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Variable Equation of State for Anisotropic Dark Energy Models in Kantowski-Sach Space Time

Amrapali P. Wasnik

Department of Mathematics, Bharatiya Mahavidyalaya, Amravati, India.

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ABSTRACT

We investigate dark energy models in an anisotropic Kantowski-Sachs space –time with a variable equation of state(EoS). The EoS for dark energy ω is found to be time dependent and its existing range for derive models is in good agreement with the recent observations. Under the suitable condition, the anisotropic models approach to isotropic scenario. We also find that during the evolution of the universe, the EoS parameter for DE changes from $\omega>-1$ to $\omega>-1$ in first model whereas from $\omega>-1$ to $\omega>-1$ in second model which is consistent with recent observations. The cosmological constant Λ is found to be a positive decreasing function of time and it approaches a small positive value at late time (i.e. the present epoch) which is corroborated by results from recent supernovae Ia observations. The physical and geometric properties are also discussed.

Introduction

In 1998, the observations of Type Ia (SNeIa) established that our universe is currently accelerating [1-3] and recent observations of SNeIa of high confidence level [4-6] have further confirmed this. In addition, measurements of the cosmic microwave background (CMB) [7] and large scale structure (LSS) [8] strongly indicate that our universe is dominated by a component with negative pressure, dubbed as dark energy. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests 73 % content of the universe in the form of dark energy, 23% in the form of non-baryonic dark-matter and the rest 4% in the form of the usual baryonic dark-matter as well as radiation.

High-precision measurements of expansion of the universe are required to understand how the expansion rate changes over time. In general relativity, the evolution of the expansion rate is parameterized by the cosmological equation of state (the relationship between temperature, pressure, and combined matter, energy and vacuum energy density for region of space). Measuring the equation of state for dark energy is one of the biggest efforts in observational cosmology today. The DE model has been characterized in a conventional manner by the equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ which is not necessarily constant, where ρ is the energy density and p is the fluid pressure [9]. The present data seem to slightly favour an evolving dark energy with EoS $\omega>-1$ around the present epoch and $\omega>-1$ in the near past. Obviously, ω cannot cross -1 for quintessence or phantom alone. Some efforts have been made to build a dark energy model whose EoS can cross the phantom divide. The simplest DE candidate is the vacuum energy ($\omega=-1$), which is mathematically equivalent to the cosmological constant (Λ). The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence($\omega>-1$)[10], phantom energy ($\omega>-1$) [11] and quintom (that can across from phantom region to quintessence region as evolved) and have time dependent EoS parameter. Some other limits obtained from observational results coming from SNe Ia data [12] and combination of SNe Ia data with CMBR anisotropy and galaxy clustering statistics [13] are $-1.67 < \omega < -0.62$ and $-1.33 < \omega - 0.79$, respectively. The latest results in 2009, obtained after a combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high redshift type Ia supernovae and galaxy clustering, constrain the dark energy EoS to $-1.44 < \omega < -0.92$ at 68% confidence level [14,15],However, it is not at all obligatory to use a constant value of ω. Due to lack of the observational evidence in making a distinction between constant and variable ω, usually the equation of state parameter is considered as a constant [16,17,18] with phase wise value $-1, 0, -\frac{1}{3}, \text{and} + 1$ for vacuum fluid, dust fluid, radiation and stiff dominated universe respectively. But in general, ω is a function of time or redshift [19,20,21]. Some literature are also available on models with varying fields, such as cosmological models with variable EoS parameter in Kaluza-Klein metric and wormhole [22,23]. In recent years various form of time dependent ω have been used for variable Λ models by Mukhopadhyay et al. [24]. Setare [25a,25b.25c] and Setare and Saridakis [26] have also studied the DE models in different contexts. In well –known reviews on modified gravity [27,28],it is clearly indicated that any modified gravity may be represented as effective fluid with time dependent ω.

Tele:

E-mail address: amra_math@rediffmail.com

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behavior of universe and such models have been widely studied in framework of General Relativity in search of a realistic picture of the universe in its early stages. Though Bianchi type-I universe is the prime candidate for studying the possible effects of an anisotropy in the early universe on present-day observations, there are few other models (for example, $B-\mathrm{VI}_0$), which describe an anisotropic space-time and generate interest among physicists.[29-32]. Pradhan and Bali [33] and Bali et al. [34] have studied $B-\mathrm{VI}_0$ space time in connection with massive strings. Recently Amirhashchi et al.[35] presented dark energy models in an anisotropic $B-\mathrm{VI}_0$ space –time by considering constant deceleration parameter. In this paper, we have investigated two new Kantowski-Sach DE models with variable equation of state EoS . The out line of the paper is as follows:

In section.2, the metric and the field equations are described. In section 3 deals with the solutions of the field equations in two different cases .Section 4 deals with physical and geometric behavior of the models. In section 5, we describe an other dark energy model and its physical aspects. Finally, conclusions are summarized in the last Section 6.

2. Metric and Field Equations:

We consider the Kantowski-Sachs space-time in the form

$$ds^2=-dt^2+a^2dr^2+b^2(d\theta^2+\sin^2 d\varphi^2) \quad (1)$$

where the scale factors a and b are functions of cosmic time only. By preserving the diagonal form of the energy momentum tensor in a consistent way with the above metric, the simplest generalization of EoS parameter of perfect fluid may be to determine it separately on each spatial axis. Therefore the energy momentum tensor of perfect fluid is taken as

$$T_i^j=\mathrm{diag}[T_0^0,T_1^1,T_2^2,T_3^3] \quad (2)$$

Thus, one may parameterize it as follows

$$T_i^j=\mathrm{dia}[\rho,-p_x,-p_y-p_z]$$
$$=\mathrm{dia}[1,-\omega_x,-\omega_y,-\omega_z]\rho$$
$$=\mathrm{dia}[1,-\omega,-(\omega+\delta),-(\omega+\delta)]\rho \quad (3)$$

where ρ is the energy density of the fluid p_x,p_y,p_z are the pressures and $\omega_x,\omega_y,\omega_z$ are the directional EoS parameters along the x,y,z respectively, $\omega(t)=p/\rho$ is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_x=\omega$ and then introducing skewness parameter δ which is the deviation from ω along both *y* and *z*-axes. ω and δ are not necessarily constants and might be function of the cosmic time t.

The Einstein's field equations are

$$R_{ij}-\frac{1}{2}Rg_{ij}=-T_{ij} \quad (4)$$

where the symbols have their usual meaning.

By adopting commoving coordinates, Einstein's field equation (4), for the Kontowski-Sachs space-time, the field equations take the form

$$\frac{2a_4b_4}{ab}+\frac{1}{b^2}+\frac{b_4{}^2}{b^2}=\rho \quad (5)$$

$$\frac{a_{44}}{a}+\frac{b_{44}}{b}+\frac{a_4b_4}{ab}=-(\omega+\delta)\rho \quad (6)$$

$$\frac{2b_{44}}{b}+\frac{1}{b^2}+\frac{b_4{}^2}{b^2}=-\omega\rho \quad (7)$$

where a subscript 4 indicates differentiation with respect to t.

3. Solutions of the field equations

The spatial volume for the model (1) is given by

$$V=R^3=ab^2 \quad (8)$$

where R is the mean scale factor. The mean Hubble parameter H is given as

$$H=\frac{R_4}{R}=\frac{1}{3}\frac{V_4}{V}=\frac{1}{3}\left(\frac{a_4}{a}+2\frac{b_4}{b}\right) \quad (9)$$

The directional Hubble parameters in the directions of *x,y and z* respectively may be defined as

$$H_x=\frac{a_4}{a}\text{and}H_y=H_z=\frac{b_4}{b} \quad (10)$$

The volumetric deceleration parameter$q$, the scalar expansion , shear scalar $\sigma^2$and the average anisotropy parameter A_m are defined by

$$q=-RR_{44}R_4{}^2 \quad (11)$$

$$\theta=\frac{a_4}{a}+2\frac{b_4}{b} \quad (12)$$

$$\sigma^2=13\left(\frac{a_4}{a}-\frac{b_4}{b}\right)^2 \quad (13)$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \quad (14)$$

where $H_i(i=x,y,z)$ represents the directional Hubble parameter in the direction of $x, y\text{and}z$, respectively. $A_m=0$ corresponds to isotropic expansion.

Initially we apply the law of variation for Hubble parameter for the Kantowski-Sach metric may be given by

$$H=D(ab^2)^{-\frac{n}{3}} \quad (15)$$

where $D>0\text{and}n\geq 0$ are constants. Such type of relations have firstly been considered by Berman [36], Berman and Gomide [37] for solving FRW models. Letter on many authors have used this law to study FRW and Bianchi type models.

On integrating, after equating (9) and (15), we obtain

$$ab^2=(nDt+c_1)^{\frac{3}{n}} \quad \text{for} \quad n\neq 0 \quad (16)$$

here c_1 is positive constants of integration. The values for the deceleration parameter for the mean scale factor as:

$$q=n-1 \quad (17)$$

which is constant. The sign of q indicates where the model inflates or not. The positive sign of q i.e. $0\leq n<1$ indicates inflation. It is remarkable to mention here that through the current observations of SNe Ia and CMBR favors accelerating models $(q<0)$, but both do not altogether rule out the deceleration ones which are also consistent with these observations [38]

Now we assume that the expansion (θ) is proportional to shear (σ), this condition lead to

$$\frac{a_4}{a}-\frac{b_4}{b}=\alpha_0\sqrt{3}\left(\frac{a_4}{a}+2\frac{b_4}{b}\right)$$

which yields to

$$\frac{a_4}{a}=m\frac{b_4}{b}$$

where $m=\frac{2\alpha_0\sqrt{3}+1}{1-\alpha_0\sqrt{3}}$ and α_0 are arbitrary constants. Above equation, after integration, reduces to

$$a=\beta b^m$$

where β is an integrating constant. Here , for simplicity and without any loss of generality, we assume $\beta = 1$. Hence we have

$$a = b^m \tag{18}$$

The motivation behind assuming this condition is explained with reference to Thorne[39], the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within 30 percent [40,41]. To put more precisely, red- shift studies place the limit

$$\frac{\sigma}{H} \le 0.3,$$

On the ratio of shear σ to Hubble constant H in the neighborhood of our Galaxy today. Collins et al. [42] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

Using (18) (8) in (9), we obtain the expressions for metric function as follows

$$b = (\mathrm{nDt} + c_1)^{\frac{1}{\mathrm{ml}}} \tag{19}$$

$$a = (\mathrm{nDt} + c_1)^{\frac{1}{l}} \tag{20}$$

where c_1 is an integrating constant

Hence the model (1) reduces to

$$ds^2 = -dt^2 + (\mathrm{nDt} + c_1)^{\frac{2}{l}} dr^2 + (\mathrm{nDt} + c_1)^{\frac{2}{\mathrm{ml}}} (d\theta^2 + \sin^2\theta d\varphi^2) \tag{21}$$

4. Physical aspects of dark energy model

The expressions for the Hubble parameter H, scalar of expansion θ ,shear scalar σ and the average anisotropy parameter A_m for the model (21) are given by

$$\theta = 3H = \frac{3L}{(\mathrm{nDt} + c_1)} \tag{22}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{\mathrm{nD}(m-1)}{\mathrm{ml}}\right]^2 (\mathrm{nDt} + c_1)^{-2} \tag{23}$$

$$A_m = \frac{(m^2+2)(\mathrm{nD})^2 - 2\mathrm{nDLml}(m+2)}{3m^2 l^2 L^2} + 1 \tag{24}$$

Using equation (5) , the energy density of the fluid is obtain as

$$\rho = (2m+1)\frac{(\mathrm{nD})^2}{(\mathrm{ml})^2}(\mathrm{nDt} + c_1)^{-2} + (\mathrm{nDt} + c_1)^{-2\mathrm{ml}} \tag{25}$$

Using equation (7) , the EoS parameter is obtained as

$$\omega = \frac{(2\mathrm{ml}-3)\left(\frac{\mathrm{nD}}{\mathrm{ml}}\right)^2 (\mathrm{nDt}+c_1)^{-2} - (\mathrm{nDt}+c_1)^{-2\mathrm{ml}}}{(2m+1)\left(\frac{\mathrm{nD}}{\mathrm{ml}}\right)^2 (\mathrm{nDt}+c_1)^{-2} + (\mathrm{nDt}+c_1)^{-2\mathrm{ml}}} \tag{26}$$

Using equation (6), the skewness parameter δ are computed as

$$\delta = \frac{[2+l(m-1)-m(m+1)]\left(\frac{\mathrm{nD}}{\mathrm{ml}}\right)^2 (\mathrm{nDt}+c_1)^{-2} + (\mathrm{nDt}+c_1)^{-2\mathrm{ml}}}{(2m+1)\left(\frac{\mathrm{nD}}{\mathrm{ml}}\right)^2 (\mathrm{nDt}+c_1)^{-2} + (\mathrm{nDt}+c_1)^{-2\mathrm{ml}}} \tag{27}$$

From (26), it is observed that the equation of state parameter ω is time dependent, it can be function of redshift z or scale factor R as well. The redshift dependence of ω can be linear like

$$\omega(z) = \omega_0 + \omega' z \tag{28}$$

with $\omega' = \frac{d\omega}{dz}\big|_{z=0}$ (see [43,44]) or nonlinear as

$$\omega(z) = \omega_0 + \frac{\omega_1 z}{1+z} \tag{29}$$

[45,46]. So as for as the scale factor dependence of ω is concern. The parametrization

$$\omega(R) = \omega_0 + \omega_R(1-R) \tag{30}$$

where ω_0 is the present value $(R = 1)$ and ω_R is the measure of the time variation ω' is widely used in the literature [47]

So, if the present work is compare with experimental results [11,12,13,14], then one can conclude hat the limit of ω provided by (26) may accommodated with the acceptable range of EoS parameter . Also it is observed that at $t = t_c, \omega$ vanishes, where t_c is a critical time given by

$$t_c = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{2\mathrm{ml} - 3}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{31}$$

Thus, for this particular time , our model represents a dusty universe. We also note that earlier real matter at $t \le t_c$, where $\omega \ge 0$ later on at $t \ge t_c$, where $\omega > 0$ converted to the dark energy dominated phase of universe.

For the value of ω to be in consistent with observation [11], we have the following general condition

$$t_1 < t < t_2,$$

where

$$t_1 = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+3.34\right)+0.67\right]}{2.67}}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{32}$$

$$t_2 = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+1.24\right)-0.38\right]}{1.62}}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{33}$$

For this constrain, we obtain $-1.67 < \omega < -0.62$ which is in good agreement with the limit obtained from observational results coming from SNe Ia data [11].

For the value of ω to be in consistent with observation [12] , we have the following general condition

$$t_3 < t < t_4, \tag{34}$$

where

$$t_3 = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+2.66\right)+0.33\right]}{2.33}}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{35}$$

$$t_4 = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+1.58\right)-0.21\right]}{1.79}}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{36}$$

For this constrain, we obtain $-1.33 < \omega < -0.79$ which is in good agreement with the limit obtained from observational results coming from SNe Ia data [11] with CMB anisotropy and galaxy clustering statistics [12].

For the value of ω to be in consistent with observation [13,14] , we have the following general condition

$$t_5 < t < t_6,$$

$$t_5 = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+2.88\right)+0.44\right]}{2.44}}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{37}$$

$$t_6 = \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{\frac{3\left[m\left(\frac{2}{3}l+1.84\right)-0.08\right]}{1.92}}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} - \frac{c_1}{\mathrm{nD}} \tag{38}$$

For this constrain, we obtain $-1.33 < \omega < -0.79$ which is in good agreement with the limit obtained from observational results [13,14] From (25), we note that energy density of the fluid $\rho(t)$ is a decreasing function of time and $\rho \geq 0$ when

$$t \leq \frac{1}{\mathrm{nD}}\left(\frac{\mathrm{nD}}{\mathrm{ml}}\sqrt{2m+1}\right)^{\frac{\mathrm{ml}}{\mathrm{ml}-1}} + \frac{c_1}{\mathrm{nD}} \tag{39}$$

Here ρ is a positive decreasing function of time and it approaches to zero as $t \to \infty$.

In absence of any curvature, matter energy density Ω_m and dark energy Ω_Λ are related by the equation

$$\Omega_m + \Omega_\Lambda = 1 \tag{40}$$

Where $\Omega_m = \frac{\rho}{3H^2}$ and $\Omega_\Lambda = \frac{\Lambda}{3H^2}$. Thus, (40) reduces to

$$\frac{\rho}{3H^2} + \frac{\Lambda}{3H^2} = 1 \tag{41}$$

Using (22) and (25) in (41), the cosmological constant is obtained as

$$\Lambda = \left[3L^2 - (2m+1)\left(\frac{\mathrm{nD}}{\mathrm{ml}}\right)^2\right](\mathrm{nDt}+c_1)^{-2} - (\mathrm{nDt}+c_1)^{-\frac{2}{\mathrm{ml}}} \tag{42}$$

From (42), we observe that Λ is decreasing function of time and it is always positive when

$$t > \frac{1}{\mathrm{nD}}\left[3L^2 - (2m+1)\left(\frac{\mathrm{nD}}{\mathrm{ml}}\right)^2\right]^{\frac{\mathrm{ml}}{2(\mathrm{ml}-1)}} - \frac{c_1}{\mathrm{nD}} \tag{43}$$

We observe that cosmological parameter is decreasing function of time and it approaches a small positive value at late time (i.e. At present epoch). Recent cosmological observations[1,2]; [3,5,4] suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(\mathrm{Ghc}^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ- term. Thus, the nature of Λ in our derive DE model is supported by recent observations.

From (22)-(23), it can be seen that all the kinematical parameters H, θ, and σ diverge at the initial singularity. There is a Point Type singularity [48] at $t = -\frac{c_1}{\mathrm{nD}}$ in the model. The mean anisotropic parameter is constant and it increases. Thus, the dynamics of the mean anisotropy parameter depends on the value of . Since $\frac{\sigma^2}{\theta^2} = \text{constant}$, the models does not approach isotropy through the whole evolution of the universe.

5. Other dark energy model

Now we take the following ansatz for the scale factor, where the increase in terms of time evolution is

$$R(t) = \mathrm{te}^t \tag{44}$$

By the above choice of scale factor yields a time dependent deceleration parameter. We define the deceleration parameter q as usual,

$$q = -\frac{R_{44}R}{R_4{}^2} = -\frac{R_{44}}{\mathrm{RH}^2} \tag{45}$$

Using (44) into (45), we find

$$q = -1 + \frac{1}{(1+t^2)} \tag{46}$$

Using (18) and (45) in (9), we obtain the expressions for metric functions as follows

$$a = (\mathrm{te}^t)^{\frac{3m}{2+m}} \tag{47}$$

$$b = (\mathrm{te}^t)^{32+m} \tag{48}$$

Hence the model (1) reduces to

$$ds^2 = -dt^2 + (\mathrm{te}^t)^{\frac{6m}{m+2}}dr^2 + (\mathrm{te}^t)^{\frac{6}{m+2}}(d\theta^2 + \sin^2\theta d\varphi^2) \tag{49}$$

The expressions for the Hubble parameter H, scalar of expansion θ, shear σ and the average anisotropy parameter A_m for the model (49) are given by

$$\theta = 3H = 3\left(\frac{R_4}{R}\right) = 3\left(\frac{t+1}{t}\right) \tag{50}$$

$$\sigma^2 = \frac{(m-1)^2}{m^2r^2}\left(\frac{t+1}{t}\right)^2 \tag{51}$$

$$A_m = \frac{(m^2+2)-2mr(m+2)}{3mr^2} + 1 \tag{52}$$

Where $r = \frac{m+2}{3m}$. Since $\frac{\sigma^2}{\theta^2} \neq 0$ for all values of m except for $m = 1$, hence the model is anisotropic except for $m = 1$. The dynamics of the mean anisotropic parameter depends on the value of m. The mean anisotropic parameter is constant. We observed that when $m = 0, A_m \to \infty$ and for $m = 1, A_m = 0$. Thus, the observed isotropy of the universe can be achieved in phantom model.

The energy density of the fluid can be find by using (47) and (48) in (5)

$$\rho = \frac{(2m+1)}{m^2r^2}\left(\frac{t+1}{t}\right)^2 + (\mathrm{te}^t)^{\frac{-2}{\mathrm{rm}}} \tag{53}$$

Using (47), (48) and (53) in (7), the EoS parameter ω is obtained as

$$\omega = -\frac{\left[\frac{3(t+1)^2-2\mathrm{mr}}{t^2m^2r^2} + (\mathrm{te}^t)^{-\frac{2}{\mathrm{rm}}}\right]}{\frac{(2m+1)}{r^2m^2}\left(\frac{t+1}{t}\right)^2 + (\mathrm{te}^t)^{-\frac{2}{\mathrm{rm}}}} \tag{54}$$

Using (47), (48), (53) and (54) in (6), the skew parameter δ are computed as

$$\delta = -\frac{\left[\frac{(m^2+m-2)}{m^2r^2}\left(\frac{t+1}{t}\right)^2 + \frac{(1-m)}{\mathrm{mrt}^2} - (\mathrm{te}^t)^{-\frac{2}{\mathrm{rm}}}\right]}{\frac{(2m+1)}{r^2m^2}\left(\frac{t+1}{t}\right)^2 + (\mathrm{te}^t)^{-\frac{2}{\mathrm{rm}}}} \tag{55}$$

So, if the present work is compared with experimental results [11,12,13,14], then one can conclude that the limit of ω provided by (54) may accommodated with the acceptable range of EoS parameter. Also it is observed that at $t = t_c$, ω vanishes, where t_c is a critical time given by the following relation

$$\frac{3(t+1)^2-2\mathrm{mr}}{m^2r^2t_c{}^2} + (t_c e^{t_c})^{-\frac{2}{\mathrm{rm}}} = 0 \tag{56}$$

Thus for this particular time, our model represents a dusty universe, We also note that the earlier real matter at $t \leq t_c$, where $\omega \geq 0$ later on at $t > t_c$, where $\omega < 0$ converted to the dark energy dominated phase of universe.

From (53), we note that energy density of the fluid $\rho(t)$ is a decreasimg function of time and $\rho \geq 0$ when

$$(te^t)^{2rm}\left(\frac{t+1}{t}\right)^2 \geq \frac{m^2 r^2}{2m+1} \quad (57)$$

Here ρ is a positive decreasing function of time and it approaches to zero as $t \to \infty$.

Using (50),(53) in (41), the cosmological constant is obtained as

$$\Lambda = \left[3 - \frac{9(2m+1)}{(m+2)^2}\right]\left(\frac{t+1}{t}\right)^2 - (te^t)^{\frac{-2}{rm}} \quad (58)$$

From (58), we observe that Λ is a decreasing function of time and it is always positive when

$$\left(\frac{t+1}{t}\right)(te^t)^{\frac{6}{m+2}} \geq \frac{(m+2)^2}{3(m-1)^2} \quad (59)$$

We observe that cosmological parameter is decreasing function of time and it approaches a small positive value at late time. Thus ,the nature of Λ in this derived DE model is also in good agreement with recent observations [1,2,3,5,4].

6. Conclusion

A new class of anisotropic Kantowski -Sach Dark energy models with variable EoS parameter ωhas been investigated by using time dependent deceleration parameter. In literature it is a plebeian practice to consider constant deceleration parameter. Now for a universe which was decelerating in past and acceleration at present epoch, the DP must show signature flipping as already discussed in Section 2 . Therefore our consideration of DP to be variable is physically justified.

Our Power law solution represents the singular model where the spatial scale factors and volume scalar vanish at $t = -\frac{c_2}{nD}$. All the physical parameters are infinite at this initial epoch and tend to zero as $t \to \infty$. There is a point Type singularity [48] at $t = -\frac{c_2}{nD}$ in the model.

The existence of DE, in whatever form , is needed to reconcile the measured geometry of space with the total amount of matter in the universe, DE models present the dynamics of EoS parameter ω provided by (26) and (68) whose range is in good agreement with the acceptable range by the recent observations [11,12,13,14] .It can be easily seen that in both DE models . Thus , our both anisotropic parameter vanishes at $m = 1$. Thus, our both anisotropic models approach to isotropy at $m = 1$. It is already discussed in previous section , we obtain cosmological constant dominated universe, quintessence and phantom fluid dominated universe [45], representing different phases of the universe through- out the evolving process.

Our DE model is of great importance in the sense that the nature of decaying vacuum energy $\Lambda(t)$ is supported by recent cosmological observations. Though there are many suspects (candidates) such as cosmological constant, vacuum energy, scalar field, brane world, cosmological nuclear-energy, etc. as reported in the vast literature for DE, the proposed model in this paper favors EoS parameter as a possible suspect for the DE.

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