if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$,for all x in X.

iii)$[M] \cap [N]$=

$\{\langle x, [\min\{M^-(x), N^-(x)\}, \min\{M^+(x), N^+(x)\}]\rangle / x \in X\}$

iv)$[M] \cup [N]$=

$\{\langle x, [\max\{M^-(x), N^-(x)\}, \max\{M^+(x), N^+(x)\}]\rangle / x \in X\}$

$[M]^C = [1] - [M] = \{\langle x, [1 - M^+(x), 1 - M^-(x)\rangle / x \in X\}$

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1.3 Definition

The union of two (S,Q)-fuzzy sets A and B of a set X is defined by $([A] \cup [B])(x,q) = \max\{[A](x,q), [B](x,q)\}$ for all x in X and q in Q.

1.4 Definition

The intersection of two (S,Q)-fuzzy sets A and B of a set X is defined by $([A] \cap [B])(x,q) = \min\{[A](x,q), [B](x,q)\}$ for all x in X and q in Q.

1.5 Definition

Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subset [M] of R is said to be an interval valued Q -fuzzy subhemiring (IVQFSHR) of R if the following conditions are satisified:

(i) $[M](x+y,q) \geq \min([M](x,q), [M](y,q))$

(ii) $[M](xy,q) \geq \min([M](x,q), [M](y,q))$, for all x and y in R, and q in Q.

1.6 Definition

Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subhemiring [A] of R is said to be an interval valued Q-fuzzy normal subhemiring (IVQFNSHR) of R if [A](xy,q)=[A](yx,q) , for all x and y in R and q in Q.

1.7 Definition

Let (R, +, .) and (R', +, .) be any two hemirings. Then the function f:R—>R' is called a hemiring homomorphism if it satisfies the following axioms:

i) $f(x+y) = f(x) + f(y)$,

ii) $f(xy) = f(x)f(y)$, for all x and y in R.

1.8 Definition

Let (R, +, .) and (R', +, .) be any two hemirings. Then the function f:R—>R' is called a hemiring anti-homomorphism if it satisfies the following axioms:

i) $f(x+y) = f(y) + f(x)$,

ii) $f(xy) = f(y)f(x)$, for all x and y in R.

1.9 Definition

Let (R, +, .) and (R', +, .) be any two hemirings. Then the function f:R—>R' be a hemiring homomorphism. If f is one-to-one and onto ,then f is called a hemiring isomorphism.

1.10 Definition

Let (R, +, .) and (R', +, .) be any two hemirings. Then the function f:R—>R' be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a hemiring anti-isomorphism.

1.11 Definition

Let A be a Q-fuzzy subset of X and $\alpha \in [0,1 - \operatorname{Sup}\{[A](x,q): x \in X, 0 < [A](x,q) < 1\}]$.

Then $T = T_{\alpha}^{A}$ is called a interval valued Q-fuzzy translation of [A] if $T(x,q) = [A]((x,q) + \alpha)$, for all x in X.

2. Isomorphism And Antiisomorphism In Interval Valued Q- Fuzzy Translation Interval Valued Q- Fuzzy Subhemirings Of A Hemiring

2.1 Theorem

Let (R, +,.) and (R', +, .) be any two hemirings. The interval valued Q-fuzzy normal subhemiring of f(R)= R' under the anti-homomorphic preimage is an interval valued Q-fuzzy normal subhemiring of R.

Proof

Let (R, +,.) and (R', +, .) be any two hemirings.Let f:R—>R' be an anti- homomorphism. Then $f(x+y) = f(y) + f(x)$, and $f(xy) = f(y)f(x)$ for all x and y in R. Let [V] be a interval valued Q-fuzzy normal subhemiring of f(R)= R' and A be an anti-homomorphic pre-image of [V] under f. we have to prove that [A] is an interval valued Q-fuzzy normal subhemiring of hemiring R. Let x and y in R and q in Q. Then clearly [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R. Since [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring R'.

Now, $[A](xy,q) = [V]((f(xy),q))$,

since $[A](x,q) = [V]((f(x),q)) =$

$([V]\ (f(y)f(x),q))$

as f is an anti-homomorphism $= [V](f(x)f(y),q) = [V]((f(yx),q))$ as f is an anti-homomorphism=$[A](yx,q)$, since $[A](xy,q) = [A](yx,q)$, which implies that $[A](xy,q) = [A](yx,q)$ for all x and y in R and q in Q.Hence [A] is an interval valued Q-fuzzy normal subhemiring of hemiring R.

In the following Theorem $\circ$ is the composition operation of functions:

2.2 Theorem

Let [A] be an interval valued Q-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemi ring R onto H. If [A] be an interval valued Q-fuzzy normal subhemiring of hemiring H,then [A]$\circ$ f is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

Proof

Let x and y in R and q in Q and [A] be an interval valued Q-fuzzy normal subhemiring of the hemiring H. Then we have Clearly [A]$\circ$ f is an interval valued Q-fuzzy subhemiring of the hemiring R.

Now,

$([A] \circ f)((xy,q)) = [A]((f(xy),q)) =$

$[A]((f(x)f(y),q)$

as f is an isomorphism $=[A](f(y)f(x),q) = [A]((f(yx),q))$ as f is an isomorphism = $([A] \circ f)((yx,q))$ for all x and y in R and q in Q. Therefore [A]$\circ$f is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

2.3 Theorem

Let [A] be a interval valued Q-fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemi ring R onto H. If [A] be a interval valued Q-fuzzy normal subhemiring of hemiring H, then [A]$\circ$ f is a interval valued Q-fuzzy normal subhemiring of the hemiring R.

Proof

Let x and y in R and q in Q and [A] be a interval valued Q-fuzzy normal subhemiring of hemiring H. Then we have clearly [A]$\circ$ f is an interval valued Q-fuzzy subhemiring of the hemiring R.

Now, $([A] \circ f)((xy,q)) = [A]((f(xy),q))$

$= ([A](f(y)f(x),q)$ as f is an anti- isomorphism$=$ $([A](f(x)f(y),q) = [A]((f(yx),q))$ as f is an anti-isomorphism $= ([A]\circ f)((yx,q))$, which implies that $([A]\circ f)((xy,q)) = ([A]\circ f)((yx,q))$, for all x and y in R and q in Q. Therefore [A] of is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

2.4 Theorem

If [M] and [N]are two interval valued Q-fuzzy translations of interval valued Q-fuzzy normal subhemiring [A] of a hemiring (R, +, .),then their intersection $[M]\cap[N]$ is an interval valued Q-fuzzy translation of A.

Proof

It is trivial.

2.5Theorem

The intersection of family of interval valued Q-fuzzy translations of interval valued Q-fuzzy normal subhemiring [A] of a hemiring (R, +,.) is an interval valued Q-fuzzy translation of A.

Proof

It is trivial.

2.6Theorem

If [M] and [N] are two interval valued Q-fuzzy translations of interval valued Q-fuzzy normal subhemiring [A] of a hemiring (R ,+, .),then their union $[M]\cup[N]$ is an interval valued Q-fuzzy translation of A.

Proof

It is trivial.

2.7Theorem

The union of family of interval valued Q-fuzzy translations of interval valued Q-fuzzy normal subhemiring [A] of a hemiring (R ,+, .) is an interval valued Q-fuzzy translation of A.

Proof

It is trivial.

2.8 Theorem

Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If f:R—>R' is a homomorphism, Then interval valued Q-fuzzy translation of a interval valued Q-fuzzy normal subhemiring [A] of R under the homomorphic image is an interval valued Q-fuzzy normal subhemiring of f(R)=R'.

Proof

Let (R, +, .) and (R', +,.) be any two hemirings and Q be a non-empty set and f:R—>R' be homomorphism. That is f(x +y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let $T = T_\alpha{}^A$ be the interval valued Q-fuzzy translation of an interval valued Q-fuzzy normal subhemiring of [A] of R and [V] be the homomorphic image of [T] under f. We have to prove that [V] is an interval valued Q-fuzzy normal subhemiring of R'. Now, f(x) and f(y) in R' and q in Q. Then clearly, [V] is an interval valued Q-fuzzy subhemiring of the hemiring R' .

Now, $[V](f(x)f(y),q) = [V](f(xy),q) \geq$ $[T](xy,q) = [A]((xy,q)+\alpha) = [A]((yx,q)+\alpha) =$ $[T](yx,q) \leq [V](f(yx),q) = [V](f(y)f(x),q)$

,which implies that $[V](f(x)f(y),q) = [V](f(y)f(x),q)$ for all f(x) and f(y) in R' and q in Q. Therefore [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring R'.

2.9 Theorem

Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If f:R—>R' is a homomorphism, Then interval valued Q-fuzzy translation of a interval valued Q-fuzzy normal subhemiring [V] of f(R)= R' under the homomorphic pre-image is an interval valued Q-fuzzy normal subhemiring of R.

Proof

Let (R, +, .) and (R', +,.) be any two hemirings and Q be a non-empty set and f:R—>R' be homomorphism. That is f(x +y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let $T = T_\alpha{}^V$ be the interval valued Q-fuzzy translation of an interval valued Q-fuzzy normal subhemiring of [V] of R' and [A] be the homomorphic pre-image of [T] under f. We have to prove that [A] is an interval valued Q-fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then clearly [A] is an interval valued Q-fuzzy subhemiring of the hemiring R.

Now, $[A](xy,q) = [T](f(xy),q) = [V]((f(xy),q)+\alpha) = [V]((f(x)f(y),q)+\alpha) = [V]((f(y)f(x),q)+\alpha) = [V]((f(yx),q)+\alpha) = [T](f(yx),q) = [A](yx,q)$

, which implies that $[A](xy,q) = [A](yx,q)$, for all x and y in R and q in Q. Therefore, [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

2.10 Theorem

Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If f:R—>R' is a anti-homomorphism,Then interval valued Q-fuzzy translation of a interval valued Q-fuzzy normal subhemiring [A] of R under the anti-homomorphic image is interval valued Q-fuzzy normal subhemiring of f(R)=R'.

Proof

Let (R, +, .) and (R', +,.) be any two hemirings and Q be a non-empty set and f:R—>R' be an anti-homomorphism. That is f(x +y) = f(y)+f(x) and f(xy)=f(y)f(x), for all x and y in R. Let $T = T_\alpha{}^A$ be the interval valued Q-fuzzy translation of a interval valued Q-fuzzy normal subhemiring of [A] of R and [V] be the anti- homomorphic image of $T_\alpha{}^A$ under f. We have to prove that [V] is an interval valued Q-fuzzy normal subhemiring of f(R)= R'.Now for ,f(x) and f(y) in R and q in Q. Then clearly, [V] is an interval valued Q-fuzzy subhemiring of the hemiring R'.

Now, $[V]((f(x)f(y),q)) = [V](f(yx),q) \geq$ $[T](yx,q) = [A]((yx,q)+\alpha) = [A]((xy,q)+\alpha) = [T](xy,q) \leq [V](f(xy),q) =$ $[V]((f(y)f(x),q))$

,which implies that

$[V]((f(x)f(y),q)) =$ $[V]((f(y)f(x),q))$, for all f(x) and f(y) in R' and q in Q.

Therefore [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring R'.

2.11 Theorem

Let (R, +,.) and (R', +,.) be any two hemirings and Q be a non-empty set. If f:R—>R' is a anti-homomorphism, then interval valued Q-fuzzy translation of a interval valued Q-fuzzy normal subhemiring [V] of f(R)= R' under the anti-homomorphic pre-image is an interval valued Q-fuzzy normal subhemiring of R.

Proof

Let (R, +, .) and (R', +,.) be any two hemirings and Q be a non-empty set and f:R—>R' be an anti-homomorphism. That is f(x +y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let $T = T_\alpha{}^V$ be the interval valued Q-fuzzy translation of a interval valued Q-fuzzy normal subhemiring of [V] of R' and [A] be the anti- homomorphic pre-image of [T] under f. We have to prove that [A] is an interval valued Q-fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then Clearly, [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

Now

$$[A](xy,q) = [T](f(xy),q) = [V]\big((f(xy),q) + \alpha\big) = [V]\big((f(y)f(x),q) + \alpha\big) = [V]\big((f(x)f(y),q) + \alpha\big) = [V](f(yx),q) + \alpha) = [T](f(yx),q) = [A](yx,q)$$

, which implies that $[A](xy,q) = [A](yx,q)$, for all x and y in R and q in Q. Therefore, [A] is an interval valued Q-fuzzy normal subhemiring of R.

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