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# $\tau^{+}$*-Generalized closed sets in simple extended topological spaces

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ABSTRACT

In this paper, we introduce a new class of sets called $\tau^{+}$*-closed sets and $\tau^{+}$*-generalized closed sets in topological spaces.



**Introduction**

The notion of closed set plays a fundamental role in the study of topological spaces. Levine[8], in 1970 introduced the concept of generalized closed sets in a topological space by comparing the closure of a subset with its open supersets. This has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting concepts. New and interesting applications have been found in the field of Economics, Biology and Robotics etc. Generalized closed sets remain as an active and fascinating field within mathematicians.

**Origin**

The basic sets α-open[12], semi-open[6], pre-open[11],semi pre-open[1] sets were introduced by O.Njastad, M.Levine, A.S.Mashhoar and D.Andrijevic. Levine[8], found the concept of generalized closed sets ( g closed sets) in a topological space.The generalization of the above closed sets are given by S.P.Arya and T.Nour [2],Dontchev.J[4], H.Maki R.Devi and K.Balachandran[9],[10],P.Bhattacharyya and B.K.Lahiri[3] namely generalized semi closed, generalized semi pre closed, α generalised closed and generalized α closed sets, semi generalized closed sets.

In 1963,N.Levine[7] introduced the concept of simple extension of a topologies, $\tau^{+}(B)=\{O\cup(O'\cap B)\ /\ O,O'\in\tau,\ B\notin\tau\}$ and $\tau_p^{+}$ generalized closed sets[13] are given by F.Nirmala Irudayam and Sr.I.Arokia Rani. W.Dunham[5]found the cl* and τ* of a set in topology. its generalization τ*g [15] is given by A.Pushpalatha, S.Eswaran and P.Rajarubi.

In this paper, we impose the concepts of extension to the τ*, τ*g sets and study their properties.

**1.Preliminaries**

**Definition 1.1**. A subset A of a topological space (X, τ) is Called

(i) α-closed[12] if cl(int(cl(A))) ⊆ A.

(ii) semi-closed[6] if int(cl(A)) ⊆ A.

(iii) pre-closed[11] if cl(int(A)) ⊆ A.

(iv) sp-closed[1] if int(cl(int(A))) ⊆ A.

(v) g-closed[8]set if cl(A))) ⊆ G whenever A ⊆ G and G is open in X.

(vi) gs-closed[2] set if scl(A))) ⊆ G whenever A ⊆ G and G is open in X.

(vii) gsp-closed[4] set if spcl(A))) ⊆ G whenever A ⊆ G and G is open in X.

(viii) α g-closed[9] set if αcl(A))) ⊆ G whenever A ⊆ G and G is open in X.

(ix) g α –closed[10] set if αcl(A))) ⊆ G whenever A ⊆ G and G is α open in X.

(x) s g closed[3] set if scl(A) ⊆ G whenever A ⊆ G and G is semi open in X.

(xi) $\alpha^{+}$-closed[14] if $cl^{+}(int(cl^{+}(A)))$ ⊆ A where A is a open set in $\tau^{+}$.

(xii) semi $^{+}$-closed[14] if $int(cl^{+}(A))$ ⊆ A where A is a open set in τ+.

(xiii) $pre^{+}$-closed[14] if $cl^{+}(int(A))$ ⊆ A where A is a open set in τ+.

The complements of the above mentioned sets are called their respective open sets.

**Definition 1.2.** For the subset A of a topological X, the generalized closure operator cl*[5] is defined by the intersection of all g-closed sets containing A.

**Definition 1.3.** For the subset A of a topological X, the topology τ*[5] is defined by $\tau^* = \{G : cl^*(G^c) = G^c\}$

**Definition 1.4.** A subset A of a topological space X is called τ*-generalized closed set (briefly τ*-g-closed) [15] if cl*(A) ⊆ G whenever A ⊆ G and G is τ*-open.

The complement of τ*-generalized closed set is called the τ*-generalized open set

(briefly τ*-g-open).

**2.Simple Extension of Set**

**Definition 2.1**. A subset A of a topological space (X, $\tau^{+}$) is said to be

(i) $sp^{+}$-closed if $int(cl^{+}(int(A)))$ ⊆ A where A is a open set in τ+.

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(ii) $g^+$-closed set if $cl^+(A))) \subseteq G$ whenever $A \subseteq G$ and G is open in $\tau^+$.
(iii) $gs^+$-closed set if $scl^+(A))) \subseteq G$ whenever $A \subseteq G$ and G is open in $\tau^+$.
(iv) $gsp^+$-closed set if $spcl^+(A))) \subseteq G$ whenever $A \subseteq G$ and G is open in $\tau^+$.
(v) $\alpha\ g^+$-closed set if $\alpha cl^+\ (A))) \subseteq G$ whenever $A \subseteq G$ and G is open in $\tau^+$.
(vi) $g\ \alpha^+$ -closed set if $\alpha cl^+\ (A))) \subseteq G$ whenever $A \subseteq G$ and G is $\alpha^+$ open in $\tau^+$.
(vii) $s\ g^+$ closed set if $scl^+(A) \subseteq G$ whenever $A \subseteq G$ and G is semi open in $\tau^+$.
(viii)$\tau_p^+$ generalized closed ($\tau_p^+$ g closed)[12] if $\tau^+$ $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre open in $(X,\tau^+)$.
The complements of the above mentioned sets are called their respective open sets.

**Definition 2.2.** For the subset A of a topological $(X, \tau^+)$
(i) the semi-closure$^+$ of A (briefly $scl^+(A)$) is defined as the intersection of all semi$^+$-closed sets containing A.
(ii) the semi pre-closure$^+$ of A (briefly $spcl^+(A)$) is defined as the intersection of all semipre $^+$-closed sets containing A.
(iii) the $\alpha^+$ -closure of A (briefly $\alpha cl^+(A)$) is defined as the intersection of all $\alpha^+$ -closed sets containing A.

**Definition 2.3.** For the subset A of a topological $(X,\tau^+)$ the generalized closure operator $cl^{+*}$ is defined by the intersection of all $g^+$-closed sets containing A.

**Definition 2.4.** For the subset G of a topological $(X, \tau^+)$ the topology $\tau^{+*}$ is defined by $\tau^{+*} = \{G : cl^{+*}(G^c) = G^c\}$

**Definition 2.5.** A subset A of a topological space $(X, \tau^+)$ is called $\tau^{+*}$-generalized closed set (briefly $\tau^{+*}$-g-closed sets) if $cl^{+*}(A) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^{+*}$-open.
The complement of $\tau^{+*}$-generalized closed set is called the $\tau^{+*}$-generalized open set
(briefly $\tau^{+*}$-g-open).

**Theorem 2.6**
Every closed set in $( X , \tau^+ )$ is $\tau^{+*}$-g-closed.

**Proof**
Let A be a closed set. Let us assume that $A \subseteq G$ and G is $\tau^{+*}$ open. Since A is closed in $( X , \tau^+ )$, $cl^+(A) = A \subseteq G$. But $cl^{+*}(A) \subseteq cl^+(A)$. Thus, we have $cl^{+*}(A) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^{+*}$-open. Therefore A is $\tau^{+*}$-g-closed.
The converse of the above theorem is need not be true as seen from the following example.

**Example 2.7**
Consider the topological space X = {a, b, c, d} with topology τ = {X, φ, {b},{a, d},{a, b, d}} and B = {c}.
Then, $\tau^+$ = {X, φ,{b],{c},{b, c},{a, b, d},{a, c, d}}.
Then the sets{a},{d},{a, b},{a, c} are $\tau^{+*}$-g-closed but not closed.

**Theorem 2.8**
Every $\tau^{+*}$-closed set in X is $\tau^{+*}$-g-closed.

**Proof**
Let A be a $\tau^{+*}$-closed set. Let us assume that $A \subseteq G$ where G is $\tau^{+*}$-open. Since A is $\tau^{+*}$-closed, $cl^{+*}(A) = A \subseteq G$. Thus, we have $cl^{+*}(A) \subseteq G$ whenever $A \subseteq G$ and G is $\tau^{+*}$-open.Therefore A is $\tau^{+*}$-g-closed.
The converse of the above theorem is need not be true as seen from the following example.

**Example 2.9**
Consider the topological space X = {a, b, c, d} with τ = { X, φ,{c},{d},{c, d}}
Let B={a},then $\tau^+$ ={X, φ,{a},{c},{d},{c ,d},{a, c},{a, d},{a, c, d}}.
{a, b, c} is not $\tau^{+*}$ closed but $\tau^{+*}$ g closed set.

**Theorem 2.10**
Every $g^+$-closed set in X is a $\tau^{+*}$-g-closed set.

**Proof**
Let A be a $g^+$-closed set. Assume that $A \subseteq G$, G is $\tau^{+*}$-open in X. Then $cl^+(A) \subseteq G$, since A is $g^+$-closed. But $cl^{+*}(A) \subseteq cl^+(A)$. Therefore $cl^{+*}(A) \subseteq G$. Hence A is $\tau^{+*}$-g-closed.
The converse of the above theorem is need not be true as seen from the following example.

**Example 2.11**
Consider the topological space X = {a, b, c, d} with topology τ = {X, φ, {b},{a, d},{a, b, d}} and B = {c}.
Then, $\tau^+$ = {X, φ,{b},{c},{b, c},{a, b, d},{a, c, d}}.
Then the set {b, c} are $\tau^{+*}$-g-closed but not $g^+$ closed set.

**Remark 2.12**
The following examples gives the relationship between $\tau^{+*}$-g-closed set with other existing sets ($sg^+$ closed set, $gs^+$ closed set, $gsp^+$ closed set, $\alpha g^+$ closed set, $g\alpha^+$ closed set,
$sp^+$ closed set, $\alpha^+$ closed set, $pre^+$ closed set)

**Example 2.13**
**(i)** Consider the topological space X = {a, b, c,d} with topology τ = {X, φ, {a},{b},{a,b}} and B = {c}.Then, $\tau^+$={X,φ,{a},{b],{c},{a,b},{a,c},{b,c},{a,b,c}}.
- {a},{b},{c},{a,b},{a,c} are $gs^+$ closed sets but not $\tau^{+*}$-g-closed sets.
- {a,b,d} is $\tau^{+*}$-g-closed set but not $sg^+$ closed set.
- {a},{b},{c},{a,b} are $sg^+$ closed sets but not $\tau^{+*}$-g-closed set.

**(ii)** Consider the topological space X = {a, b, c ,d} with topology
τ = {X, φ, {a}, {b} ,{a,b}} and B = {c}.
Then, $\tau^+$ = {X, φ, {a},{b},{c},{a,b},{a,c}, {b,c}, {a,b,c}}.
- {a},{b},{c} are $gsp^+$ closed sets but not $\tau^{+*}$-g-closed sets.
- {c},{a,c} are $\alpha g^+$ closed set but not $\tau^{+*}$-g-closed set.
- {c} is a $g\alpha^+$ closed set but not $\tau^{+*}$-g-closed set.
- {a},{b},{c},{a,b} are $sp^+$ closed sets but not $\tau^{+*}$-g-closed sets.
- {c},{a,b} are $\alpha^+$ closed set but not $\tau^{+*}$-g-closed set.
- {c},{a,b} are $pre^+$ closed sets set but not $\tau^{+*}$-g-closed sets.
- {a, d} is $\tau^{+*}$-g closed set but not $gsp^+$ closed set.
- {a} is $\tau^{+*}$-g closed set but not $\alpha g^+$ closed set.
- {a}, {a, d}, {a, c, d} are $\tau^{+*}$-g closed sets but not $g\alpha^+$ closed sets.

**(iii)** Consider the topological space X = {a, b, c ,d} with topology
τ = {X, φ, {c}, {d} ,{c,d}} and B = {a}.
Then, $\tau^+$ = {X, φ, {a},{c},{d},{c,d},{a,c}, {a,d}, {a,c,d}}.
- {d}is $\tau^{+*}$-g-closed set but not $\alpha^+$ closed set.
- {a}is $\tau^{+*}$-g-closed set but not $pre^+$ closed set.

**(iv)** Consider the topological space X = {a, b, c, d} with the topology
τ = {X, φ,{a}, {a, b}}. Let B = {c}. Then, $\tau^+$ = {X, φ,{a}, {c}, {a, c}, {a, b}, {a, b, c}}.
- {b}, {b, d} are $\tau^{+*}$-g-closed sets but not semi $pre^+$ closed sets.

**(v)** Consider the topological space X = {a, b, c} with the topology
τ = {X, φ,{c}}. Let B = {b}. Then, $\tau^+$ = {X, φ,{b}, {c}, {b, c}}.
- {b}, {c} are $gs^+$-closed sets but not $\tau^{+*}$-g closed sets.

**Theorem 2.14**

For any two sets A and B, $\mathbf{cl}^{+}*(A \cup B) = cl^{+}*(A) \cup cl^{+}*(B)$

**Proof**

Let A and B be any two sets. Since $A \subseteq A \cup B$ , we have $cl^{+}*(A) \subseteq cl^{+}*(A \cup B)$and since $B \subseteq A \cup B$ , we have $cl^{+}*(B) \subseteq cl^{+}*(A \cup B)$.

Therefore $cl^{+}*(A) \cup cl^{+}*(B) \subseteq cl^{+}*(A \cup B)$. …………(1)

Also , $cl^{+}*(A)$ and $cl^{+}*(B)$ are the closed sets Therefore $cl^{+}*(A) \cup cl^{+}*(B)$ is also a closed set.

Again, $A \subseteq cl^{+}*(A)$ and $B \subseteq cl^{+}*(B)$ implies $A \cup B \subseteq cl^{+}*(A) \cup cl^{+}*(B)$.

Thus, $cl^{+}*(A) \cup cl^{+}*(B)$ is a closed set containing $A \cup B$. Since $cl^{+}*(A \cup B)$ is the smallest closed set containing $A \cup B$ we have $cl^{+}*(A \cup B) \subseteq cl^{+}*(A) \cup cl^{+}*(B)$ ………(2).

From (1) and (2), we have $cl^{+}*(A \cup B) = cl^{+}*(A) \cup cl^{+}*(B)$.

**Theorem 2.15**

Union of two $\tau^{+}*$ g-closed sets in X is a $\tau^{+}*$- g-closed set in X.

**Proof**

Let A and B be any two $\tau^{+}*$ g-closed sets. Let $A \cup B \subseteq G$, where G is $\tau^{+}*$-open.

Since A and B are $\tau^{+}*$-g-closed sets, $cl^{+}*(A) \cup cl^{+}*(B) \subseteq G$. But by Theorem 2.14,

$cl^{+}*(A) \cup cl^{+}*(B) = \mathbf{cl}^{+}*(A \cup B)$. Therefore $\mathbf{cl}^{+}*(A \cup B) \subseteq G$.

Hence $A \cup B$ is a $\tau^{+}*$- g-closed set.

**Theorem 2.16**

A subset A of X is $\tau^{+}*$-g-closed if and only if $cl^{+}*(A) – A$ contains no non-empty $\tau^{+}*$ -closed set in $\tau^{+}$.

**Proof**

Let A be a $\tau^{+}*$-g-closed set. Suppose that F is a nonempty $\tau^{+}*$-closed subset of $cl^{+}*(A) – A$. Then, $F \subseteq cl^{+}*(A) – A$. Thus, $F \subseteq cl^{+}*(A) \cap A^{c}$.Therefore $F \subseteq cl^{+}*(A)$ -------(1) and $F \subseteq A^{c}$. Since $F^{c}$ is a $\tau^{+}*$-open set and A is a $\tau^{+}*$-g-closed, $cl^{+}*(A) \subseteq F^{c}$.

That is $F \subseteq [cl^{+}*(A)]^{c}$ -----------(2) Hence, from (1) and (2), $F \subseteq cl^{+}*(A) \cap [cl^{+}*(A)]^{c} = \varphi$. That is $F = \varphi$, a contradiction. Thus $cl^{+}*(A) – A$ contain no non-empty $\tau^{+}*$-closed set in X.

Conversely, assume that $cl^{+}*(A) – A$ contains no nonempty $\tau^{+}*$-closed set.

Let $A \subseteq G$, G is $\tau^{+}*$-open. Suppose that $cl^{+}*(A)$ is not contained in G, then $cl^{+}*(A) \cap G^{c}$ is a non-empty $\tau^{+}*$-closed set of $cl^{+}*(A) – A$ which is a contradiction. Therefore $cl^{+}*(A) \subseteq G$ and hence A is $\tau^{+}*$-g-closed.

**Corollary 2.17**

A subset A of X is $\tau^{+}*$ g-closed if and only if $cl^{+}*(A) – A$ contains no non-empty closed set in X.

**Proof**

The proof follows from the Theorem 2.16 and the fact that every closed set is $\tau^{+}*$- closed set in X.

**Corollary 2.18**

A subset A of X is $\tau^{+}*$-g-closed if and only if $cl^{+}*(A) – A$ contain no non-empty open set in X.

**Proof**

The proof follows from the Theorem 2.16 and the fact that every open set is $\tau^{+}*$-open set in X.

**Theorem 2.19**

If a subset A of X is $\tau^{+}*$-g-closed and $A \subseteq B \subseteq cl^{+}*(A)$, then B is a $\tau^{+}*$-g-closed set in $\tau^{+}$.

**Proof**

Let A be a $\tau^{+}*$-g-closed set such that $A \subseteq B \subseteq cl^{+}*(A)$. Let U be a $\tau^{+}*$-open set of X such that $B \subseteq U$. Since A is $\tau^{+}*$-g-closed, we have $cl^{+}*(A) \subseteq U$.

Now $cl^{+}*(A) \subseteq cl^{+}*(B) \subseteq cl^{+}*[ cl^{+}*(A)] = cl^{+}*(A) \subseteq U$. That is $cl^{+}*(B) \subseteq U$, U is $\tau^{+}*$-open. Therefore B is $\tau^{+}*$-g-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

**Example 2.20**

Consider the topological space $(X, \tau^{+})$, where X = {a, b, c,d}.τ = {X , φ, {a},{b},{a,b}} and B = {c}. Then, $\tau^{+}$ = {X , φ,{a},{b},{c},{a,c},{a,b},{b,c},{a,b,c}}

Let A = {d} and B = {a,d}. B are $\tau^{+}*$-g-closed sets in $(X, \tau^{+})$ and $cl^{+}*(A) = cl^{+}*\{d\} = \{d\}$.

But $A \subseteq B$ is not a subset of $cl^{+}*\{d\}$.

**Theorem 2.21**

Let A be a $\tau^{+}*$-g-closed in $(X, \tau^{+})$. Then A is $g^{+}$-closed if and only if $cl^{+}*(A) – A$ is $\tau^{+}*$-open.

**Proof**

Suppose A is $g^{+}$-closed in X. Then $cl^{+}*(A) = A$ and so $cl^{+}*(A) – A = \varphi$ which is $\tau^{+}*$- open in X. Conversely, suppose $cl^{+}*(A) – A$ is $\tau^{+}*$-open in X. Since A is $\tau^{+}*$-g-closed, by the Theorem 2.16, $cl^{+}*(A) – A$ contains no non-empty $\tau^{+}*$-closed set in X. Then $cl^{+}*(A) – A = \varphi$

Hence A is $g^{+}$-closed.

**Theorem 2.22**

For $x \in X$, the set $X – \{x\}$ is $\tau^{+}*$-g-closed or $\tau^{+}*$ open.

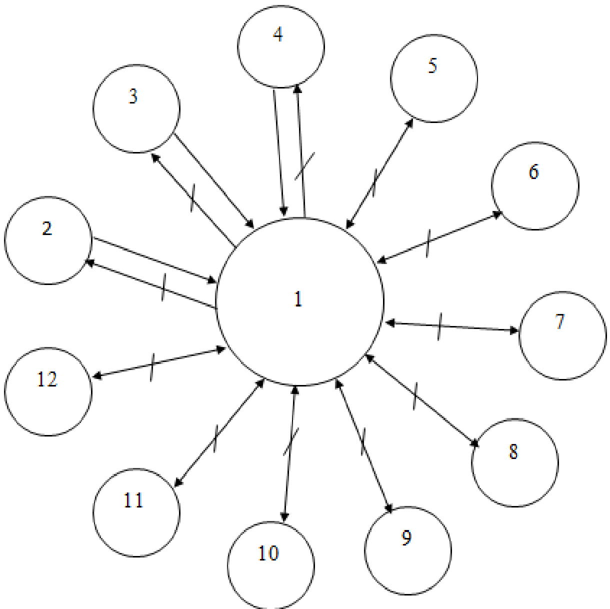
**Proof**

Suppose $X – \{x\}$ is not $\tau^{+}*$-open. Then X is the only $\tau^{+}*$-open set containing $X – \{x\}$. This implies $cl^{+}*( X – \{x\}) \subseteq X$.

Hence $X – \{x\}$ is a $\tau^{+}*$-g-closed in X.

**Remark 2.23**

From the above discussion, we obtain the following implications.




1 ⟶ 2 means 1 implies 2 , 1 ⟶̸ 2 means 1 does not imply 2 and

1 ⟷̸ 2 means 1 and2 are independent sets.

| | |
|---|---|
| $1 \Rightarrow \tau^{+}*$-g-closed set. | $7 \Rightarrow \alpha g^{+}$closed set. |
| $2 \Rightarrow$ closed$^{+}$ set. | $8 \Rightarrow$ pre$^{+}$-closed set. |
| $3 \Rightarrow g^{+}$ closed set. | $9 \Rightarrow$ semi$^{+}$-closed set. |
| $4 \Rightarrow \tau^{+}*$ closed set. | $10 \Rightarrow sg^{+}$ closed set. |
| $5 \Rightarrow \alpha^{+}$ closed set. | $11 \Rightarrow sp^{+}$ closed set. |
| $6 \Rightarrow g\alpha^{+}$-closed set. | $12 \Rightarrow gs^{+}$ closed set. |

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